

Uncertainty Shocks in Real-Business-Cycle and New-Keynesian Frameworks

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Abstract: This note surveys two papers¹ on uncertainty shocks in the model of real business cycle and of effective demand. We introduce to this topic with definition of uncertainty shock and establishment of some stylized fact. An identified uncertainty shock in the data causes significantly negative comovement in output, consumption, investment, and hours worked. We first show that uncertainty shocks alone in standard RBC models (or in general-equilibrium models with flexible prices) cannot reproduce this comovement. By contrast, we subsequently show that uncertainty shocks can easily generate comovement with countercyclical markups through sticky prices in a New-Keynesian framework. We conclude with analysis on different transmission mechanism in different frameworks.

1 Introduction and Stylized Facts

The surge in research interest in uncertainty has been driven by several factors. First, the jump in uncertainty in 2008 and its likely role in shaping the Great Recession has focused policy attention onto the topic. Second, the increased availability of empirical proxies for uncertainty, such as panels of firm-level outcomes, online news databases, and surveys, has facilitated empirical work. Third, the increase in computing power has made it possible to include uncertainty shocks directly in a wide range of models, allowing economists to abandon assumptions built on “certainty equivalence”, which refers to the amount of money that would be required as compensation for risk.

Given this broad definition of uncertainty, it should be unsurprising that there is no

¹1. Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., & Terry, S. J. (2012). Really uncertain business cycles (No. w18245). National Bureau of Economic Research.

2. Basu, S., & Bundick, B. (2017). Uncertainty shocks in a model of effective demand. *Econometrica*, 85(3), 937-958.

perfect measure but instead a broad range of proxies. Throughout this section the volatility of the stock market or GDP is used as a measure of uncertainty for empirical purposes, and apparently this single concept of uncertainty might be a stand-in for a mixture of risk and uncertainty².

Generally we assume that a firm, indexed by j , produces output in period t according to the following production function: $y_{j,t} = A_t z_{j,t} k_{j,t}^\alpha n_{j,t}^v$, $\alpha + v < 1$

where $k_{t,j}$ and $n_{t,j}$ denote idiosyncratic capital and labor employed by the firm. Each firm's productivity is a product of two separate processes: an aggregate component, A_t , and an idiosyncratic component, $z_{j,t}$.

We assume that the aggregate and idiosyncratic components of business conditions follow autoregressive processes:

$$\log(A_t) = \rho^A \log(A_{t-1}) + \sigma_{t-1}^A \epsilon_t \text{ (macroeconomic shocks)}$$

$$\log(z_{j,t}) = \rho^Z \log(z_{j,t-1}) + \sigma_{t-1}^Z \epsilon_{j,t} \text{ (microeconomic shocks)}$$

We allow σ_t^A and σ_t^Z to vary over time, generating periods of low and high macro and micro uncertainty.

These two shocks are driven by different statistics. Volatility in $z_{j,t}$ implies that cross-sectional dispersion-based measures of firm performance (output, sales, stock market returns, etc.) are time-varying, while volatility in A_t induces higher variability in aggregate variables like GDP growth and the S&P500 index.

In Appendix, Figure 1 to Figure 4 show empirical facts related to micro- and macro-uncertainty.

In addition to uncertainty shock in production, some literature also models uncertainty by imposing time-varying second moment to the preference shocks.

2 Modeling Uncertainty Shocks I: RBC

The General Equilibrium Model

In Blooms(2012), the model departs from frictionless standard RBC models in three ways:

1. Uncertainty is time-varying: inclusion of shocks to both the level of technology (first moment) and its variance (second moment), at both microeconomic and macroeconomic levels;
2. Heterogeneous firms, subject to idiosyncratic shocks;

²Bloom, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives*, 28(2), 153-76.

3. Non-convex adjustment costs in both capital and labor.

- **Production technology:** diminishing returns to scale :

$$y_{j,t} = A_t z_{j,t} k_{j,t}^\alpha n_{j,t}^v, \alpha + v < 1$$

y : firm's output; k & n : idiosyncratic capital & labor;

Productivity: A_t , aggregate component; $z_{j,t}$, idiosyncratic component.

- AR(1) processes of two components (first moment):

$$\log(A_t) = \rho^A \log(A_{t-1}) + \sigma_{t-1}^A \epsilon_t \text{ (macroeconomic shocks)}$$

$$\log(z_{j,t}) = \rho^Z \log(z_{j,t-1}) + \sigma_{t-1}^Z \epsilon_{j,t} \text{ (microeconomic shocks)}$$

- We allow σ_t^A and σ_t^Z to vary over time according to a two-state Markov chain. (second moment)

The two-state Markov chain process of uncertainty³:

$$\sigma_t^A \in [\sigma_L^A, \sigma_H^A], \text{ where } Pr(\sigma_{t+1}^A = \sigma_j^A | \sigma_t^A = \sigma_k^A) = \pi_{k,j}^\sigma$$

$$\sigma_t^Z \in [\sigma_L^Z, \sigma_H^Z], \text{ where } Pr(\sigma_{t+1}^Z = \sigma_j^Z | \sigma_t^Z = \sigma_k^Z) = \pi_{k,j}^\sigma$$

There are six uncertainty parameters: $\sigma_L^A, \sigma_H^A, \sigma_L^Z, \sigma_H^Z, \pi_{L,H}^\sigma, \pi_{H,L}^\sigma$

- **Capital Law of Motion:**

$$k_{j,t+1} = (1 - \delta_k) k_{j,t} + i_{j,t}$$

where δ_k denotes depreciation rate of capital and i_t denotes net investment.

- subject to capital adjustment cost:

$$\text{if } i > 0, AC^k = y(z, A, k, n) F^K;$$

$$\text{if } i < 0, AC^k = y(z, A, k, n) F^K + S|i|;$$

where F^K is a fixed disruption cost, $S|i|$ is resale loss for disinvestment (when $i < 0$).

