Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity

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Highlight

- Macro model studying credit shock see also
  - as disturbance to asset collateral value (Jermann & Quadrini 12')
  - with rich firm heterogeneity
  - qualitatively different recession from tfp-driven ones
- Firm dynamic model with see also
  - real and financial frictions
  - inefficient capital allocation
  - non-trivial macroeconomic effects
- First DSGE model combining
  - firm heterogeneity
  - real frictions
  - financial frictions (Kiyotaki & Moore 97')
- Numerical method of independent merit

Failure of Neoclassical Investment Model

- A standard neoclassical firm's problem:

$$max \quad k_{it}^{\alpha} - i_{i,t} - \frac{1}{2}\phi(i_{it}/k_{it})^2 k_{it} + \frac{1}{1+r} v(k_{it+1})$$

$$s.t. \quad k_{it+1}^{\alpha} = (1-\delta)k_{it} + i_{i,t} \quad (multiplier : q_{it})$$

f.o.cs

$$q_{it} = v'(k_{it+1})$$

$$q_{it} = 1 + \phi(i_{it}/k_{it})$$

- Two implications of the q-theory model:
  1. q_{it} is the marginal value of capital to the firm;
  2. investment (ratio) is positively related to q_{it}:

$$i_{it}/k_{it} = \phi^{-1}(q_{it} - 1)$$

Failure of Neoclassical Investment Model

- Proxy for q (under constant returns):

$$v'(k_{it}) = \frac{v(k_{it})}{k_{it}}$$

$$q_{it} = \frac{v(k_{it+1})}{k_{it+1}} = \frac{1}{1+r}\sum_s \left(\frac{1-\delta}{1+r}\right)^s [\alpha k_{it+s+1}^{\alpha-1} + \phi_{it+s+1}]$$

- Empirical regression:

$$\frac{i_{it}}{k_{it}} = \alpha_i + \beta q_{it} + \mathbf{B} ctrvar_{it} + \varepsilon_{it}$$

- Failures of neoclassical investment model:
  - Coefficient β is estimated to be small and unstable;
  - Coefficients on *ctrvars*, especially cash flow, are large and significant.
- Lessons from failures of neoclassical investment model:
  - Real frictions (non-convex adjustment costs etc.) are important;
  - Financial frictions (borrowing constraints etc.) are important.

Frictionless Economy

Two-period model:

$$\max_{k_{i1},b_{i1}} d_{i0} + \frac{1}{R}E[d_{i1}]$$

$$d_{i0} = x_{i0} + \frac{1}{R}b_{i1} - k_{i1}$$

$$d_{i1} = z_{i1}k_{i1}^{\alpha} - b_{i1}$$

Solution (*MM theorem*):

$$k_{i1} = \left(\frac{\alpha E[z_{i1}]}{R}\right)^{\frac{1}{1-\alpha}}$$

$\rightarrow$ any finite b and d optimal
$\Rightarrow$ Frictionless model makes no prediction about financial variables

Financial Frictions

- Common frictions to equity finance:
  - Cannot raise new equity: $d_{i0} \geq 0$
  - Costly to raise new equity: pay some cost if $d_{i0} < 0$
  - Dividend adjustment cost: $\phi(d_{i0}, d^*)$
- Common frictions to debt finance:
  - Collateral constraint: $b_{i0} \leq$ (some measurement of) collateral value
  - Limited commitment: default risk $\rightarrow$ risk premium

$\Rightarrow$ Non-trivial effects of financial variables for investment!

- Frictions in this paper:

  a. (equity) cannot raise new equity: $d_{i0} \geq 0$

  b. (debt) collateral constraint: $b_{i0} \leq$ collateral value

Firm Heterogeneity $\{k, b, \varepsilon\}$

Firm Heterogeneity:

- k: predetermined capital
  - some degree of specificity
  - partial investment irreversibility
  - when $i > 0$, $k' = (1-\delta)k + i$
    when $i < 0$, $\theta_k k' = \theta_k(1-\delta)k + i$, $\theta_k < 1$
- b: constrained borrowing
  - current capital as collateral
  - taken specificity into account
  - borrowing constraint

$$b' \leq \zeta_l \theta_k k$$

- ε: idiosyncratic productivity
  - production function

$$y = z\varepsilon F(k, n)$$

  - persistent shocks to z
  - persistent shocks to ε

Immediate Messages

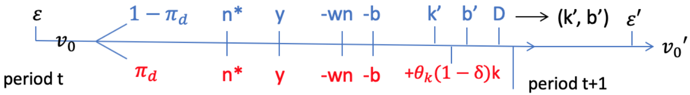
Frictions[1] + Heterogeneity:

