Week 1: Dynare Basics

Computation Study Group

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Dynare: Intro

It is

- a popular platform for handling DSGE and OLG models.
- replied on the rational expectations hypothesis
- user-friendly and intuitive, embedded in Matlab

It can

- compute the steady state of a model;
- compute the first, second order or higher order approximation to solutions of stochastic models;
- estimate parameters of DSGE models using either a maximum likelihood or a Bayesian approach.

99% questions answered at: https://www.dynare.org/manual/

Dynare: Installation

- have Matlab installed
- download at https://www.dynare.org/download/
- install (manual 2.2.1)
- configure (manual 2.4)
  - use command (not recommend):
    'addpath c:/dynare/4.x.y/matlab'
  - use menu entries
    "Set Path" > "File" > "Add Folder…" > *matlab subdirectory* of your Dynare installation > save
  - done !

Dynare: .mod

Structure of a .mod file

- Preamble
  - parameters
  - variables
- Model
  - nonlinear, yes!
- Initial Value, End Value
- Steady State
  - Dynare can do it for you
  - Do it yourself if you can
  - Give initial value if you can
- Shocks
- Computation

To run a .mod code, type “dynare xxxx.mod” in command window

Deterministic v.s. Stochastic models

The important question to ask when using Dynare:
Is your model stochastic or deterministic?

- The distinction hinges on whether future shocks are known.
- **Deterministic Models:** the occurrence of all future shocks is known exactly at the time of computing the model's solution (Perfect foresight).
- **Stochastic Models:** only the distribution of future shocks is known.
- The solution methods for these two types differ significantly.

Deterministic Simulation: Fiscal Policies in a Growth Model (RMT Ch.11)

A Deterministic Growth Model (From RMT Ch.11)

- Household

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

s.t.
$(1+\tau_{ct})c_t + k_{t+1} - (1-\delta)k_t + B_t =$
$(\eta_t - \tau_{kt}(\eta_t - \delta))k_t - \tau_t + R_t B_{t-1}$

- Firm

$$\max_{k_t} k_t^{\alpha} - \eta_t k_t$$

- Government

$$g_t + R_t B_{t-1} = \tau_{ct} c_t + \tau_{kt}(\eta_t - \delta)k_t + \tau_t + B_t$$

Dynamics

- Euler Equation

$$c_t^{-\gamma} = \beta \frac{1+\tau_{ct}}{1+\tau_{ct+1}} c_{t+1}^{-\gamma} [(1-\tau_{kt+1})(\alpha k_{t+1}^{\alpha-1} - \delta) + 1] \quad (1)$$

- Resource Constraint

$$k_{t+1} + c_t + g_t = (1-\delta)k_t + k_t^{\alpha} \quad (2)$$

- Two endogenous variables: c and k; Three exogenous variables: τ_c, τ_k and g.
- Steady State

$$1 = \beta[(1-\tau_k)(\alpha k^{\alpha-1} - 1) - \delta) + 1] \quad (3)$$

$$c = k^{\alpha} - \delta k - g \quad (4)$$

Deterministic Simulation

- A general formation
  Suppose x is the vector of endogenous variables and u is the vector of exogenous variable.
  The dynamics of the system is governed by a system of difference equations $f(x_{t+1}, x_t, x_{t-1}, u_t) = 0$.
- Idea of deterministic simulation.
  Given (1) x_0 (initial value of endogenous variable) (2) x_{T+1} (end value of endogenous variable) and (3) $\{u_t\}_{t=0}^{T}$ (the whole path of exogenous variables), we simulate $\{u_t\}_{t=1}^{T}$ (the path of endogenous variables) by solving the following system of equations:

$$\begin{cases} f(x_2, x_1, x_0, u_1) = 0 \\ f(x_3, x_2, x_1, u_2) = 0 \\ \ldots\ldots \\ f(x_{T+1}, x_T, x_{T-1}, u_T) = 0 \end{cases}$$

What we usually do

In the calculation of the transition path in the perfect foresight equilibrium, there are some typical cases.