- **Hours Law of Motion:**

$$n_{j,t+1} = (1 - \delta_n) n_{j,t} + s_{j,t}$$

where δ_n denotes exogenous destruction rate of hours worked (for example illness, retirement etc.)

$s_{j,t}$ denotes net flows into hours worked.

- subject to labor adjustment cost:

$$\text{if } |s| > 0, AC^n = y(z, A, k, n) F^L + |s|Hw;$$

where F^L is a fixed disruption cost, $|s|Hw$ is a linear hiring/firing cost (Hw is aggregate wage).

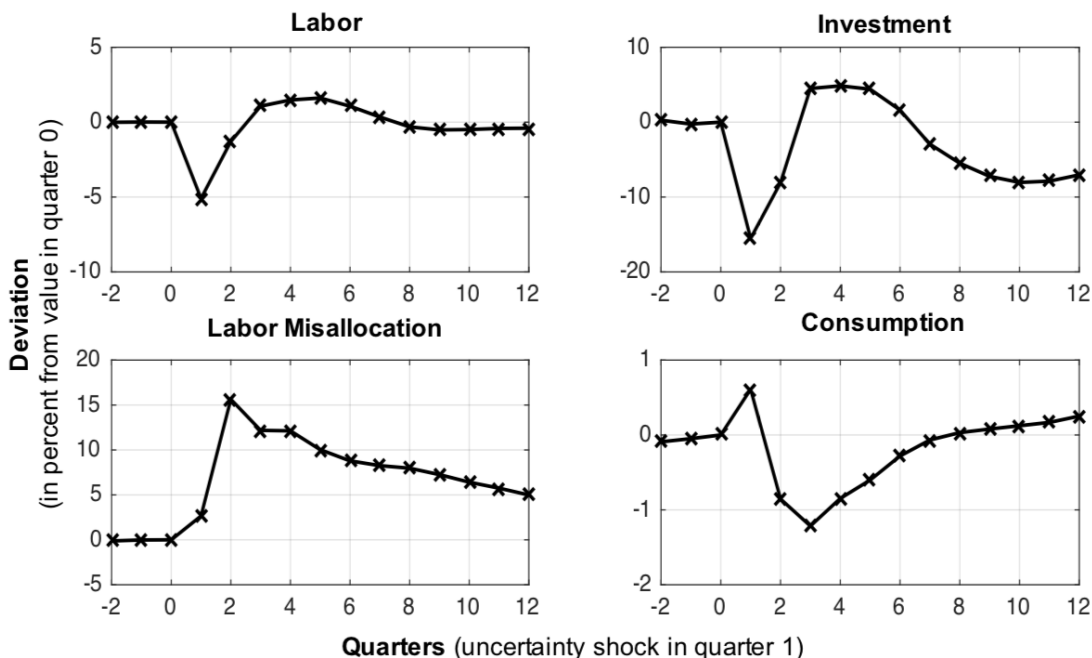
³We assume micro- and macro- uncertainty follow the same process.

Quantitative Analysis

A pure uncertainty shock leads to real effect on macroeconomic aggregates due to existence of non-convex adjustment cost. (Second moment shock alone has limited effect on macroeconomic aggregates if there is no adjustment cost.) Large uncertainty shocks produce a rapid drop and rebound in output, employment, productivity growth and investment. With absence of adjustment cost, such fluctuations are mostly harmless. With adjustment cost, however, fluctuations become costly so that firms temporarily pause their investment and hiring activities. Given a surge in uncertainty, negative comovements in macroeconomic aggregates are expected⁴.

In Bloom (2012), within one quarter after a pure uncertainty shock, there is a drop in output for about 2.5%, and then there is a recovery back to normal levels in one year after the shock. This significant fall is the first key results of the paper as it shows that uncertainty shocks generate business cycles in this general equilibrium framework.

So, what's mechanism behind the path of output? Figure below further depicts the general effects of a pure uncertainty shock.



There are at least three channels: labor, capital and mis-allocation of factors of production. First, labor. When uncertainty increases, most firms pause recruitment, and because workers continue to leave for illness, maternity or retirement without

⁴Bloom, N. (2009). The impact of uncertainty shocks. *econometrica*, 77(3), 623-685.

being replaced, total hours drop. Similarly, investment has drops rapidly due to the increase in uncertainty. Since investment falls but capital continues to depreciate, there will be a drop in the capital stock as well.

The channel of misallocation of resources is a bit sophisticated: In normal times, unproductive firms contract in size by layoff or by cutting down branches, and productive firms continue to expand through recruitment and setting up new branches, and this mechanism helps maintain high levels of aggregate productivity. But when uncertainty is high, both productive and unproductive firms reduce expansion and contraction, which shuts off the mechanism of reallocation for economic adjustment. In the lower-right panel we plot the time profile of consumption. Clearly, the rise in consumption is an unattractive feature of this pure uncertainty shock, because it is against our intuition that when uncertainty is higher people postpone consumptions, especially consumption on durable goods. One explanation here is that misallocation lowers the expected return on savings, making consumption more attractive relatively. So, households save less and consume more. As we will soon discuss, the difficulty in producing comovement in consumption is inevitable in models with flexible price.

So, the author proposes at least three solutions. The first is an open economy approach, which is to allow save in other technologies besides capital, for example, in foreign assets⁵. In an open economy model a domestic uncertainty shock induces agents to increase their savings abroad. But for closed economy model this approach seems to be implausible. Another approach is to use a different type of utility function. Here again we want to use a utility function where consumption and hours are complementary. This method should work in a closed model but would be very complicated for computation. Another option would be to model precautionary behavior from households in the wake of an uncertainty shock, as Basu and Bundick (2017) do in a New Keynesian environment with demand-determined output. We proceed our discuss with their approach.

