Week 2: High-Order Approximation

Computation Study Group

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Why (when) to Do High-Order Approximation

- 1st gen of DSGE models usually solved at 1st-order perturbation
  - i.e. *gensys* system Sims (2001)
  - suitable for linear rational expectations systems
- high-order approximation popularized after financial crisis
  - more accurate
  - appropriate for welfare analysis(*)
  - state-dependence simulation
  - second-moment shocks(*)
- Dynare uses 2-nd order approximation by default

High-Order Approximation: Warnings (optional)

- 1st-order approximation won't explode
- high-order approximation can explode
- example (optional)

$$y_t = \rho y_{t-1}^2$$

Starting from a given initial condition y_0 we have:

$$y_1 = \rho y_0^2$$

$$y_2 = \rho^3 y_0^4, \ldots$$

Even if the model looks stationary because $\rho < 1$, the generated path may diverge if $|y_0| > 1$ (*dependence of path on history*)

Welfare Analysis

A Planner's Problem

- social planner solves

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

s.t. law of motion

$$K_{t+1} = A_t K_t^{\alpha} - C_t + (1-\delta) K_t \tag{1}$$

and TFP shock

$$\log A_t = \rho \log A_{t-1} + \sigma \varepsilon_t \tag{2}$$

- F.O.C.s

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta \right) \tag{3}$$

Auxiliary Equations

- output
$$Y_t = A_t K_t^{\alpha} \tag{4}$$
- investment
$$I_t = Y_t - C_t \tag{5}$$
- return
$$R_t = \alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta \tag{6}$$
- welfare
$$V_t = \log(C_t) + \beta E_t V_{t+1} \tag{7}$$
- parameters:
$\beta = 0.99$, $\delta = 0.025$, $\alpha = 1/3$, $\rho = 0.95$, $\sigma = 0.01$.

Steady State

$$A = 1$$

$$R = \frac{1}{\beta}$$

$$K = [\frac{R - 1 + \delta}{\alpha}]^{1/(\alpha-1)}$$

$$Y = K^{\alpha}$$

$$I = \delta K$$

$$C = Y - I$$

$$V_{ss} = \frac{\log C}{1 - \beta}$$

Stochastic Simulation (1st-order)

1st-order simulation

stoch_simul(order=1,irf=20);

- compare simulated ergo mean of V against steady state V_{ss}
  - '='
- set $\sigma = 0.02$, compare simulated ergo mean of V with benchmark model
  - '='
- interpretation

Stochastic Simulation (2nd-order)

2nd-order simulation

stoch_simul(order=2,irf=20);

- compare simulated mean of V against steady state V
- set $\sigma = 0.02$, compare simulated mean of V with $\sigma = 0.01$
- what if household are more risk averse?

Implement Higher-Order Approximation

- smaller shock
  - 'degree of freedom'?
- pruning
  - remove the explosiveness related to initial condition
- example (optional)

$$y_t = \gamma y_{t-1} + \rho y_{t-1}^2$$

can be rewritten as (*not equivalently*)

$$z_t = \rho z_{t-1}$$

and

$$y_t = \gamma y_{t-1} + \rho z_{t-1}^2$$

Stochastic Simulation (2nd-order *w. pruning*)

2nd-order simulation

stoch_simul(order=2,irf=20, pruning);

- compare IRF of V against simulation w/o pruning
- set $\sigma = 0.05$, compare IRF of V against simulation w/o pruning

Second Moment Shock

Uncertainty Shock

Usually we assume TFP follows an AR(1) process with *time-invariant* persistence and volatility

$$\log A_t = \rho \log A_{t-1} + \sigma \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

Now let's assume the volatility σ_t can be time-varying

$$\sigma_t = (1-\rho_\sigma)\sigma_{ss} + \rho_\sigma \sigma_{t-1} + \sigma_\sigma e_t, \quad e_t \sim N(0,1) \tag{8}$$

where e_t captures *uncertainty shock*.

Uncertainty Shock

- second moment shock ($\sigma_t = 0.01 \rightarrow \sigma_t = 0.02$)
- impulse response

$$V\_irf1 = \frac{V(\sigma = 0.02) - V_{ss}(\sigma = 0.00)}{V_{ss}(\sigma = 0.0)} \tag{9}$$

$$V\_irf2 = \frac{V(\sigma = 0.02) - V(\sigma = 0.01)}{V(\sigma = 0.01)} \tag{10}$$

- which one is reported by default ?
  - V_irf1
- which one is desired ?
  - V_irf2
- how to replicate V_irf2? (PS2)