- Unexpected temporary shock:
  $x_0 = x_{T+1} =$ steady-state value, $u_1 =$ value of the shock, u_t $=$ steady-state value $(t \neq 1)$.
- Unexpected permanent shock:
  $x_0 =$ old steady-state value, $x_{T+1} =$ new steady-state value, $u_t =$ new permanent level $(t \geq 1)$.
- Expected temporary shock:
  $x_0 = x_{T+1} =$ steady-state value, $u_{t_0} =$ value of the shock, u_t $=$ steady-state value $(t \neq t_0)$.
- Expected permanent temporary shock:
  $x_0 =$ old steady-state value, $x_{T+1} =$ new steady-state value,
  $$u_t = \begin{cases} old\ \ steady - state\ \ level, & t \leq t_0 \\ new\ \ permanent\ \ level, & t > t_0 \end{cases}$$

Simulate in Dynare

Functions of modules

- initval
  Set $t = 0$ values for x_t and u_t. If there is no endval module or steady module, they are also $t = T + 1$ values for x_t and u_t.
- initval + steady
  The values provided by initval module is only served as the initial value for calculating steady-state value, which is used as $t = 0$ and $t = T + 1$ values for x_t and u_t.
- endval
  endval sets $t = T + 1$ values for x_t and u_t
- endval + steady
  The values provided by endval module is served as the initial value for calculating new steady-state value, which is used as $t = T + 1$ values for x_t and u_t.

Simulate in Dynare

Functions of modules (Cont.)

- shock: it specifies the value of u_t from $t = 1$ to $t = T$.
  var [name of the exogenous variable, e.g. g]
  periods [e.g. 1, 10:12 20]
  values [e.g. 0.1 0.2 0.1]
  The unspecified values from $t = 1$ to $t = T$ are set same as $t = T + 1$ value.

Examples

Here are some examples from Recursive Macroeconomics 4ed Chapter 11.

- Figure 11.9.1: Response to foreseen once-for-all increase in g.
- Figure 11.9.4: Response to foreseen once-for-all increase in τ_c.
- Figure 11.9.5: Response to foreseen once-for-all increase in τ_k.
- Figure 11.9.6: Response to foreseen one-time pulse increase in g.

Use command "simul(periods=T)", where T specifies the length of the simulation periods. Simulated paths are stored in "oo_.endo_simul" and "oo_.exo_simul", in the form of $1 \times (T+2)$ and $(T+2) \times 1$ vectors respectively.

Stochastic Simulation: Benchmark RBC Model

Firm

Representative firm maximizes its profit with production technology of a Cobb-Douglas form ($0 < \alpha < 1$):

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{5}$$

There is perfect competition for labor and capital, in other word, the factors of production are paid their marginal products.
TFP, a_t, follows an AR(1) process given by ($0 < \rho < 1$):

$$\log(A_t) = \rho \log(A_{t-1}) + \sigma\varepsilon_t, \quad \varepsilon_t \sim N(0,1) \tag{6}$$

Household

Representative household maximizes life time utility given by:

$$\max E_t \sum_{s=0}^{\infty} \beta^t [\log(C_t) - \psi \frac{N_t^{1+\gamma}}{1+\gamma}]$$

where $0 < \beta < 1$, $\gamma > 0$.
subject to budget constraint:

$$W_t N_t + R_t K_t = C_t + I_t$$

and capital law of motion ($0 < \delta < 1$):

$$K_{t+1} = (1-\delta) K_t + I_t \tag{7}$$

Equilibrium

$$\alpha \frac{Y_t}{K_t} = R_t \tag{8}$$

$$(1-\alpha)\frac{Y_t}{N_t} = W_t \tag{9}$$

$$\psi N_t^{\theta} = \frac{1}{C_t} W_t \tag{10}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}(R_{t+1} + 1 - \delta) \tag{11}$$

$$Y_t = C_t + I_t \tag{12}$$

Equation (5)-(12) solve $\{Y_t, C_t, I_t, K_t, N_t, W_t, R_t, A_t\}$.

Stochastic Simulation in Dynare

Suppose y_t is the vector of endogenous variables; u_t is the vector of exogenous variables. A general form of the model is

$$E_t f(y_{t+1}, y_t, y_{t-1}, u_t) = 0 \tag{13}$$

Idea: (1) Solves a policy function $y_t = g(y_{t-1}, u_t)$;
(2) Use policy function to calculate impulse response functions.

- How to solve the policy function?
  – By perturbation methods: perform Taylor expansion on policy functions around the deterministic steady state.
  For example, by first-order Taylor expansion, we can approximate a linear policy function around the steady state: $y_t - y^{ss} = A(y_{t-1} - y^{ss}) + Bu_t$.
- After we obtain policy function, IRF can be calculated by iterating forward the initial shock.