⁵Fernndez-Villaverde, J., Guerrn-Quintana, P., Rubio-Ramirez, J. F., & Uribe, M. (2011). Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6), 2530-61.

3 Modeling Uncertainty Shocks II: NK

Why Does RBC Fail?

Under reasonable assumptions, an increase in uncertainty about the future induces precautionary saving and lowers consumption. If households supply labor inelastically, then total output remains constant since the level of technology and capital stock remain unchanged in response to the uncertainty shock. Unchanged total output and reduced consumption together imply that investment must rise. If households can adjust their labor supply and consumption and leisure are both normal goods, an increase in uncertainty also induces precautionary labor supply, or a desire for the household to supply more labor for any given level of the real wage. As current technology and the capital stock remain unchanged, the competitive demand for labor remains unchanged as well. Thus, higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple neoclassical models subject to uncertainty fluctuations.

Equivalently speaking, a large class of one-sector business-cycle models can be characterized by a few key equations:

$$Y_t = C_t + I_t; \tag{1}$$

$$Y_t = F(K_t, Z_t N_t); \tag{2}$$

$$\frac{W_t}{P_t} U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t); \tag{3}$$

$$\frac{W_t}{P_t} = Z_t F_2(K_t, Z_t N_t). \tag{4}$$

When prices are flexible, firm labor demand in Equation (4) only depends on the level of the capital stock K_t and technology Z_t , neither of which changes in response to higher uncertainty. Through the production function, higher labor supply with unchanged capital and technology means that output must rise. Higher output with lower consumption implies that investment must rise via the national income accounts identity. Thus, higher uncertainty under flexible prices lowers consumption but causes an expansion in output, investment, and hours worked.

The Sticky Price Model

Uncertainty shocks can easily generate comovement by adding countercyclical markups through sticky prices⁶:

Equation (1) to (4) revisited: When prices adjust slowly, by contrast, aggregate demand determines output in the short run, which reverses the causal ordering of these equations. Higher uncertainty reduces the demand for consumption goods, which lowers output directly in Equation (1). Lower output reduces the benefit to owning capital, since the marginal revenue product of capital falls. The decline in the desired capital stock is reflected in a lower level of investment. Since consumption and investment both fall, output and hours worked both decline, since labor is the only input to production that can change in response to higher uncertainty.

In sticky price models, equation (4) can be written as:

$$\frac{W_t}{P_t} = \frac{1}{\mu_t} Z_t F_2(K_t, Z_t N_t). \quad (5)$$

where μ_t is markup in price over marginal cost. Following previous paragraph, mechanically, precautionary labor supply reduces firm marginal cost, which increases the markup when prices are sticky. Thus, equilibrium hours worked may fall as a result of the shifts in labor supply and labor demand, when firm markups increase enough to produce a decrease in equilibrium hours worked in response to a rise in uncertainty.

More specifically, the model features optimizing households and firms and a central bank that follows a Taylor rule to stabilize inflation and offset adverse shocks. We allow for sticky prices using the quadratic-adjustment-cost specification of Rotemberg (1982). (The derivation of the model is attached as in Appendix.B .)

⁶The authors document some empirical evidence in support for countercyclical markup, although identifying movements in markups remains difficult, even in a setting where we can likely identify the true movements in macroeconomic aggregates.

