Problem Set 3

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Task 1: Hopenhayn (1992)

Hopenhayn (1992) is workhorse PE model of industry dynamics, which features endogenous stationary distribution with entry-and-exit, no aggregate uncertainty and frictionless economy (except a fixed operation cost).

Settings

- discrete and infinite time horizon
  - discount factor: β
- continuum of firms
  - law of large numbers holds
- homogeneous product
  - exogenous aggregate demand for output
  - single input: labor
  - exogenous aggregate supply of input
- entry and exit
  - potential entrants are *ex ante* identical
  - incumbents are heterogeneous in idiosyncratic productivity

Firm's Problem

- production technology:

$$f(a, n) = ay(n) = an^{\alpha}$$

  - a: idiosyncratic productivity, Markov process: a $\rightarrow$ a'
  - labor input
  - $\alpha < 1$: decreasing return to scale $\rightarrow$ optimal size
- role of fixed cost: c^f

- generating endogenous exit

- operating profit:

$$\pi(a, p, w) = \max_n pf(a, n) - wn - c^f$$

  - optimal output denoted as $q^* := f(a, n^*)$
  - optimal input denoted as $n^* := n(a, p.w)$

Incumbent's Problem

- two decisions:
  - size of employment: one-to-one mapping from productivity (a)
  - exit
- exit decision:
  - if exit: 0
  - if not exit: *expected* operating profit
- value function:

$$v_t(a; \mu) = \pi(a, p, w) + \beta max\{0, \int v_{t+1}(a'; \mu')F(da'|a)\}$$

  - μ : aggregate state (i.e., distribution, thus prices)
  - exit cut-off value a^*:

$$0 = \int v_{t+1}(a'; \mu')F(da'|a^*) \quad \text{or}$$
$$a^* = inf\{a \in A : \int v_{t+1}(a'; \mu')F(da'|a^*) \geq 0\}$$

Entrant's Problem

- size of potential entrants: M_t
- one decision:
  - entry, after paying a sunk entry cost c^e
- entry decision:
  - enter if

$$\int v_t(a, \mu)g(da) \geq c^e$$

  - free entry:

$$\int v_t(a, \mu)g(da) = c^e \text{ if } M_t > 0$$

Distribution Law of Motion:

$$\mu_{t+1}([0,a']) = \underbrace{\int_{a \geq a^*} F(a'|a)\mu_t(da)}_{ContinuingIncumbent} + \underbrace{M_{t+1}G(a')}_{Entrants} \tag{1}$$

Define

$$\hat{P}_t = \begin{cases} \int_{a \in A} F(a'|a) & if \quad a \geq a^* \\ 0 \quad otherwise \end{cases}$$

⇒ Law of Motion:

$$\mu_{t+1} = \hat{P}_t \mu_t + M_{t+1} g \tag{2}$$

Equilibrium

- aggregate supply (endogenous)

$$Q^s(\mu_t) = \int q_t(a,\mu)\mu_t(da)$$

- aggregate demand (exogenous)

$$Q^s$$

- aggregate labor demand (endogenous)

$$N^d(\mu_t) = \int n_t(a,\mu)\mu_t(da)$$

- aggregate labor supply (exogenous)

$$N^s$$

- both markets clear at equilibrium
- focus on stationary equilibrium
  - constant distribution over time

Distribution

- Stationary Distribution:

$$\mu^* = \hat{P}\mu^* + M^* g \tag{3}$$

$$\Rightarrow \quad \mu^* = M^*(I - \hat{P})^{-1} g \tag{4}$$

  - stationary distribution is linearly homogeneous in m (scalar)
  - stationary distribution can be found by simulation as well.

Calibration. Following the paper.

Task. 1. Solve the stationary equilibrium of the model. 2. Evaluate the effect of an increase in entry cost c_e on (1) exit threshold (a^*); (2). entrants' mass (m^*); (3) output price (p^*); (4) entry/exit rate (m^*/μ^*); (5) overall firm distribution.

Reference .

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. Econometrica: Journal of the Econometric Society, 1127-1150.

Task 2: Khan and Thomas (2008)

Khan and Thomas (2008) is workhorse GE model of industry dynamics. The model features fixed adjustment cost of investment.

Task. See a homework by Prof. Winberry at Wharton:
https://www.thomaswinberry.com/teaching/phd_lectures/homework2.pdf

Reference .

Khan, A., Thomas, J. K. (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. Econometrica, 76(2), 395-436.