Uncertainty Shock in a Tractable Heterogeneous Agent Model with Flexible Price

Ding Dong

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Preview

- uncertainty shock
  - high-order approximation
  - stochastic steady state
- tractable hetero-agent model
  - aggregation done with paper and pencil
  - solved as rep-agent model in Dynare
- aside: discretize AR(1) into Markov process
  - grid value
  - stationary distribution

Model

Firms

There are a continuum of firms

- CRS technology with labor(n_{jt}) (exogenous A_t)

$$y_{jt} = A_t z_{jt} n_{jt} \tag{1}$$

- z_{jt} :persistent idiosyncratic shock subject to uncertainty shock ρ_t

$$z_{jt+1} = \begin{cases} z_{jt} & w.p. \quad \rho_t \\ \tilde{z} & w.p. \quad 1-\rho_t \end{cases} \tag{2}$$

- Bellman equation:

$$V_t(z_{jt}, \tau_{jt}) = \max_{n_{jt}} \tau_{jt} A_t z_{jt} n_{jt} - W_t n_{jt} + \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \tag{3}$$

s.t. working capital constraint

$$w_t n_{jt} \leq \theta_t \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta_t B_{jt} \tag{4}$$

where τ: idiosyncratic i.i.d distortion; M_{t+1}: SDF $\sim$ household problem

Firms

There exists a cut-off $\tau_{jt}^* = \frac{W_t}{A_t z_{jt}}$ beyond which firms produce

- labor policy

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\theta_t B_{jt}}{W_t}, & \textit{if} \quad \tau_{jt} \geq \tau_{jt}^* \\ 0, & \textit{otherwise} \end{cases} \tag{5}$$

- labor demand for each type z_j

$$N_{jt} \equiv \frac{\theta_t B_{jt}}{W_t} \left[1 - F(\tau_{jt}^*)\right] \tag{6}$$

- firm value:

$$V_t(z_{jt}, \tau_{jt}) \equiv \max \left\{ \frac{\tau_{jt}}{\tau_{jt}^*} - 1, 0 \right\} \theta_t B_{jt} + B_{jt} \tag{7}$$

- expected firm value

$$\bar{V}_t(z_{jt}) = \int V_t(z_{jt}, \tau) dF(\tau) = \underbrace{[1 + \theta_t \int_{\tau_{jt}^*} (\frac{1}{\tau_{jt}^*} \tau - 1) dF(\tau)]}_{\equiv \Phi(\tau_{jt}^*)} B_{jt} \tag{8}$$

Firms

- expected discounted firm value

$$B_{jt} \equiv \beta E_t \left[\rho_t \bar{V}_{jt+1} + (1-\rho_t) \sum_{i=1}^{I} \pi_i \bar{V}_{it+1} \right] \tag{9}$$

- market value

$$\bar{V}_t \equiv \sum_{i=1}^{I} \pi_i \bar{V}_{it} \tag{10}$$

- labor market clearing condition (N_t exogenous)

$$\sum \pi_j N_{jt} = N_t \tag{11}$$

- endogenous TFP

$$Z_t \equiv \frac{Y_t}{A_t N_t} = \frac{\sum_{j=1}^{I} \pi_j z_{jt} N_{jt}}{N_t} \tag{12}$$

Stationary Equilibrium

- endogenous variables
  - $\{N_j, W, Z, \tau_j^*, \bar{V}_j, \bar{V}, B_j\}$
  - B is function of $\bar{V}_j$
  - Z is function of N_j
  - τ_j^* is function of W
  - reduced to $\{W, \bar{V}_j, \bar{V}\}$
- key equations

$$\bar{V} = \sum_{i=1}^{I} \pi_i \bar{V}_i$$

$$\sum \pi_j N_{jt} = N_t$$

$$\bar{V}_j = \beta \Phi(\frac{W}{Az_j})[\rho \bar{V}_j + (1-\rho)\bar{V}] \Rightarrow \bar{V}_j = \frac{\beta(1-\rho)\Phi(\frac{W}{Az_j})}{1-\beta\rho\Phi(\frac{W}{Az_j})}\bar{V} \equiv g_j(W)\bar{V}$$

- single equation that solves W

$$\sum_j \pi_j g_j(W) = 1$$

Stationary Equilibrium

- cut-off solved

$$\tau_j^* = \frac{W}{Az_j}$$

- borrowing capacity B_j is linear to $\bar{V}$

$$B_j \equiv \beta\left[\rho g_j(W)\bar{V} + (1-\rho)\bar{V}\right] \equiv b_j(W)\bar{V}$$

- labor market clearing condition pins down $\bar{V}$

$$\sum_j \frac{\theta_t b_j(W)}{W}\left[1 - F(\tau_j^*)\right]\bar{V} = N$$

- idiosyncratic firm value

$$\bar{V}_j = g_j(W)\bar{V}$$

Codes: Discretize AR(1)

$$\log(z_t) = \mu_z + \rho_z \log(z_{t-1}) + \sigma_z \varepsilon_{zt}$$

- Tauchen method
  - number of grid points
  - value of z_j at each point
  - transition matrix
  - stationary distribution: π_j
- codes:
  $[logz, Pi] = tauchen(muz, rhoz, sigz, pngridz);$
  $z = exp(logz);$
  $pi = limitdist(Pi)';$

Codes: Macro-Processor in Dynare

- for all j = 1, 2, 3, ..., 10
  - variables
  - parameters
  - models
  - steady state
- codes:
  see attached files

Codes: Uncertainty Shock

- stochastic steady state (SSS)
    - all shocks set to zero
    - variance non-zero
- simulation
    - turn on uncertainty shock
- IRF
    - percentage deviation from SSS