Qualitative Results: Comovement

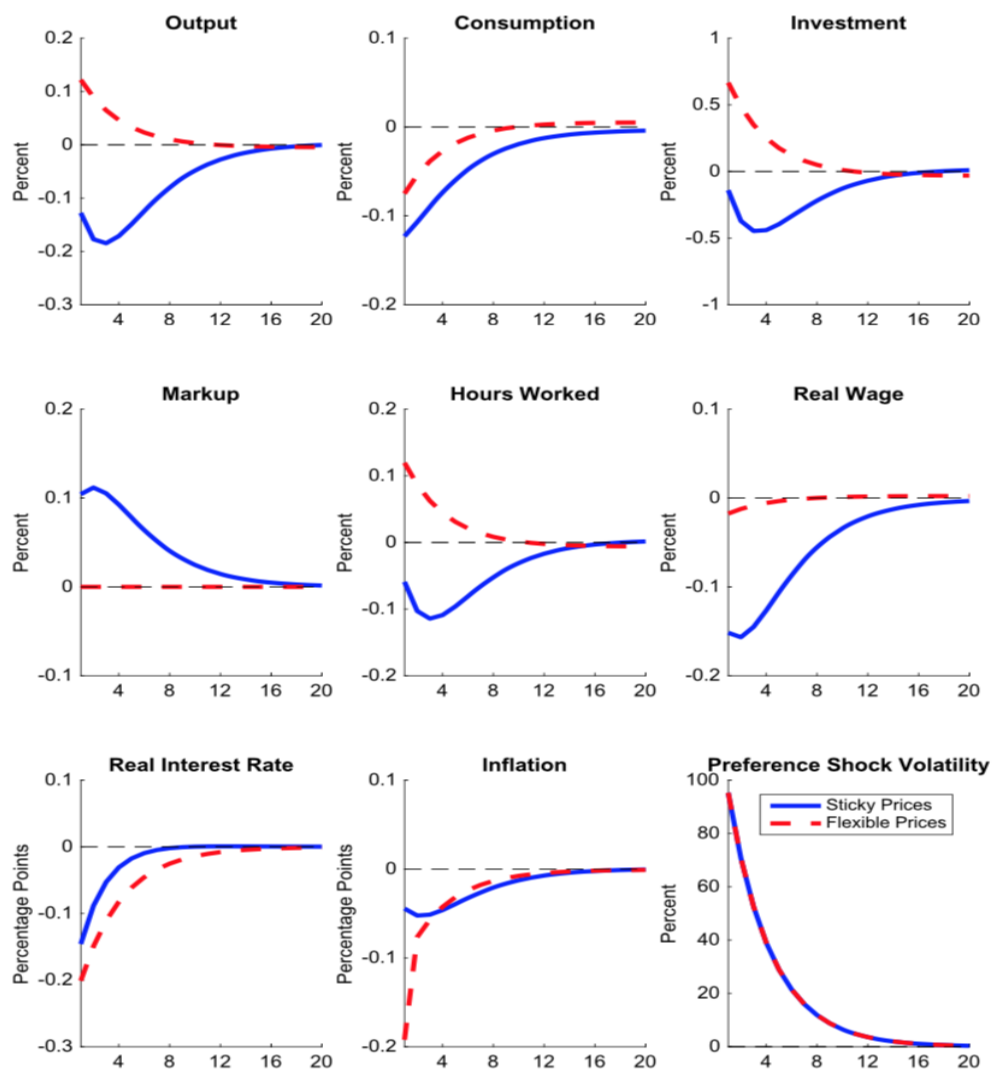


FIGURE 3.—Model-implied impulse responses under flexible and sticky prices.

Figure 3 plots the impulse responses of the model to a demand uncertainty shock under both flexible and sticky prices. Households want to consume less and save more when uncertainty increases in the economy. To save more, households would like to reduce consumption and increase hours worked. In a model where output is always at its flexible price or natural level, this higher desired saving translates into higher actual saving and investment rises. Higher labor supply with unchanged capital and technology means that output must rise. Through the national income accounting identity, higher output with lower consumption implies that investment must rise. Thus, higher uncertainty under flexible prices lowers consumption but

causes an expansion in output, investment, and hours worked. With sticky prices, however, aggregate demand determines output in the short run. Higher uncertainty reduces the demand for consumption goods, which lowers output directly. Lower output reduces the benefit to owning capital, since the marginal revenue product of capital falls. The decline in the desired capital stock is reflected in a lower level of investment. Since consumption and investment both fall, output and hours worked both decline, since labor is the only input to production that can change when the shock is realized. Finally, Figure 3 shows that consumption falls further when prices are sticky. The slow adjustment of prices creates a prolonged period of lower inflation, which raises the real interest rate relative to the flexible-price benchmark and further depresses consumption.

Quantitative Results

TABLE II
EMPIRICAL AND MODEL-IMPLIED VOLATILITY IN
MACROECONOMIC AGGREGATES^a

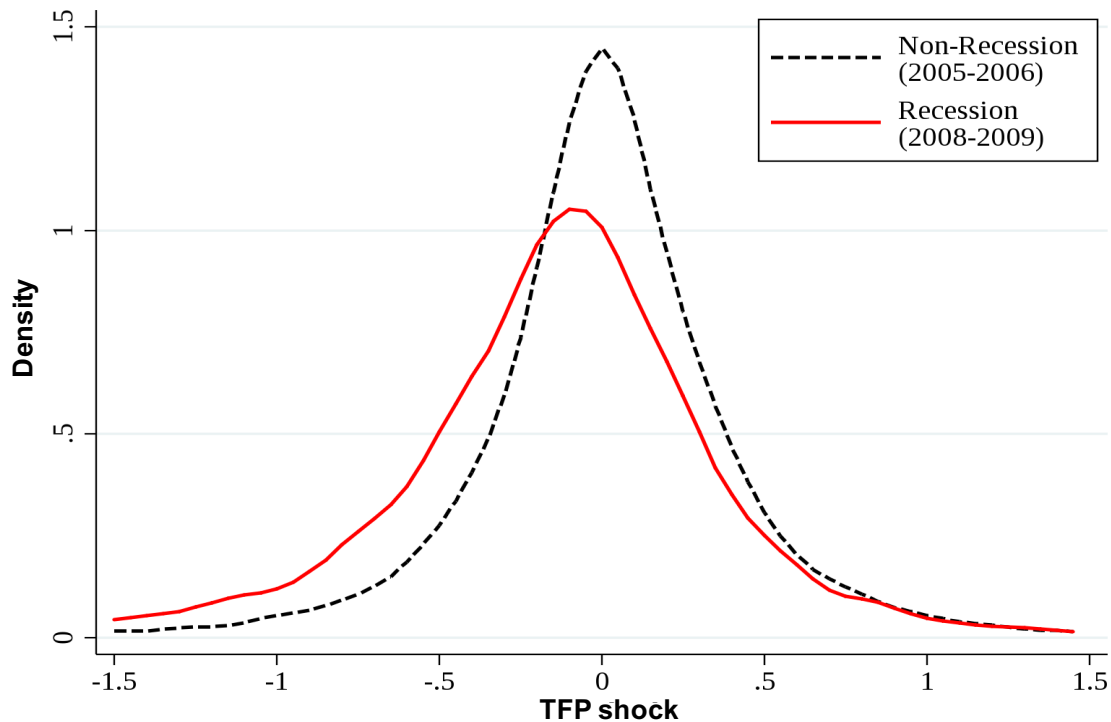
Moment	Percent		Relative to Output	
	Data	Model	Data	Model
Unconditional Volatility				
Output	1.1	1.0	1	1
Consumption	0.7	0.8	0.6	0.7
Investment	3.8	4.7	3.4	4.5
Hours Worked	1.4	0.8	1.3	0.8
Stochastic Volatility				
Output	0.4	0.2	1	1
Consumption	0.2	0.2	0.5	0.7
Investment	1.6	1.2	3.6	5.0
Hours Worked	0.5	0.2	1.0	0.9

^aUnconditional volatility is measured with the sample standard deviation. We measure stochastic volatility using the standard deviation of the time-series estimate for the 5-year rolling standard deviation. The empirical sample period is 1986–2014. See Appendix A for additional details.

The model closely matches the volatility of output, consumption, and investment we observe in the data. As with many other standard macroeconomic models, however, the model does struggle to generate sufficient fluctuations in hours worked relative to output.