- (real) partial irreversibility:
  - lumpiness: frequency of large investment
  - persistence: positive auto-corr of investment
  - investment rules of (S,s) type
- (real) partial irreversibility+ idiosyncratic shocks: large but unproductive firms cannot adjust to optimal level
- (financial) borrowing constraint + idiosyncratic shocks: small but productive firms cannot adjust to optimal level $\Rightarrow$ disproportionate capital stock to productivity.
- Does such misallocation amplify credit shock?

[1] There is no frictions in labor market: so that same $(k, \varepsilon) \rightarrow$ same (n,y).

Firm's Problem

Timeline:



Expected value *before* the beginning of each period:

$$v_0(k,b,\varepsilon;s,\mu) = (1-\pi_d)v(k,b,\varepsilon;s,\mu) + \pi_d \max_n [z\varepsilon F(k,n) - \omega(s,\mu)n + \theta_k(1-\delta)k - b] \quad (1)$$

Value of continuation at the beginning of each period:

$$v(k,b,\varepsilon;s,\mu) = \max\{v^u(k,b,\varepsilon;s,\mu), v^d(k,b,\varepsilon;s,\mu)\} \quad (2)$$

Firm's Problem

Upward Adjusting Firm:

$$v^u(k, b, \varepsilon; s, \mu) = \max_{n,k',b',D} [D + E_{s'} d_{s'} E_{\varepsilon'} v_0(k', b', \varepsilon'; s', \mu')] \qquad (3)$$

s.t.

$$k' \geq (1-\delta)k$$

$$b' \leq \zeta_l \theta_k k$$

$$D = z\varepsilon F(k,n) - \omega(s,\mu)n + q(s,\mu)b' - b - [k' - (1-\delta)k] \geq 0$$

$$\mu' = \Gamma(s,\mu)$$

Firm's Problem

Downward Adjusting Firm:

$$v^d(k, b, \varepsilon; s, \mu) = \max_{n,k',b',D} [D + E_{s'} d_{s'} E_{\varepsilon'} v_0(k', b', \varepsilon'; s', \mu')] \qquad (4)$$

s.t.

$$k' \leq (1-\delta)k$$

$$b' \leq \zeta_l \theta_k k$$

$$D = z\varepsilon F(k, n) - \omega(s, \mu)n + q(s, \mu)b' - b - \theta_k[k' - (1-\delta)k] \geq 0$$

$$\mu' = \Gamma(s, \mu)$$

Household's Problem

Utility Function:

$$V^h(\lambda, \phi; s, \mu) = \max_{c, n^h, \phi', \lambda'} [U(c, 1 - n^h) + \beta E_{s'} V^h(\lambda', \phi'; s', \mu')] \qquad (5)$$

s.t.

$$c + q\phi' + \int_S \rho_1 \lambda'(d[k' \times b' \times \varepsilon']) \leq [\omega n^h + \phi + \int_S \rho_0 \lambda(d[k \times b \times \varepsilon])]$$

$$\mu' = \Gamma(s, \mu)$$

where: current share holding: λ, value of current share: ρ_0;
where: matured bond: ϕ;
where: future share holding: λ', value of current share: ρ_1;
where: future bond: ϕ', bond price: $1/q$.

$\Rightarrow C^h(\lambda, \phi; s, \mu)$; $N^h(\lambda, \phi; s, \mu)$; $\Phi^h(\lambda, \phi; s, \mu)$; $\Lambda^h(k', b', \varepsilon'; \lambda, \phi; s, \mu)$

