Group Assignment 2

June 2021

Task 1: Uncertainty Shock with Sticky Price

Setting. We consider a (slightly) simplified model of Leduc and Liu (2016). The model features a stylized search-and-matching framework with sticky price.

Household. The representative household consumes a basket of retail goods. The utility function is given by

$$E\sum_{t=0}^{\infty}\beta^t\left[\ln\left(C_t\right)-\chi N_t\right]$$

subject to the sequence of budget constraints

$$C_t+\frac{B_t}{P_tR_t}=\frac{B_{t-1}}{P_t}+w_tN_t+d_t,\quad \forall t\geq 0$$

where P_t denotes the price level, B_t denotes holdings of a nominal risk-free bond, R_t denotes the nominal interest rate, w_t denotes the real wage rate, d_t denotes profit income from ownership of intermediate goods producers and of retailers. Optimal bond-holding decisions are described by the intertemporal Euler equation

$$1=\mathrm{E}_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_t}{\pi_{t+1}}$$

where $\pi_t=\frac{P_t}{P_{t-1}}$ is inflation rate, and Λ_t is marginal utility of consumption.

Aggregation sector. Denote by Y_t the final consumption good, which is a basket of differentiated retail goods. Denote by $Y_t(j)$ a type j retail good for $j\in[0,1]$. We assume that

$$Y_t=\left(\int_0^1 Y_t(j)\frac{\eta-1}{\eta}\right)^{\frac{\eta}{\eta-1}}$$

where $\eta>1$ is the elasticity of substitution between differentiated products. Expenditure minimizing implies that demand for a type j retail good is inversely related to the relative price, with the demand schedule given by

$$Y_t^d(j)=\left(\frac{P_t(j)}{P_t}\right)^{-\eta}Y_t$$

$$P_t=\left(\int_0^1 P_t(j)^{\frac{1}{1-\eta}}\right)^{1-\eta}$$

Retail goods producers. There is a continuum of retailers, each producing a differentiated product using a homogeneous intermediate good as input. The production function of a retail good of type $j \in [0, 1]$ is given by

$$Y_t(j) = X_t(j)$$

where $X_t(j)$ is the input of intermediate goods used by retailer j and $Y_t(j)$ is the output. The retail goods producers are price takers in the input market and monopolistic competitors in the product markets, where price adjustments are subject to the quadratic cost

$$\frac{\Omega_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t,$$

where the parameter $\Omega_p \geq 0$ measures the cost of price adjustments and π denotes the steady-state inflation rate. Price adjustment costs are in units of aggregate output. A retail firm that produces good j chooses $P_t(j)$ to maximize the profit

$$\mathrm{E}_t \sum_{i=0}^{\infty} \frac{\beta^i \Lambda_{t+i}}{\Lambda_t}\left[\left(\frac{P_{t+i}(j)}{P_{t+i}} - q_{t+i}\right) Y_{t+i}^d(j) - \frac{\Omega_p}{2}\left(\frac{P_{t+i}(j)}{\pi P_{t+i-1}(j)} - 1\right)^2 Y_{t+i}\right]$$

where q_t denotes the relative price of intermediate goods. The optimal price-setting decision implies that, in a symmetric equilibrium with $P_t(j) = P_t$ for all j, we have

$$q_t = \frac{\eta - 1}{\eta} + \frac{\Omega_p}{\eta}\left[\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) - \mathrm{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}}{Y_t}\frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\right].$$

Labor Market. In the beginning of period t, there are u_t unemployed workers searching for jobs and there are v_t vacancies posted by firms. The matching technology is described by the Cobb-Douglas function

$$m_t = \mu u_t^{\alpha} v_t^{1-\alpha}$$

where m_t denotes the number of successful matches and the parameter $\alpha \in (0, 1)$ denotes the elasticity of job matches with respect to the number of searching workers. The parameter μ scales the matching efficiency. The probability that an open vacancy is matched with a searching worker (i.e., the job filling rate) is given by

$$q_t^v = \frac{m_t}{v_t}$$

The probability that an unemployed and searching worker is matched with an open vacancy (i.e., the job finding rate) is given by

$$q_t^u = \frac{m_t}{u_t}$$

In the beginning of period t, there are N_{t-1} workers. A fraction ρ of these workers lose their jobs. Thus, the number of workers who survive the job separation is $(1 - \rho)N_{t-1}$. At the same time, m_t new matches are formed. Thus, aggregate employment in period t evolves according to

$$N_t = (1 - \rho)N_{t-1} + m_t.$$

With a fraction ρ of employed workers separated from their jobs, the number of unemployed workers searching for jobs in period t is given by