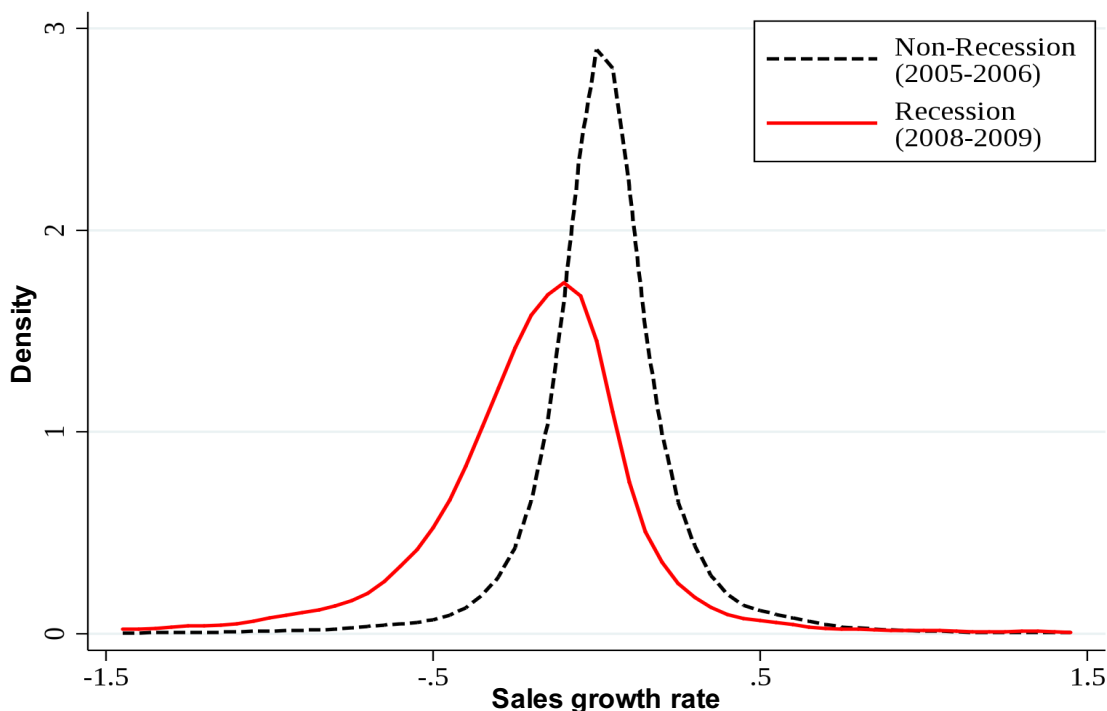
Appendix

Figure 1: The variance of establishment-level TFP shocks increased by 76% in the Great Recession



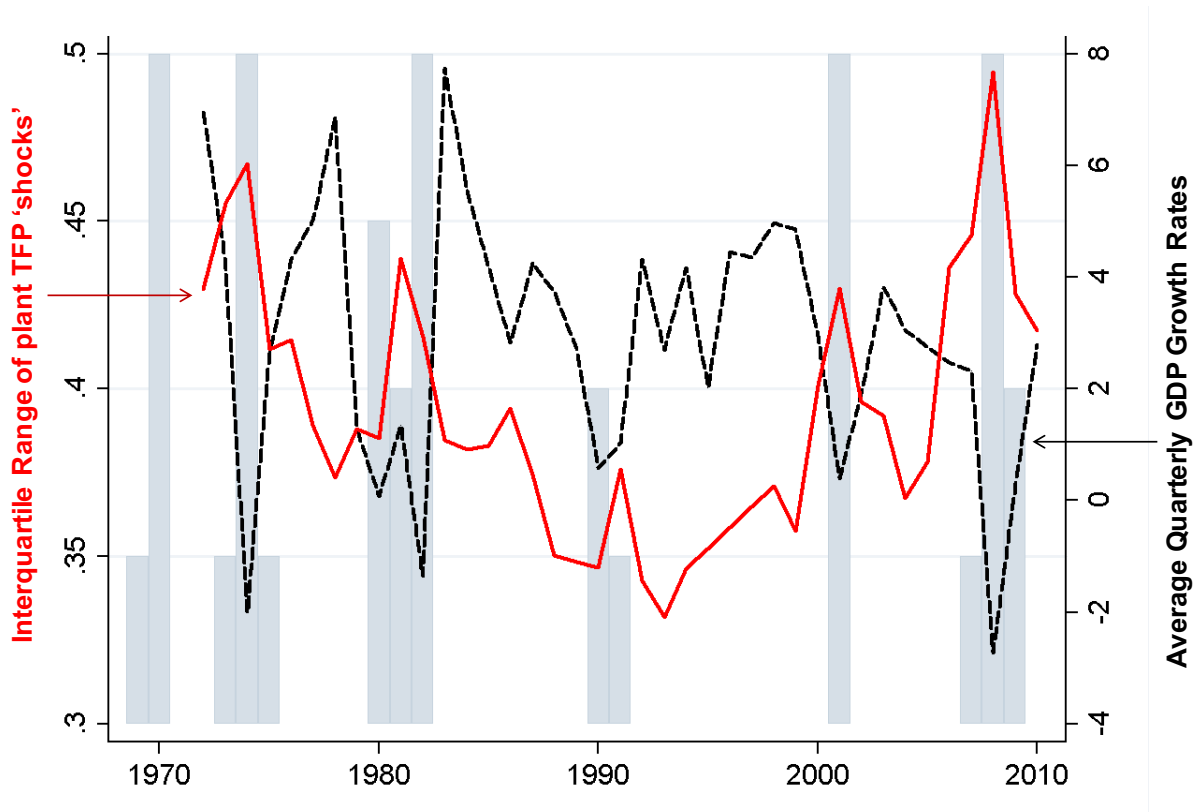
Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures using a balanced panel of 15,752 establishments active in 2005-06 and 2008-09. TFP Shocks are defined as residuals from a plant-level log(TFP) AR(1) regression that also includes plant and year fixed effects. Moments of the distribution for non-recession (recession) years are: mean 0 (-0.166), variance 0.198 (0.349), coefficient of skewness -1.060 (-1.340) and kurtosis 15.01 (11.96). The year 2007 is omitted because according to the NBER the recession began in 12/2007, so 2007 is not a clean "before" or "during" recession year.