Recursive Equilibrium

Market Clearing Conditions:

$$\Lambda^h(k', b', \varepsilon'; \lambda, \phi; s, \mu) = \mu'(k', b', \varepsilon'; s, \mu)$$

$$N^h(\lambda, \phi; s, \mu) = \int_S [N(k, \varepsilon; s, \mu)]\mu(d[k \times b \times \varepsilon])$$

$$C^h(\lambda, \phi; s, \mu) = \int_S [y - (1 - \pi_d)IC + \pi_d(\theta_k(1 - \delta)k - k_0)]\mu(d[k \times b \times \varepsilon])$$

$$\Phi^h(\lambda, \phi; s, \mu) = \int_S [B(k, b, \varepsilon; s, \mu)]\mu(d[k \times b \times \varepsilon])$$

Solving the Heterogeneous Model

Outline:

- Subsume household's problem into the firm's problem
  - replacing prices of labor, bond, output and discount factors

- Solve firm's decision rules on dividend, capital and debt
  - sorting firms to two types: constrained and unconstrained
  - constrained firms exposed to binding borrowing constraint
  - unconstrained firms permanently free from borrowing constrained

- Krusell-Smith algorithm to solve the problem numerically
  - nonlinear, iterative and computationally intensive
  - we do have better algorithm now

Subsume household's problem into the firm's problem

Step 1

- output price[2]:

$$p(s,\mu) = D_1 U(C, 1-N) \tag{6}$$

- real wage: = MRS(c,n)

$$\omega(s,\mu) = \frac{D_2 U(C,1-N)}{D_1 U(C,1-N)} = \frac{D_2 U(C,1-N)}{p(s,\mu)} \tag{7}$$

- bond price: = expected gross real interest rate

$$q(s,\mu) = \frac{\beta E_s D_1 U(C',1-N')}{D_1 U(C,1-N)} = \frac{\beta E_s D_1 U(C',1-N')}{p(s,\mu)} \tag{8}$$

- firm's discount factor: consistent with MRSc,n

$$d(s,\mu) = \beta D_1 U(C',1-N')/D_1 U(C,1-N)$$

[2] We implicitly assume that firms discount by the same factor as households

Reformulate firm's problem

Step 2

- Expected value *before* the beginning of each period[3]:

$$V_0(k,b,\varepsilon;s,\mu) = (1-\pi_d)V(k,b,\varepsilon;s,\mu) + \pi_d \max_n p(s,\mu) \times [z\varepsilon F(k,n) - \omega(s,\mu)n + \theta_k(1-\delta)k - b] \quad (9)$$

- Expected value *at* the beginning of each period:

$$V(k,b,\varepsilon;s,\mu) = \max_{n,k',b',D} [p(s,\mu)D + \beta E_{s'} E_{\varepsilon'} V_0(k',b',\varepsilon';s',\mu')] \quad (10)$$

s.t.

$$D \geq 0$$

$$z\varepsilon F(k,n) - \omega n + qb' - b - J(k' - [1-\delta]k)[k' - (1-\delta)k] - D \geq 0 \quad (11)$$

$$\zeta_l \theta_k k - b' \geq 0 \quad (12)$$

[3] J(x)=1 if $x \geq 0$; $J(x) = \theta_k$ if $x < 0$;
and we exploit the fact that labor decision is static, independent of k' and b'.

Reformulate firm's problem

Step 2 (cont'd)

- Firms solve eq(9)-(12), taken $\{p, \omega, q\}$ as given
- Static labor choice:

$$z\varepsilon D_2 F(k, n^*) = \omega$$

- Profit:

$$\pi(k, b, \varepsilon; s, \mu) = z\varepsilon F(k, n^*) - \omega n^* - b \qquad (13)$$

- Determination of $[D, k', b']$
  - most challenging objects
  - sort firms into two types
  - constrained firms: D=0 $\leftrightarrow$ k' $\rightarrow$ b'
  - unconstrained firms: k' unaffected by borrowing limits

Unconstrained Firms

- Multiplier on borrowing constraints are zero
  $\rightarrow$ sufficient capital to circumvent collateral constraint
  $\rightarrow$ capital choice independent of financial position
- Indifferent b/w saving and dividends[4]
  $\rightarrow$ indifferent about b'
  $\rightarrow$ mv of firm's retained earning (saving) = household (p)
- b affecting value only through profit $\pi(k, b, \varepsilon; s, \mu)$

$$W(k, b, \varepsilon) = W(k, 0, \varepsilon) - pb$$

- Minimum saving policy:

