INFLATION DISAGREEMENTS AND THE TRANSMISSION OF MONETARY POLICY

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Abstract. Forecasters often disagree about inflation outlooks. We examine the implications of inflation disagreements for the transmission of monetary policy in a New Keynesian framework, generalized to incorporate heterogeneous beliefs about the central bank's inflation target. This tractable framework provides a microeconomic foundation for a discounted intertemporal Euler equation, mitigating the forward-guidance puzzle. The model implies that inflation disagreements weaken the macroeconomic effects of both forward guidance policy and the conventional interest rate policy. The model's mechanism and predictions are supported by empirical evidence.

I. Introduction

Survey data about inflation expectations reveal frequent time variations not only in the consensus (mean) but also in disagreements (dispersion) across individuals (Mankiw et al., 2003; Weber et al., 2022; Bhandari et al., 2024). Figure 1 shows the cross-sectional dispersion of inflation forecasts, measured by the interquartile range of inflation expectations over the one-year horizon across individual forecasters in the University of Michigan Survey of Consumers. Evidently, inflation disagreements fluctuate over time, spiking during the early periods of the global financial crisis and again during the post-pandemic period when inflation surged.

While the role of inflation expectations in monetary policy transmission has been well studied, less is known about the role of inflation disagreements. This paper studies how

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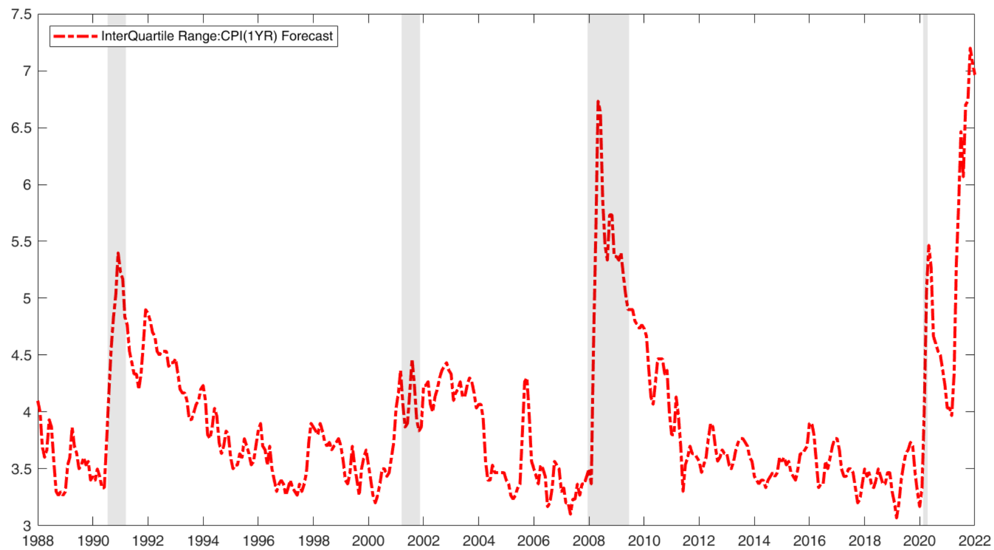

FIGURE 1. Inflation Forecast Dispersion from the Michigan Survey of Consumers.

This figure shows the time series of inflation disagreements, measured by the interquartile range (i.e., the differences between the 75th percentile and the 25th percentile) of inflation forecasts over the one-year horizon from Michigan Survey of Consumers. The mean, persistence and standard deviation of this time series from July 1991 to December 2022 are 4.062, 0.866 and 0.983 respectively.

inflation disagreements affect the transmission of conventional and unconventional monetary policy (such as forward guidance) in a tractable general equilibrium framework with heterogeneous inflation expectations.