Figure 2: The variance of establishment-level sales growth rates increased by 152% in the Great Recession



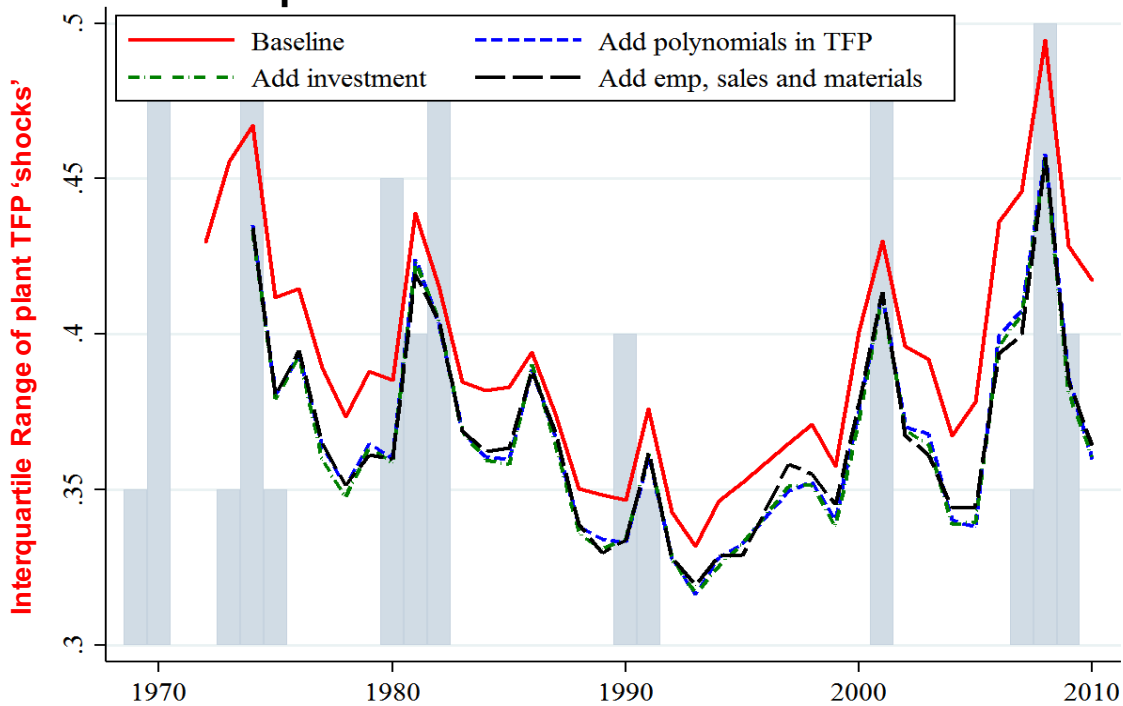
Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures using a balanced panel of 15,752 establishments active in 2005-06 and 2008-09. Moments of the distribution for non-recession (recession) years are: mean 0.026 (-0.191), variance 0.052 (0.131), coefficient of skewness 0.164 (-0.330) and kurtosis 13.07 (7.66). The year 2007 is omitted because according to the NBER the recession began in 12/2007, so 2007 is not a clean "before" or "during" recession year.

Figure 3: TFP 'shocks' are more dispersed in recessions



Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures establishments, using establishments with 25+ years to address sample selection. Grey shaded columns are the share of quarters in recession within a year.

Figure 4: Robustness test: different measures of TFP 'shocks' are all more dispersed in recessions



Notes: Constructed from the Census of Manufactures and the Annual Survey of Manufactures establishments, using establishments with 25+ years to address sample selection. Grey shaded columns are share of quarters in recession within a year. The four lines are: *Baseline*: Interquartile Range of plant TFP 'shocks' (as in Figure 3). *Add polynomials in TFP*: includes the first, second and third lags of log TFP, and their 5 degree polynomials in the AR regression which is used to recover TFP shocks. *Add investment*: includes all the controls from the previous specification plus the first, second and third lags of investment rate, and their 5 degree polynomials. *Add emp, sales and materials*: includes all the controls from the previous specification plus the second and third lags of log employment, log sales, and log materials, as well as their 5 degree polynomials.

The baseline model shares many features of the models of Ireland (2003), Ireland (2011), and Jermann (1998). The model features optimizing households and firms and a central bank that follows a Taylor rule to stabilize inflation and offset adverse shocks. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). Our baseline model considers technology and household discount rate shocks. The discount rate shocks have a time-varying second moment, which we interpret as the degree of uncertainty about future demand.

B.1. Households

In our model, the representative household maximizes lifetime utility given Epstein–Zin preferences over streams of consumption C_t and leisure $1 - N_t$. The key parameters governing household decisions are its risk aversion σ over the consumption-leisure basket and its intertemporal elasticity of substitution ψ . The parameter $\theta_V \triangleq (1 - \sigma)(1 - 1/\psi)^{-1}$ controls the household's preference for the resolution of uncertainty. The household receives labor income W_t for each unit of labor N_t supplied to the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and holds equity shares S_t and one-period risk-less bonds B_t issued by representative intermediate goods firm. Equity shares have a price of P_t^E and pay dividends D_t^E for each share S_t owned. The risk-less bonds return the gross one-period risk-free interest rate R_t^R . The household divides its income from labor and its financial assets between consumption C_t and holdings of financial assets S_{t+1} and B_{t+1} to carry into next period. The discount rate of the household β is subject to shocks via the stochastic process a_t .