$$B^w(k, \varepsilon; s, \mu) = \min_{\{\varepsilon_j | \pi_{ij} > 0 \ and \ s_m | \pi^s_{lm} > 0\}} \widetilde{B}\Big(K^w(k, \varepsilon), \varepsilon_j; s_m, \Gamma(s, \mu)\Big),$$

$$\widetilde{B}(k, \varepsilon; s, \mu) = z\varepsilon F(k, N(k, \varepsilon)) - \omega N(k, \varepsilon) + q \min\Big\{B^w(k, \varepsilon; s, \mu), \zeta\theta_k k\Big\}$$
$$-\mathcal{J}\Big(K^w(k, \varepsilon) - (1-\delta)k\Big)\Big[K^w(k, \varepsilon) - (1-\delta)k\Big]$$

---

[4] We have to impose additional assumptions on saving policy rule (minimum saving) to guarantee so in all future dates and states.

Unconstrained Firms

- Target capital stocks (k^*)

$$k_u^*(\varepsilon) = arg\max_{k'}[-pk' + \beta E_{s'}E_{\varepsilon'}W_0(k', \varepsilon'; s', \mu')] \tag{14}$$

$$k_d^*(\varepsilon) = arg\max_{k'}[-p\theta_k k' + \beta E_{s'}E_{\varepsilon'}W_0(k', \varepsilon'; s', \mu')] \tag{15}$$

- Capital decision rule: (S, s) form

$$K^w(k, \varepsilon; s, \mu) = \begin{cases} k_u^*(\varepsilon; s, \mu), & \text{if } k_u^* > (1-\delta)k \\ (1-\delta)k, & \text{if } k_u^* < (1-\delta)k < k_d^* \\ k_d^*(\varepsilon; s, \mu), & \text{if } k_d^* < (1-\delta)k \end{cases} \tag{16}$$

- $D^w(k, b, \varepsilon; s, \mu)$ is implied given the decision rule for k and b.

Constrained Firms

- Value function of constraint firm:

$$V^c(k, b, \varepsilon; s, \mu) = max\{V^u(k, b, \varepsilon; s, \mu), V^d(k, b, \varepsilon; s, \mu)\} \quad (17)$$

- Given (k, ε), find a cut-off debt level where
  - non-negative investment is possible
  - borrowing constraint is not violated
  - avoid negative dividends
- max b with $k' = (1 - \delta)k$ and $D \geq 0$:

$$\hat{b} = q\zeta\theta_k k + z\varepsilon F(k, n^*) - \omega n^*$$

- $b > \hat{b} \rightarrow$ downward adjustment: $V^d(k, b, \varepsilon; s, \mu)$
- $b < \hat{b} \rightarrow$ upward adjustment: $V^u(k, b, \varepsilon; s, \mu)$

Distinction b/w Unconstrained and Constrained Firms

- If a firm can:
  - adopt capital rule of unconstrained firm
  - hold debt level within saving function
  - pay non-negative dividend
- The firm is indistinguishable from unconstrained firm with (k, ε)

$$\begin{aligned} V(k, b, \varepsilon; s, \mu) &= W(k, b, \varepsilon; s, \mu) \quad , iff \quad D^w(k, b, \varepsilon; s, \mu) \geq 0 \\ &= V^c(k, b, \varepsilon; s, \mu) \quad , otherwise \end{aligned} \tag{18}$$

Solve the Problem (K-S algorithm)

Step 3:

- Computational challenges
    - presence of investment irreversibility
    - collateral constraint
    - firm level productivity shocks
- Curse of dimensionality:
    - individual state variable: $\{k, b, \varepsilon\}$
    - necessity to track their joint distribution: μ
    - aggregate state variable: $\{s, \mu\} = \{z, \zeta; \mu\}$
    - high-dimensional object
- Approximation of aggregate state
    - $\{s, \mu\} \rightarrow \{s, m, \nu_1, \nu_2\}$
    - m: unconditional mean of capital
    - ν_1, ν_2: lagged indicators of credit crisis

Solve the Problem (K-S algorithm)

Step 3 (cont'd): In each iteration,

- solve value function in an inner loop
  - m' and p taken as given
  - interpolation of functions at knots of individual and aggregate states
  - piece-wise polynomial cubic splines at off-knots points
- solve quantity and prices at outer loop
  - over 10,000 simulations
  - using value functions from inner loop
  - using actual distribution of firms
- update forecasting rules for m' and p

Heterogeneous response to macro shocks

Gertler and Gilchrist (1994):

- Heterogeneous response to monetary shock
  - Do financial constraints amplify aggregate response to monetary policy?
- Test using cross-sectional implication: constrained firms more responsive
  - proxy for financial constraints with size [5]
- Sales + Inventory investment decline more for *small* firms following monetary tightening
- Small firms more bank dependent
  - large firms have more long-term debt + commercial paper
- Financial variables matters for cyclical response.