$$u_t = 1 - (1 - \rho)N_{t-1}$$

The unemployment rate is given by

$$U_t = u_t - m_t = 1 - N_t$$

Labor Demand A firm can produce only if it successfully hires a worker. The production function for a firm with one worker is given by

$$x_t = Z_t,$$

where x_t denotes output. The term Z_t denotes an aggregate technology shock, which follows the stationary stochastic process

$$\ln Z_t = \rho_z \ln Z_{t-1} + \sigma_{zt}\varepsilon_{zt}$$

The parameter $\rho_z \in (-1, 1)$ measures the persistence of the technology shock. The term ϵ_{zt} is an i.i.d. innovation to the technology shock and is a standard normal process. The term σ_{zt} is a time-varying standard deviation of the innovation, which we interpret as a technology uncertainty shock. We assume that the uncertainty shock follows the stationary stochastic process

$$\ln \sigma_{zt} = (1 - \rho_{\sigma_z}) \ln \sigma_z + \rho_{\sigma_z} \ln \sigma_{z,t-1} + \sigma_{\sigma_z}\varepsilon_{\sigma_z,t},$$

where the parameter $\rho_{\sigma_z} \in (-1, 1)$ measures the persistence of the uncertainty shock, the term $\varepsilon_{\sigma_2,t}$ is an i.i.d. standard normal process, and the parameter $\sigma_{\sigma_z} > 0$ is the standard deviation of the innovation to technology uncertainty.

If a firm finds a match, it obtains a flow profit in the current period after paying the worker. In the next period, if the match survives (with probability $1 - \rho$), the firm continues; if the match breaks down (with probability ρ), the firm posts a new job vacancy at a fixed cost κ, with the value V_{t+1}. The value of a firm with a match (denoted by J_t^F) is therefore given by the Bellman equation

$$J_t^F = q_t Z_t - w_t + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[(1 - \rho) J_{t+1}^F + \rho V_{t+1} \right].$$

If the firm posts a new vacancy in period t, it costs κ units of final goods. The vacancy can be filled with probability q_t^v, in which case the firm obtains the value of the match. Otherwise, the vacancy remains unfilled and the firm goes into the next period with the value V_{t+1}. Thus, the value of an open vacancy is given by

$$V_t = -\kappa + q_t^v J_t^F + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1 - q_t^v) V_{t+1}$$

Free entry implies that $V_t = 0$, so that

$$\frac{\kappa}{q_t^v} = J_t^F$$

Labor Supply. If a worker is employed, he obtains wage income but suffers a utility cost of working. In period $t + 1$, the match is separated with probability ρ and the separated worker can find a new match with probability q_{t+1}^u. Thus, with probability $\rho \left(1 - q_{t+1}^u\right)$, a separated worker fails to find a new job in period $t + 1$ and enters the unemployment pool. Otherwise, the worker continues to be employed. The (marginal) value of an employed worker (denoted by J_t^W) therefore satisfies the Bellman equation

$$J_t^W = w_t - \frac{\chi}{\Lambda_t} + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left\{ \left[1 - \rho \left(1 - q_{t+1}^u\right)\right] J_{t+1}^W + \rho \left(1 - q_{t+1}^u\right) J_{t+1}^U \right\}$$

where J_t^U denotes the value of an unemployed worker. An unemployed worker obtains nothing and can find a new job in period $t+1$ with probability q_{t+1}^u. Thus, the value of an unemployed worker satisfies the Bellman equation

$$J_t^U = E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[q_{t+1}^u J_{t+1}^W + \left(1 - q_{t+1}^u\right) J_{t+1}^U \right]$$

Wage Determination. Firms and workers bargain over wages. The Nash bargaining problem is given by

$$\max_{w_t} \left(J_t^W - J_t^U\right)^b \left(J_t^F\right)^{1-b}$$

where $b \in (0,1)$ represents the bargaining weight for workers. Define the total surplus as

$$S_t = J_t^F + J_t^W - J_t^U$$

Then the bargaining solution is given by

$$J_t^F = (1-b) S_t, \quad J_t^W - J_t^U = b S_t$$

It then follows that

$$b S_t = w_t^N - \frac{\chi}{\Lambda_t} + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[(1-\rho) \left(1 - q_{t+1}^u\right) b S_{t+1} \right]$$

Given the bargaining surplus S_t, which itself is proportional to the match value J_t^F, this last equation determines the Nash bargaining wage w_t^N.

If the equilibrium real wage equals the Nash bargaining wage, then we can obtain an explicit expression for the Nash bargaining wage.

$$w_t^N = (1-b) \left[\frac{\chi}{\Lambda_t} + \phi \right] + b \left[q_t Z_t + \beta (1-\rho) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{\kappa v_{t+1}}{u_{t+1}} \right].$$

We follow the literature to formalize sticky wage by assuming that

$$w_t = w_{t-1}^{\gamma} \left(w_t^N\right)^{1-\gamma}$$

where $\gamma \in (0,1)$ represents the degree of real wage rigidity.