The representative household maximizes lifetime utility by choosing C_{t+s} , N_{t+s} , B_{t+s+1} , and S_{t+s+1} for all $s = 0, 1, 2, \dots$ by solving the following problem:

$$V_t = \max \left[a_t (C_t^\eta (1 - N_t)^{1-\eta})^{(1-\sigma)/\theta_V} + \beta (\mathbb{E}_t V_{t+1}^{1-\sigma})^{1/\theta_V} \right]^{\theta_V/(1-\sigma)}$$

subject to its intertemporal household budget constraint each period,

$$C_t + \frac{P_t^E}{P_t} S_{t+1} + \frac{1}{R_t^R} B_{t+1} \leq \frac{W_t}{P_t} N_t + \left(\frac{D_t^E}{P_t} + \frac{P_t^E}{P_t} \right) S_t + B_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\frac{\partial V_t}{\partial C_t} = \lambda_t, \tag{S.1}$$

$$\frac{\partial V_t}{\partial N_t} = \lambda_t \frac{W_t}{P_t}, \tag{S.2}$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\}, \tag{S.3}$$

$$1 = R_t^R \mathbb{E}_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \right\}, \tag{S.4}$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. Epstein–Zin preferences imply the following relationships:

$$\begin{aligned} \frac{\partial V_t}{\partial C_t} &= a_t V_t^{1-(1-\sigma)/\theta_V} \eta \frac{(C_t^\eta (1-N_t)^{1-\eta})^{(1-\sigma)/\theta_V}}{C_t}, \\ \frac{\partial V_{t+1}}{\partial C_{t+1}} &= a_{t+1} V_{t+1}^{1-(1-\sigma)/\theta_V} \eta \frac{(C_{t+1}^\eta (1-N_{t+1})^{1-\eta})^{(1-\sigma)/\theta_V}}{C_{t+1}}, \\ \frac{\partial V_t}{\partial C_{t+1}} &= \beta V_t^{1-(1-\sigma)/\theta_V} (\mathbb{E}_t V_{t+1}^{1-\sigma})^{1/\theta_V-1} \mathbb{E}_t \left\{ V_{t+1}^{-\sigma} \left(\frac{\partial V_{t+1}}{\partial C_{t+1}} \right) \right\} \\ &= \beta V_t^{1-(1-\sigma)/\theta_V} (\mathbb{E}_t V_{t+1}^{1-\sigma})^{1/\theta_V-1} \mathbb{E}_t \left\{ V_{t+1}^{-\sigma} a_{t+1} V_{t+1}^{1-(1-\sigma)/\theta_V} \eta \frac{(C_{t+1}^\eta (1-N_{t+1})^{1-\eta})^{(1-\sigma)/\theta_V}}{C_{t+1}} \right\}. \end{aligned}$$

Thus, we define the household stochastic discount factor M between periods t and $t+1$:

$$M_{t+1} \triangleq \left(\frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \right) = \left(\beta \frac{a_{t+1}}{a_t} \right) \left(\frac{C_{t+1}^\eta (1-N_{t+1})^{1-\eta}}{C_t^\eta (1-N_t)^{1-\eta}} \right)^{(1-\sigma)/\theta_V} \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{V_{t+1}^{1-\sigma}}{\mathbb{E}_t[V_{t+1}^{1-\sigma}]} \right)^{1-1/\theta_V}.$$

Using the stochastic discount factor, we can eliminate λ and simplify Equations (S.1)–(S.4):

$$\frac{1-\eta}{\eta} \frac{C_t}{1-N_t} = \frac{W_t}{P_t}, \quad (\text{S.5})$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ M_{t+1} \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\}, \quad (\text{S.6})$$

$$1 = R_t^R \mathbb{E}_t \{ M_{t+1} \}. \quad (\text{S.7})$$

Equation (S.5) represents the household intratemporal optimality condition with respect to consumption and leisure, and Equations (S.6) and (S.7) represent the Euler equations for equity shares and one-period risk-less firm bonds.

B.2. Intermediate Goods Producers

Each intermediate goods-producing firm i rents labor $N_t(i)$ from the representative household to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. The intermediate goods firms own their capital stocks $K_t(i)$, and face convex costs of changing the quantity of installed capital. Firms also choose the rate of utilization of their installed physical capital $U_t(i)$, which affects its depreciation rate. Each firm issues equity shares $S_t(i)$ and one-period risk-less bonds $B_t(i)$. Firm i chooses $N_t(i)$, $I_t(i)$, $U_t(i)$, and $P_t(i)$ to maximize firm cash flows $D_t(i)/P_t(i)$ given aggregate demand Y_t and price P_t of the finished goods sector. The intermediate goods firms all have the same constant returns to scale Cobb–Douglas production function, subject to a fixed cost of production Φ and their level of productivity Z_t .