[5] Some recent works provide new/direct measurement.

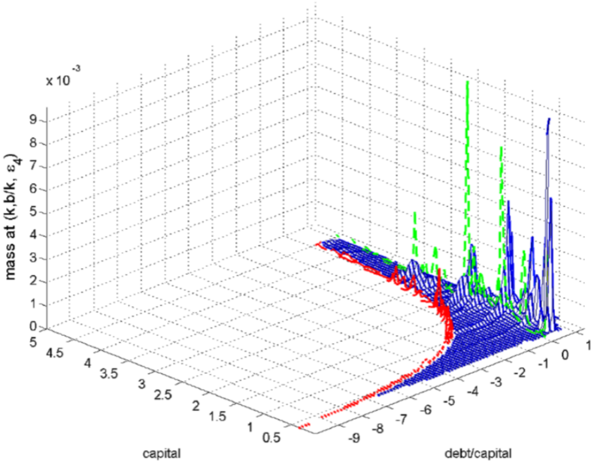
Heterogeneous response to macro shocks

Crouzet and Mehrotra (2017):

- Heterogeneous response to business cycles
- Test using micro-data.
- Some evidence small firms are more sensitive.
  - small firms are more bank-dependent and have more short-term debt
  - small firms also have more short-term assets
- Different cyclical responsiveness for monetary shocks vs. recessions.
  - unimportant for aggregate dynamics
  - weighting of firms matters
- Cyclical sensitivity *not* driven by financial variables.

Steady State

FIGURE 1. Steady state distribution: median productivity



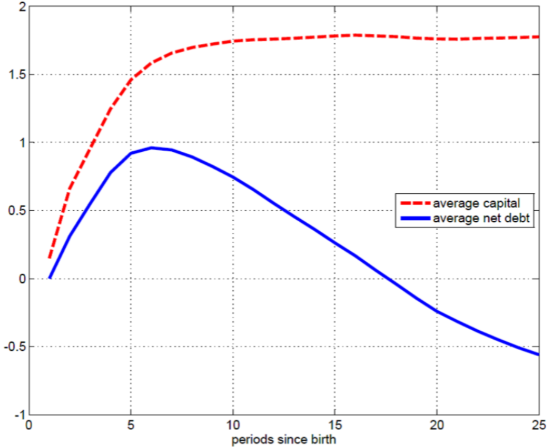
Steady State

- Inverse relation b/w firm's capital stocks and their financial savings
  - unconstrained, older, wealthier firms → minimum saving policy
  - constrained firms have lower capital or lower saving
  - no-constraint[6] firms adopt b/k levels in proportion to k (*assumed*)
- Entrants with common $\Phi(\varepsilon)$ but low (b, k)
  - absence of borrowing constraint → jump to k^u with same ε
  - with borrowing constraint → gradual adjustment of k
  - borrow to grow at maximum = binding borrowing constraint
  - long survival = unconstrained firms
- Firm dynamics
  - firm size distribution is right-skewed
  - age ↑ → employment growth ↓
  - larger and older firms pay more dividends
  - "age effects"

[6] We identify no-constraint firms as a type that never faces borrowing constraint.