We build on the standard New Keynesian framework and generalize it to incorporate belief heterogeneity. Agents in the model hold different beliefs about the central bank's inflation target, in part reflecting imperfect credibility of monetary policy. At a given nominal interest rate, an agent with a higher perceived inflation target also has a lower perceived real interest rate. Thus, the agent chooses to consume more or, equivalently, the agent has a high marginal propensity to consume (MPC). In contrast, an agent with a lower perceived inflation target will choose to save more, with a lower MPC. High-MPC agents finance consumption using both internal funds and external debt, subject to a borrowing constraint. In a state with greater inflation disagreements, there would be more agents lying the upper tail of the belief distribution, and those agents have high MPCs and thus face binding borrowing constraints.

Constrained agents cannot adjust consumption freely in response to external shocks.[1] Thus, greater disagreements about future inflation lead to more borrowing constrained agents; with more constrained agents, aggregate consumption adjusts less to changes in wealth or in the real interest rate. This mechanism provides a microeconomic foundation for a discounted Euler equation, which helps resolve the forward guidance puzzle.

The model mechanism relies on two key ingredients: heterogeneous beliefs and borrowing constraints. With more dispersed beliefs about future inflation, more agents would perceive a lower ex ante real interest rate and would thus hit the borrowing constraints. When a larger share of agents are borrowing constrained, aggregate consumption adjusts less to changes in both the current and the future real interest rate induced by conventional monetary policy or unconventional policy such as forward guidance.

The model mechanism is consistent with empirical evidence on the positive relations between individual inflation expectations and current consumption spending. Empirically, it is hard to identify causal effects of changes in inflation expectations on consumption spending. By exploiting a quasi-natural experiment in Germany and using a difference-in-differences approach, D'Acunto et al. (2021) document evidence that the announcement of value-added tax increases in 2005, to be implemented in 2007, raised German consumers' inflation expectations, leading to an immediate increase in consumers' readiness to buy durable goods. Crump et al. (2022) also find a positive relation between inflation expectations and current consumption using subjective inflation expectations data from the Federal Reserve Bank of New York's Survey of Consumer Expectations (SCE). Coibion et al. (2022) use a range of randomized information treatments in a large-scale survey of U.S. households to study how different types of communications affect consumers' inflation expectations and ultimately their spending decisions. They find that higher inflation expectations arising from information treatments lead to a rise in household spending on non-durable goods, although not on durable goods, over the next 6 months. Vellekoop and Wiederholt (2019) use the Dutch Household Survey data on inflation expectations, combined with administrative data, to show that households with higher inflation expectations are more likely to buy durable goods such as cars.[2]

[1] Increased inflation disagreements also mean that more agents would lie in the lower tail of the belief distribution. However, those agents can adjust consumption optimally in response to shocks because they do not face borrowing constraints.

[2] Bachmann et al. (2015) find a positive correlation between inflation expectation and willingness to spend among highly educated respondents and those who had inflation expectations close to the subsequent realization of inflation. However, those effects are absent in the full sample. Their findings highlight the role of cognitive abilities in determining the positive association between inflation expectations and the

The model mechanism is also consistent with the empirical evidence on the relation between household indebtedness and the effectiveness of monetary policy. In our model, inflation disagreements affect the transmission of monetary policy by changing the relative share of indebted agents. More dispersed inflation expectations lead to a larger share of credit-constrained agents, dampening the stimulating effects of monetary policy easing through rate cuts or forward guidance. This channel is consistent with Sufi (2015), who argues that monetary policy since the 2008-09 global financial crisis has been ineffective because policy easing channelled credit to heavily indebted households that are least likely to change their spending in response. In a related study, Alpanda and Zubairy (2019) use state-dependent local projections methods to show that monetary policy shocks have smaller effects on real activity such as GDP and consumption in a state with high household debt. Recent studies utilizing micro-data confirm that the inability of heavily indebted households to refinance mortgages has depressed spending following monetary stimulus during the Great Recession (Beraja et al., 2019; Cloyne et al., 2020).