Calibration

- $\beta = 0.96$: time discount rate
  - annualized risk-free rate
- $\alpha = 0.30$: capital share
  - labor income share
- $\delta = 0.10$: annual depreciation rate
  - steady state investment rate
- $\gamma = 2.00$: inverse Frisch elasticity
- $\rho = 0.90$: persistence of aggregate TFP shock
- $\sigma = 0.01$: volatility of aggregate TFP shock
- $\psi =?$: it needs calibration. Target: in steady state, $N = 1/3$.

Now calculate the steady state value for C, K, Y, I, N, W, R.

Steady State

$$A = 1, N = 1/3 \tag{14}$$

$$\frac{N}{K} = \left(\frac{1/\beta - 1 + \delta}{\alpha}\right)^{\frac{1}{1-\alpha}} \tag{15}$$

$$\frac{C}{K} = (\frac{N}{K})^{1-\alpha} - \delta \tag{16}$$

$$N = \left(\frac{(1-\alpha)(N/K)^{1-\alpha}}{\psi C/K}\right)^{\frac{1}{1+\gamma}} \Rightarrow \psi = \frac{(1-\alpha)(N/K)^{1-\alpha}}{N^{1+\gamma} C/K} \tag{17}$$

$$K = \frac{N}{N/K}, C = K\frac{C}{K} \tag{18}$$

$$W = (1-\alpha)(\frac{N}{K})^{-\alpha}, R = \alpha(\frac{N}{K})^{1-\alpha} \tag{19}$$

Calculate steady state in Dynare

Three ways

- Use Dynare built-in solver: "initval" + "steady"
- Directly provide steady-state values for Dynare: Use "steady_state_model"
- Write a separate "xxx_steadystate.m" file to solve the steady state. (Recommended)
  Caution: Don't use "alpha", "beta", "gamma"! Change their names; otherwise, MATLAB regards them as its built-in function names!

Do stochastic simulation in Dynare

- "shocks" module:
  Dynare assumes that the shock (exogenous variable) follows a normal distribution with mean zero. We only specify the variance of the normal distribution.
  shocks;
  var [exo variable name] = [variance];
  end;
- "stoch_simul" module
  stoch_simul([options]) [variable names];
  some [options]:
  order = 1/2/3 (default is 2) The order of Taylor expansion.
  irf= [integer] (default is 40) The period of impulse response functions Dynare simulates. [variable names]: the variables we want to calculate impulse response functions.
- **Question:** how to simulate a negative shock? Add minus sign to the exogenous variable in the "model" module.

Check simulation results

- Policy functions
  The results of first-order perturbation.
  Dynare solves the following policy function:

$$y_t - y^{ss} = A(y_{t-1} - y^{ss}) + Bu_t \tag{20}$$

  The results of decision rules are stored in "oo_.dr".
  "oo_.dr.state_var" tells us which variables are state variables.
  "oo_.dr.ys" stores y^{ss} in declaration order. "oo_.dr.ghx" stores matrix A, in the order described in "oo_.dr.order_var".
  "oo_.dr.ghu" stores the matrix of B, in declaration order.
- Impulse Response Functions
  The calculated impulse response functions are stored in "oo_.irfs".

Task

1. Write a .mod file that simulate the model against an aggregate TFP shock; Use built-in solver to solve the steady state.

2. Write a separate .m file to solve the steady state;

3. Introduce a demand shock, i.e. β increases temporarily. Simulate the model against the demand shock and discuss the impact. Assume that β follows an AR(1) process in log.

$$\log(\beta_t) = (1 - \rho_\beta)\log(\bar{\beta}) + \rho_\beta \log(\beta_{t-1}) + \sigma_\beta e_t, \quad e_t \sim N(0,1)$$

4. Simulate the economy against both TFP shock and demand shock, and plot *percentage* change in each variable in response to each shock.

Extension: Variable Capital Utilization

Set up

A social planner maximize social welfare given by:

$$\max_{C_t, N_t, K_{t+1}, e_t} E_t \sum_{s=0}^{\infty} \beta^t [\log(C_t) - \psi \frac{N_t^{1+\gamma}}{1+\gamma}]$$

subject to constraint:

$$C_t + K_{t+1} = (1-\delta_t)K_t + A_t(e_t K_t)^\alpha N_t^{1-\alpha} \tag{21}$$

where

$$\delta_t = \delta_0 \frac{e_t^{1+\theta}}{1+\theta} \tag{22}$$

and exogenous technology process:

$$\log(A_t) = \rho \log(A_{t-1}) + \sigma \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \tag{23}$$