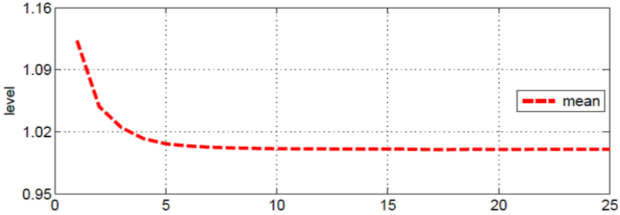
An Aside: Life-cycle of Firms (Age Effect)

FIGURE 2. Cohort in steady state



Steady State: Misallocation

FIGURE 3. Cohort Expected Discounted Return to Capital in Steady State



- Mis-allocated capital stock:
  - k of young (constrained) firms < k of old (unconstrained) firms
  - should be "=" absent financial frictions
  - old firms do not carry excess capital
  - young, small firms carry too little

Business Cycle: Benchmark vs. Full Economy

TABLE 2. BUSINESS CYCLES IN THE FULL ECONOMY

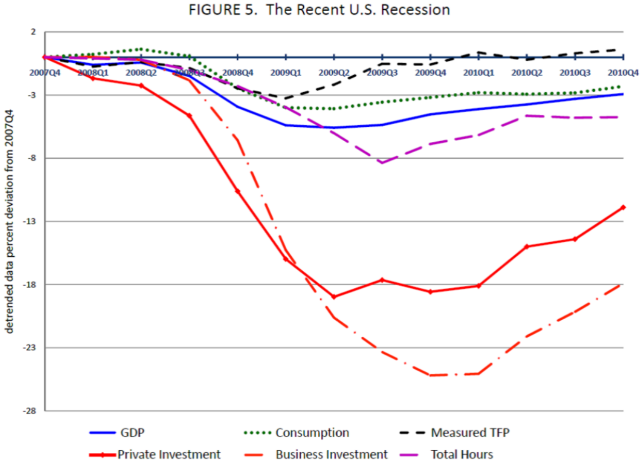
$x =$	Y	C	I	N	K	r
mean(x)	0.578	0.485	0.094	0.333	1.323	0.042
σ_x/σ_Y	(2.046)	0.514	4.106	0.599	0.517	0.467
$corr(x, Y)$	1.000	0.880	0.945	0.914	0.094	0.657

TABLE 3. BUSINESS CYCLES WITHOUT CREDIT SHOCKS

$x =$	Y	C	I	N	K	r
mean(x)	0.583	0.488	0.096	0.334	1.354	0.042
σ_x/σ_Y	(1.997)	0.503	3.860	0.562	0.485	0.453
$corr(x, Y)$	1.000	0.931	0.967	0.945	0.073	0.671

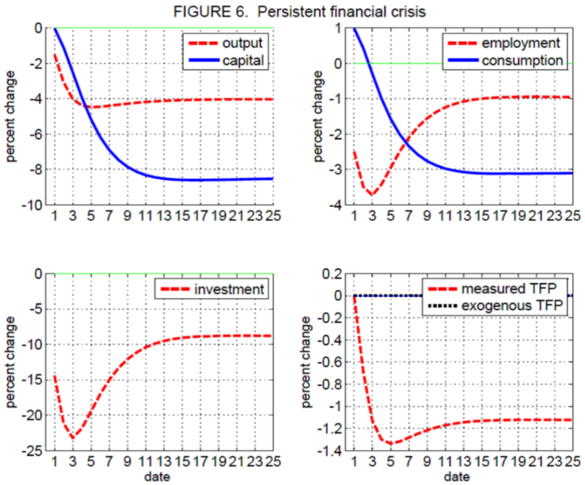
- Role of credit shocks (7 % of years):
  - reduce aggregate level of y, k and c
  - raise volatility of y, and relative volatility of c, i and n
  - weaken corr(X,y). X= [c, i, n]
  - real shocks dominates
  - more pronounced *conditional on occurrence*

Credit Crisis: Evidence



- Recent evidence in the crisis:
  - initial ↑ in [c] and ultimate ↓ in [y, n, i] unlike in RBC models
  - *noncontemporaneous* ↓ across [y, n, i, z] unlike in RBC models
  - sharp ↓ in [b] unlike in RBC models

Credit Crisis: Model



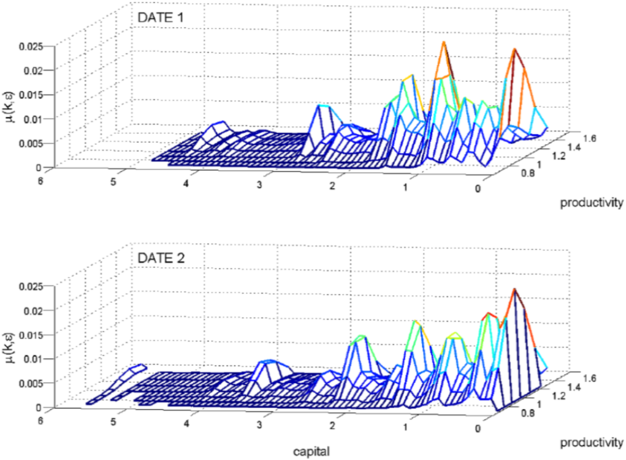
- An 88% drop in collateral value
    - 26% implied reduction in debt
    - expected duration: 3.2 yrs

