# Problem Set 1

Computation Study Group

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## Task 1: News Shock

In this question we revisit expectation-driven business cycles models (i.e. news shock). The model is a simplified version of one-sector model in Jaimovich & Rebelo (2009) taken from Wang (2012).

**Setting.** A simple version of Jaimovich and Rebelo's (2009) model can be described as a social planner's problem in which the planner solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log \left[ C_t - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

subject to a resource constraint

$$C_t + I_t = A_t \left(e_t K_t\right)^{\alpha} N_t^{1-\alpha} \tag{1}$$

and capital law of motion

$$K_{t+1} = I_t \left[ 1 - \varphi\left(\frac{I_t}{I_{t-1}}\right) \right] + \left[1 - \delta\left(e_t\right)\right] K_t \tag{2}$$

and

$$\delta(e_t) = \delta_0 \frac{e_t^{1+\theta}}{1+\theta} \tag{3}$$

where $e_t$ is capacity utilization, $\delta\left(e_t\right)$ is the rate of capital depreciation endogenously determined by capacity utilization. $I_t \varphi\left(I_t / I_{t-1}\right)$ is an adjustment cost function in investment, which satisfies the following property: $\varphi(1) = 0$, $\varphi'(1) = 0$ and $\varphi''(1) > 0$, for example,

$$\varphi\left(\frac{I_t}{I_{t-1}}\right) = \frac{s}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \tag{4}$$

Suppose aggregate TFP shock follows the process:

$$\log(A_t) = \rho \log(A_{t-1}) + \sigma^a \varepsilon_t + \sigma^u u_{t-4} \tag{5}$$

where $\varepsilon_t$ is a standard TFP shock and $u_t$ is a news shock (note that $u_t$ has an effect on $A_t$ only after 4 periods.).

**Calibration.** Set $\beta = 0.99$, $\gamma = 0.4$, $\alpha = 1/3$. Choose $\psi$ to ensure steady state labor at 1/3. Choose $\delta_0$ and $\theta$ to ensure a steady state utilization rate $e = 1$ and depreciation rate at 0.025. Choose $s$ to ensure that $\varphi''(1) = 1.15$.

**Task** .

1. Simulate this economy against news shock. Can the model generate comovement in investment and consumption?

2. This model has three distinctive features: a utility function that yields no income effect on consumption (GHH utility), variable capacity utilization, and dynamic adjustment cost in investment. Evaluate the role played by each element (you can shut down each channel).

**Reference** .


Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. The American Economic Review, 402-417.

Jaimovich, N., & Rebelo, S. (2009). Can news about the future drive the business cycle?. American Economic Review, 99(4), 1097-1118.

Wang, P. (2012). Understanding Expectation-Driven Fluctuations: A Labor-Market Approach. Journal of Money, Credit and Banking, 44(2-3), 487-506.

## Task 2: Financial Shock

In this question we revisit the effect of financial shock on business cycle. The model is taken from Jermann and Quadrini (2012).

**Setting.** The baseline model consists of a representative firm and household.

*Firms.* The firm is endowed with Cobb-Douglas technology

$$y_t = z_t k_t^{\theta} n_t^{1-\theta}$$

s.t. capital law of motion

$$k_{t+1} = i_t + (1-\delta)k_t$$

and an intertemporal budget constraint

$$b_t + w_t n_t + k_{t+1} + d_t = (1-\delta)k_t + y_t + \frac{b_{t+1}}{R_t}$$

The firm enters each period with predetermined capital $k_t$ and debt repayment $b_t$, hires labor $n_t$, and chooses investment $i_t$, equity payout (dividend $d_t$) and borrowing $b_{t+1}$ before production. Firms raise funds with an intra-temporal loan, $l_t$, to finance working capital.

$$l_t = w_t n_t + i_t + d_t + b_t - \frac{b_{t+1}}{R_t}$$

The ability to borrow (intra- and inter-temporally) is bounded by the limited enforceability of debt contracts as firms can default on their obligations. This friction on debt finance gives rise to a borrowing constraint which is assumed to be binding [1]

$$\xi_t(k_{t+1} - \frac{b_{t+1}}{R_t}) \geq l_t$$

Equity finance is also subject to an adjustment cost, such that actual cost of equity payout is

$$\varphi(d_t) = d_t + \kappa(d_t - \bar{d})^2$$

where $\kappa \geq 0$ and $\bar{d}$ is steady state equity payout.

Formally, the firm's problem is to solve the following Bellman equation:

$$V(\mathbf{s}_t; k_t, b_t) = \max_{d_t, n_t, k_{t+1}, b_{t+1}} d_t + E_t m_{t+1} V(\mathbf{s}_{t+1}; k_{t+1}, b_{t+1}) \tag{6}$$

s.t.

$$(1-\delta)k_t + y_t - w_t n_t + \frac{b_{t+1}}{R_t} = b_t + \varphi(d_t) + k_{t+1} \tag{7}$$

and

$$\xi_t(k_{t+1} - \frac{b_{t+1}}{R_t}) \geq y_t \tag{8}$$

where $\mathbf{s}_t$ summarize aggregate state, and $m_{t+1}$ is a stochastic discount factor consistent with household problem below.

[1] In the paper binding enforcement constraint is micro-founded by a tax benefit.

*Household.* The representative household maximize

$$\max_{c_t,n_t,s_{t+1}} E_0 \sum_0^\infty \beta^t \{\log c_t + \alpha \log(1-n_t)\} \tag{9}$$

subject to a borrowing constraint:

$$w_t n_t + b_t + s_t(d_t + p_t) = \frac{b_{t+1}}{(1+\tau)R_t} + s_{t+1}p_t + c_t + T_t \tag{10}$$

where $p_t$ is market price of stock, and $\tau > 0$.

**Calibration.** We set parameters as followed: $\beta = 0.98$, $\delta = 0.05$, $\theta = 0.35$, $\tau = 0.05$, $\alpha$ is set to ensure steady state $n = 0.33$. The two parameters governing degree of financial friction are set as $\xi = 0.15$ and $\kappa = 0.15$.

**Task.** Now we consider the effect of financial shock. Suppose that $\xi_t$ is stochastic and follows an AR(1) process in log:

$$\log(\xi_t) = (1-\rho)\log(\xi) + \rho\log(\xi_{t-1}) - \sigma\varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

where $\varepsilon_t$ captures *financial shock*.

(1) Simulate the economy against the financial shock;

(2) Set $\kappa = 0$, simulate the economy against the financial shock.

**Reference** .

Jermann, U., & Quadrini, V. (2012). Macroeconomic effects of financial shocks. American Economic Review, 102(1), 238-71.

Liu, Z., Wang, P., & Zha, T. (2013). Land-price dynamics and macroeconomic fluctuations. Econometrica, 81(3), 1147-1184. (baseline representative-agent model)

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