Each firm producing intermediate goods maximizes discounted cash flows using the household's stochastic discount factor:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\partial V_t / \partial C_{t+s}}{\partial V_t / \partial C_t} \right) \left[\frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} Y_t \leq [K_t(i)U_t(i)]^\alpha [Z_t N_t(i)]^{1-\alpha} - \Phi,$$

and subject to the capital accumulation equation:

$$K_{t+1}(i) = \left(1 - \delta(U_t(i)) - \frac{\phi_K}{2} \left(\frac{I_t(i)}{K_t(i)} - \delta \right) \right) K_t(i) + I_t(i),$$

where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta_\mu} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

and depreciation depends on utilization via the following functional form:

$$\delta(U_t(i)) = \delta + \delta_1(U_t(i) - U) + \left(\frac{\delta_2}{2} \right) (U_t(i) - U)^2.$$

The behavior of each firm i satisfies the following first-order conditions:

$$\begin{aligned} \frac{W_t}{P_t} N_t(i) &= (1 - \alpha) \Xi_t [K_t(i)U_t(i)]^\alpha [Z_t N_t(i)]^{1-\alpha}, \\ \frac{R_t^K}{P_t} U_t(i) K_t(i) &= \alpha \Xi_t [K_t(i)U_t(i)]^\alpha [Z_t N_t(i)]^{1-\alpha}, \\ q_t \delta'(U_t(i)) U_t(i) K_t(i) &= \alpha \Xi_t [K_t(i)U_t(i)]^\alpha [Z_t N_t(i)]^{1-\alpha}, \\ \phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[\frac{P_t}{\Pi P_{t-1}(i)} \right] & \\ &= (1 - \theta_\mu) \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} + \theta_\mu \Xi_t \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu - 1} \\ &+ \phi_P \mathbb{E}_t \left\{ M_{t+1} \frac{Y_{t+1}}{Y_t} \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right] \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} \frac{P_t}{P_t(i)} \right] \right\}, \\ q_t &= \mathbb{E}_t \left\{ M_{t+1} \left(U_{t+1}(i) \frac{R_{t+1}^K}{P_{t+1}} + q_{t+1} \left(1 - \delta(U_{t+1}(i)) - \frac{\phi_K}{2} \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} - \delta \right) \right)^2 \right. \right. \\ &\left. \left. + \phi_K \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} - \delta \right) \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) \right) \right\}, \end{aligned} \tag{S.8}$$

$$\frac{1}{q_t} = 1 - \phi_K \left(\frac{I_t(i)}{K_t(i)} - \delta \right),$$

where Ξ_t is the marginal cost of producing one additional unit of intermediate good i , and q_t is the price of a marginal unit of installed capital. R_t^K/P_t is the marginal revenue product per unit of capital services $K_t U_t$, which is paid to the owners of the capital stock. Our adjustment cost specification is similar to the specification used by [Jermann \(1998\)](#) and allows Tobin's q to vary over time.

Each intermediate goods firm finances a percentage ν of its capital stock each period with one-period risk-less bonds. The bonds pay the one-period real risk-free interest rate. Thus, the quantity of bonds $B_t(i) = \nu K_t(i)$. Total firm cash flows are divided between payments to bond holders and equity holders as follows:

$$\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu \left(K_t(i) - \frac{1}{R_t^R} K_{t+1}(i) \right).$$

Since the [Modigliani and Miller \(1958\)](#) theorem holds in our model, leverage does not affect firm value or optimal firm decisions. Leverage makes the payouts and price of equity more volatile and allows us to define a concept of equity returns in the model. We use the volatility of equity returns implied by the model to calibrate our uncertainty shock processes in Section 6.

B.3. Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{(\theta_\mu - 1)/\theta_\mu} di \right]^{\theta_\mu/(\theta_\mu - 1)} \geq Y_t.$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t . The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di,$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} Y_t.$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_\mu} di \right]^{1/(1-\theta_\mu)}.$$

B.4. Equilibrium

The assumption of Rotemberg (1982) (as opposed to Calvo (1983)) pricing implies that we can model our production sector as a single representative intermediate goods-producing firm. In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$, employ the same amount of labor $N_t(i) = N_t$, and choose the same level of capital and utilization rate $K_t(i) = K_t$ and $U_t(i) = U_t$. Thus, all firms have the same cash flows and payout structure between bonds and equity. With a representative firm, we can define the unique markup of price over marginal cost as $\mu_t = 1/\Xi_t$ and gross inflation as $\Pi_t = P_t/P_{t-1}$.

B.5. Monetary Policy

We assume a cashless economy where the monetary authority sets the net nominal interest rate r_t to stabilize inflation and output growth. Monetary policy adjusts the nominal interest rate in accordance with the following rule:

$$r_t = r + \rho_\pi(\pi_t - \pi) + \rho_y \Delta y_t, \quad (\text{S.9})$$

where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, and $\Delta y_t = \ln(Y_t/Y_{t-1})$. Changes in the nominal interest rate affect expected inflation and the real interest rate. Thus, we include the following Euler equation for a zero net supply nominal bond in our equilibrium conditions:

$$1 = R_t \mathbb{E}_t \left\{ M_{t+1} \left(\frac{1}{\Pi_{t+1}} \right) \right\}. \quad (\text{S.10})$$

B.6. Shock Processes

The demand and technology shock processes are parameterized as follows:

$$\begin{aligned} a_t &= (1 - \rho_a)a + \rho_a a_{t-1} + \sigma_a^a \varepsilon_t^a, \\ \sigma_t^a &= (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a} \sigma_{t-1}^a + \sigma^{\sigma^a} \varepsilon_t^{\sigma^a}, \\ Z_t &= (1 - \rho_Z)Z + \rho_Z Z_{t-1} + \sigma^Z \varepsilon_t^Z. \end{aligned}$$

ε_t^a and ε_t^Z are first-moment shocks that capture innovations to the level of the stochastic process for technology and household discount factors. We refer to $\varepsilon_t^{\sigma^a}$ as second-moment or ‘‘uncertainty’’ shock since it captures innovations to the volatility of the exogenous process for household discount factors. An increase in the volatility of the shock process increases the uncertainty about the future time path of household demand. All three stochastic shocks are independent, standard normal random variables.

B.7. Solution Method

Our primary focus is examining the effect of an increase in the second moment of the preference shock process. Using a standard first-order or log-linear approximation to the equilibrium conditions of our model would not allow us to examine second-moment shocks, since the approximated policy functions are invariant to the volatility of the shock processes. Similarly, second-moment shocks would only enter as cross-products with the other state variables in a second-order approximation, and thus we could not study the