Dynamics

$$K_{t+1}: \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (1 - \delta_0 \frac{e_t^{1+\theta}}{1+\theta} + \alpha A_{t+1} e_{t+1}^{\alpha} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}) \tag{24}$$

$$N_t : (1-\alpha) A_t e_t^{\alpha} K_t^{\alpha} N_t^{-\alpha} \frac{1}{C_t} = \psi N_t^{\gamma} \tag{25}$$

$$e_t : \delta_0 e_t^{\theta} K_t = A_t K_t^{\alpha} N_t^{1-\alpha} \alpha e_t^{\alpha-1} \tag{26}$$

$$C_t + K_{t+1} = (1 - \delta_0 \frac{e_t^{1+\theta}}{1+\theta}) K_t + e_t^{\alpha} A_t K_t^{\alpha} N_t^{1-\alpha} \tag{27}$$

Steady State

$$(24)+(26): e = \left(\frac{1/\beta - 1}{\delta_0 \frac{\theta}{1+\theta}}\right)^{\frac{1}{1+\theta}} \tag{28}$$

$$(26): \frac{N}{K} = \left(\frac{\delta_0 e^{\theta+1-\alpha}}{\alpha}\right)^{\frac{1}{1-\alpha}} \tag{29}$$

$$(27): \frac{C}{K} = e^{\alpha}(\frac{N}{K})^{1-\alpha} - \delta_0 \frac{e^{1+\theta}}{1+\theta} \tag{30}$$

$$(25): N = \left(\frac{(1-\alpha)e^{\alpha}(N/K)^{1-\alpha}}{\psi C/K}\right)^{\frac{1}{1+\gamma}} \tag{31}$$

Task

1. Calibrate the economy to have the same steady state as baseline RBC model.
Given the same parameter values as in baseline RBC model, to generate same steady-state value, we need $\theta = \frac{1/\beta - 1}{\delta}$, $\delta_0 = (1+\theta)\delta = 1/\beta - 1 + \delta$, $e = 1$.
2. Simulate the economy against the same TFP shock as baseline model.
3. Plot two impulse responses in the same figure. What's the role of introducing capital utilization?

(Reference: Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. The American Economic Review, 402-417.)

Extension: Alternative Preference

Set up

A social planner maximize social welfare given by:

$$\max E_t \sum_{s=0}^{\infty} \beta^t \log[C_t - \tilde{\psi} \frac{N_t^{1+\gamma}}{1+\gamma}]$$

subject to constraint:

$$C_t + K_{t+1} = (1-\delta)K_t + A_t K_t^{\alpha} N_t^{1-\alpha} \tag{32}$$

and exogenous technology process:

$$\log(A_t) = \rho \log(A_{t-1}) + \sigma \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \tag{33}$$

Dynamics

$$K_{t+1}: \frac{1}{C_t - \tilde{\psi}\frac{N_t^{1+\gamma}}{1+\gamma}} = \beta E_t \frac{1}{C_{t+1} - \tilde{\psi}\frac{N_{t+1}^{1+\gamma}}{1+\gamma}}(1 - \delta + \alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}) \tag{34}$$

$$N_t : (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} = \tilde{\psi} N_t^{\gamma} \tag{35}$$

$$C_t + K_{t+1} = (1 - \delta_0 \frac{e_t^{1+\theta}}{1+\theta}) K_t + A_t K_t^{\alpha} N_t^{1-\alpha} \tag{36}$$

Steady State

$$A = 1 \tag{37}$$

$$\frac{N}{K} = \left(\frac{1/\beta - 1 + \delta}{\alpha}\right)^{\frac{1}{1-\alpha}} \tag{38}$$

$$\frac{C}{K} = (\frac{N}{K})^{1-\alpha} - \delta \tag{39}$$

$$(1-\alpha)(\frac{N}{K})^{-\alpha} = \tilde{\psi} N^{\gamma} \tag{40}$$

$$K = \frac{N}{N/K}, C = K\frac{C}{K} \tag{41}$$

$$W = (1-\alpha)(\frac{N}{K})^{-\alpha}, R = \alpha(\frac{N}{K})^{1-\alpha} \tag{42}$$

Task

1. Calibrate the economy to have the same steady state as baseline RBC model.
Given the same parameter values as in baseline RBC model, to generate same steady-state value, we need

$$\tilde{\psi} = \frac{(1-\alpha)(\frac{N}{K})^{-\alpha}}{n^{\gamma}}$$

2. Simulate the economy against the same TFP shock as baseline model.
3. Plot two impulse responses in the same figure. What's the role of introducing GHH utility?

(Reference: Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. The American Economic Review, 402-417.)