Policy. The monetary authority follows the Taylor rule

$$R_t = r \pi^* \left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y}$$

where the parameter ϕ_π determines the aggressiveness of monetary policy against deviations of inflation from the target π^* and ϕ_y determines the extent to which monetary policy accommodates output fluctuations. The parameter r denotes the steady-state real interest rate (i.e., $r = \frac{R}{\pi}$).

In a search equilibrium, the markets for bonds, final consumption goods, and intermediate goods all clear.

Calibration. Following the paper except that $h = 0$ and $\phi = 0$.

Task. Write down the search equilibrium as a system of 16 equations for 16 variables summarized in the vector

$$\left[C_t, \Lambda_t, \pi_t, m_t, q_t^u, q_t^v, N_t, u_t, U_t, Y_t, R_t, v_t, q_t, J_t^F, w_t^N, w_t\right]$$

(1) Set $_p = 0$, so there is no price rigidity. Simulate this flexible-price model against an uncertainty shock, and discuss the result.
(2) Keep nominal rigidity and simulate the model against an uncertainty shock, and discuss the result. Contrast the results against benchmark model with flexible price.

Reference .

Leduc, S., Liu, Z. (2016). Uncertainty shocks are aggregate demand shocks. Journal of Monetary Economics, 82, 20-35.

Basu, S., Bundick, B. (2017). Uncertainty shocks in a model of effective demand. Econometrica, 85(3), 937-958.

Task 2: Uncertainty Shock with Flexible Price

In this task we solve a simplified model of Dong, Liu and Wang (2021) with flexible price. The model features heterogeneous firms and endogenous misallocation.

Setting. Consider an economy with a continuum of firms that produce with a linear technology using labor n_{jt} as single input:

$$y_{jt} = A_t z_{jt} n_{jt} \tag{1}$$

where A_t measures aggregate productivity, and z_{jt} measures idiosyncratic productivity.

The process of idiosyncratic productivity is assumed to follow the following process:

$$z_{jt+1} = \begin{cases} z_{jt} & w.p. \quad \rho_t \\ \tilde{z} & w.p. \quad 1-\rho_t \end{cases} \tag{2}$$

where $\tilde{z}$ is discrete random variable with $\tilde{z} = z_j$ occurring with probability π_j, $j = 1, 2, ..., I$. We assume that $z_1 < z_2 < ... < z_I$ without loss of generality. The process features time-invariant cross-sectional distribution of firm productivity such that, regardless of realization on ρ_t, there are always π_j fraction of firms with $z_{jt} = z_j$ in each period.

The firms' problem is given by the following Bellman equation (we suppress aggregate state in notation for simple exposition):

$$V_t(z_{jt}, \tau_{jt}) = \tau_{jt} A_t z_{jt} n_{jt} - W_t n_{jt} + \beta E_t V_{t+1}(z_{jt+1}, \tau_{jt+1}) \tag{3}$$

subject to a credit constraint

$$W_t n_{jt} \leq \theta_t \beta E_t V_{t+1}(z_{jt+1}, \tau_{jt+1}) \tag{4}$$

where θ_t is a financial shock measuring tightness of credit constraint. τ_{jt} is idiosyncratic distortion (net subsidy) on output, and is assumed to be an i.i.d. random variable with cumulative distribution function $F(\tau)$. Denote

$$\bar{V}_{jt} = \int V_t(z_{jt}, \tau_{jt}) dF(\tau), \tag{5}$$

we can write discounted future value conditional on current realization of productivity as

$$\beta E_t V_{t+1}(z_{jt+1}, \tau_{jt+1} | z_{jt} = z_j) = \beta E_t \left[\rho_t \bar{V}_{jt+1} + (1-\rho_t) \sum_{i=1}^{I} \pi_i \bar{V}_{it+1} \right] \equiv B_{jt} \tag{6}$$

Solving static profit maximization problem gives the allocations of production and credit: a firm will borrow and produce in current period if and only if net subsidy τ_{jt} is higher than a cut-off:

$$\hat{\tau}_{jt} \equiv \hat{\tau}_t(z_{jt}) = \frac{W_t}{A_t z_{jt}} \tag{7}$$

Without loss of generality we assume marginal firms operate. Firms with relatively higher productivity (z_{jt}) and subsidy (or lower tax) choose to produce and borrow up to the limit to finance wage bill. Low productivity or heavily taxed firms stay inactive and do not borrow. Therefore, idiosyncratic labor demand function is