Credit Crisis: Model

- Y $\downarrow$ immediately by 1.5%
    - capital predetermined
    - labor $\downarrow$ by 2.5% $\Leftarrow$ reduction in expected return to investment $\downarrow$
- consumption: $\uparrow \rightarrow \downarrow$
    - initial $\uparrow$: due to $\downarrow$ in return to saving
    - subsequent $\downarrow$: due to $\downarrow$ in n, y, w (as misallocation $\uparrow$)
- unconstrained firms $\rightarrow$ constrained firms
    - 17% constrained $\rightarrow$ 43% constrained
    - young firms: slower to catch up with their productivity
- TFP $\downarrow$: endogenous !

Credit Crisis: Misallocation

FIGURE 7. Persistent financial crisis: capital-productivity distribution



Credit Crisis: Misallocation

- # of medium-size firms $\downarrow$; small firms $\uparrow$ and very largest firms $\uparrow$
    - medium firm: unconstrained $\rightarrow$ constrained
    - small firm: takes longer to grow
    - largest firm: unconstrained, gain from $\downarrow$ r
- Increased efficiency from small firms
    - widened gap b/w expected investment return and interest rate
    - coexistence of $\uparrow$ in MPK and $\downarrow$ of ex post r
    - coexistence of $\uparrow$ in MPK of SME and $\downarrow$ of MPK of largest firms
- Reminiscent the finding of Eisfeldt and Rampini (06')
    - dispersion in returns to capital $\uparrow$ in recession;
    - benefit of capital reallocation $\uparrow$ in recession;
    - level of capital reallocation $\downarrow$ in recession
- Disproportionately negative impact on smaller and young firms

Wrap Up

- Firm dynamics
  - sensitivity to financial variables
  - heterogeneous response to shocks
  - "age effects" + "size effects"

- Heterogeneous firm DSGE model
  - persistent shock to $[z, \zeta; \varepsilon]$
  - heterogeneity on $[k, b, \varepsilon]$
  - real frictions: [partial irreversibility]
  - financial frictions: [borrowing constraint]

- Credit shocks qualitatively different from TFP shocks
  - gradual decline of output
  - initial rise in consumption
  - severe drop in investment, employment and GDP
  - endogenous decline of TFP
  - distribution of firms and misallocation of resources

Macro Models with Financial Shock

- Representative agent models:
  - Jermann and Quadrini (2012, AER)
  - Kiyotaki and Moore (2012)
- Heterogeneous agent models:
  - Khan and Thomas (2013, JPE)
  - Buera and Moll (2013, AEJ: Macro)
- Heterogeneous agent models with default:
  - Miao and Wang (2010)
  - Gomes, Jermann and Schmid (2016, AER)
  - Arellano, Bai and Kehoe (2016)
- Heterogeneous agent models with default and endogenous entry-exit:
  - Khan, Senga and Thomas (2016)
  - Ottonello and Winberry (2018, R&R at ECMA)
  - Gomes and Schimid (forthcoming, JF)

back

Firm Dynamic Models

- Productivity and Firm Dynamics
  - Hopenhayn (1992, ECMA)
  - Hopenhayn and Rogerson (1993, JPE)
  - Clementi and Palazzo (2016, AEJ:Macro)
- Investment and Firm Dynamics:
  - Caballero and Engel (1999, ECMA)
  - Khan and Thomas (2008, ECMA)
  - Winberry (2018, RnR at AER)
- Financial Friction and Firm Dynamics:
  - Cooley and Quadrini (2001, AER)
  - Gomes (2001, AER)
  - Clementi and Hopenhayn (2006, QJE)
  - Khan and Thomas (2013, JPE)
  - Ottonello and Winberry (2018, R&R at ECMA)

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