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\theta_t B_{jt}}{W_t}, & if \quad \tau_{jt} \geq \hat{\tau}_{jt} \\ 0, & otherwise \end{cases} \tag{8}$$

The value function in equation (3) can be re-written as

$$V_t(z_{jt}, \tau_{jt}) = \max(\frac{A_t z_{jt} \tau_{jt}}{W_t} - 1, 0)\theta_t B_{jt} + B_{jt} \tag{9}$$

Define aggregate firm value as

$$\bar{V}_t = \sum_{i=1}^{I} \pi_i \bar{V}_{it} \tag{10}$$

It follows that for $j = 1, 2, ..., I$,

$$\bar{V}_{jt} = \left[1 + \theta_t \int_{\frac{W_t}{A_t z_{jt}}} \left(\frac{A_t z_{jt}}{W_t}\tau - 1\right) dF(\tau)\right] \beta E_t \left[\rho_t \bar{V}_{jt+1} + (1-\rho_t)\bar{V}_t\right] \equiv \Phi(\frac{W_t}{A_t z_{jt}}, \theta_t) B_{jt} \tag{11}$$

where $\Phi(\frac{W_t}{A_t z_{jt}}, \theta_t) \equiv 1 + \theta_t \int_{\frac{W_t}{A_t z_{jt}}} \left(\frac{A_t z_{jt}}{W_t}\tau - 1\right) dF(\tau)$. It is clear that Φ is an increasing function of z_{jt}, θ_t, and decreasing function of W_t.
To solve the equilibrium, we need to impose a labor market clearing condition:

$$N_t = \sum_{j=1}^{I} \pi_j \int_{\tau} n_t(z_{jt}, \tau) dF(\tau) \equiv \sum_{j=1}^{I} \pi_j N_{jt}, \tag{12}$$

where N_t is exogenous labor supply and N_{jt} for $j = 1, 2, ..., I$ is

$$N_{jt} = \frac{\theta_t \beta E_t \left[\rho_t \bar{V}_{jt+1} + (1-\rho_t)\bar{V}_t\right]}{W_t} \left[1 - F(\frac{W_t}{A_t z_{jt}})\right] \tag{13}$$

The aggregate output (Y_t) is given by

$$Y_t \quad = \quad \sum_{j=1}^{I} \pi_j \frac{A_t z_{jt} \theta_t B_{jt}}{W_t}[1 - F(\frac{W_t}{A_t z_{jt}})] = A_t \sum_{j=1}^{I} \pi_j z_{jt} N_{jt} \tag{14}$$

and endogenous TFP, denoted as Z_t, is defined as

$$Z_t \equiv \frac{Y_t}{A_t N_t} = \frac{\sum_{j=1}^{I} \pi_j z_{jt} N_{jt}}{N_t} \tag{15}$$

Last equation shows that, given exogenous labor supply, endogenous TFP reflects labor misallocation.

Calibration. Assume that ε follows a log-normal distribution

$$log(\varepsilon) \sim N(\mu_t, \sigma_t), \quad \mu_t = -0.5\sigma_t^2$$

Set $\beta = 0.96$; $\sigma_{ss} = 0.80$; $\theta = 0.35$; $\rho_{ss} = 0.82$; Labor supply is 0.33. z_j is discretized into 10 states from an AR(1) process, with $\rho_z = 0.80$ and $\sigma_z = 0.20$, using Tauchen method.

Task. Write down the competitive equilibrium is defined as a sequence of variables

$$\{N_{jt}, \bar{V}_{jt}, \bar{V}_t, W_t, Y_t, Z_t\}$$

uniquely determined by equation (10) - (15), given exogenous processes of exogenous variables: $\{\rho_t, A_t, \theta_t\}$.

(1) Assume ρ_t follows the following process:

$$\log(\rho_t) = (1-\rho^\rho)\log(\bar{\rho}) + \rho^\rho \log(\rho_{t-1}) - \sigma^\rho \varepsilon_t^\rho, \quad \varepsilon_t^\rho \sim N(0,1) \tag{16}$$

where $\bar{\rho} = 0.82$, $\rho^\rho = 0.84$, $\sigma^\rho = 0.10$. Simulate the economy against this ρ_t shock.

(2) Assume σ_t follows the following process:

$$\log(\sigma_t) = (1-\rho^\sigma)\log(\bar{\sigma}) + \rho^\sigma \log(\sigma t - 1) - \sigma^\sigma \varepsilon_t^\sigma, \quad \varepsilon_t^\sigma \sim N(0,1) \tag{17}$$

where $\bar{\sigma} = 0.80$, $\rho^\rho = 0.70$, $\sigma^\rho = 0.01$. Simulate the economy against this σ_t shock.

Reference .