Our model makes two main predictions. First, in a state with higher inflation disagreements, forward guidance policy would have more muted effects on consumption spending. With a discounted Euler equation, signaling a future reduction in the real interest rate through forward guidance would have a smaller effect on current consumption than does a reduction in the current real interest rate. The magnitude of the Euler-equation discounting increases with the magnitude of inflation disagreements. Absent inflation disagreement, the Euler equation in our model coincides with that in the standard model with no discounting. In that case, a decline in the real interest rate in arbitrarily distant future would have the same stimulus effect on current consumption as does a decline in the current real interest rate, giving rise to the forward-guidance puzzle (Del Negro et al., 2012; McKay et al., 2016). In the more general case with inflation disagreement, however, current consumption responds to expected future consumption less than one-for-one (and thus the Euler equation is "discounted"). Furthermore, the sensitivity of current consumption to future consumption declines monotonically with the magnitude of inflation disagreements as our analytical results reveal.

Second, the model predicts that inflation disagreements also dampen the effects of conventional monetary policy on aggregate consumption and inflation. Higher inflation disagreements lead to more muted effects following changes in the conventional interest rate policy.

willingness to purchase durable goods, as D'Acunto et al. (2023) show using Finnish administrative and survey-based micro data.

Both predictions from the model are supported by empirical evidence. We estimate a local projections model in the spirit of Jordà (2005) to examine how the responses of real activity to changes in forward guidance policy would depend on inflation disagreements. We measure inflation disagreement using the interquartile range (IQR) of inflation forecasts over the one-year horizon from the University of Michigan Survey of Consumers. We construct forward guidance shocks following the methodology in Swanson (2021) that builds on earlier work measuring monetary policy shocks using high-frequency asset price changes around FOMC announcements (Kuttner (2001), Gürkaynak et al. (2005)). Our montly sample covers the period from July 1991 to December 2022. We find that a forward guidance shock that signals tightening of future monetary policy leads to persistently lower industrial production and higher unemployment. However, consistent with our model, the recessionary effects are more muted in periods with high inflation disagreement.

We estimate a similar local projections model to examine how the responses of real activity to changes in conventional monetary policy shocks would depend on inflation disagreements. The construction of conventional monetary policy shocks also follows the methodology in Swanson (2021). Our estimation shows that a surprise federal funds rate tightening raises unemployment and lowers industrial production, but those recessionary effects are significantly muted in periods with high inflation disagreement. This again lends empirical support to the model's predictions.

Our paper contributes to the literature on the forward guidance puzzle. In the standard New Keynesian models with rational expectations, forward guidance policy that promises changes in interest rates in the distant future would have implausibly large effects on output and inflation relative to the effects of shocks to the current interest rate (Del Negro et al., 2012; Hagedorn et al., 2019). The existing studies have shown that the forward guidance puzzle can be alleviated by introducing information frictions (Angeletos and Lian, 2018), bounded rationality (Farhi and Werning, 2019; Gabaix, 2020), or imperfect central bank credibility (Andrade et al., 2019; Campbell et al., 2019). In an important contribution, McKay et al. (2016) show that, in a heterogeneous-agent New Keyesian (HANK) framework with incomplete markets, where agents face uninsurable income risks and liquidity constraints, a precautionary-savings effect partially offsets the intertemporal substitution effects, dampening the responses of current consumption to changes in future interest rates and therefore helps resolve the forward guidance puzzle (see also McKay et al. (2017)).

Our model highlights the importance of heterogeneity in household inflation expectations, a well-documented empirical fact in survey data (Mankiw et al., 2003; Andrade et al., 2016). Relative to the HANK model of McKay et al. (2016), our model generates heterogeneity in MPCs and Euler-equation discounting through a different channel. In our model with inflation disagreements, agents with higher inflation expectations have lower perceived real interest rates. Thus, they are more likely to be borrowing constrained, resulting in more sluggish adjustments in their consumption in response to forward-guidance shocks.

In the literature, the role of inflation expectations in monetary policy transmission has been well studied (Orphanides and Williams, 2004; Galí, 2015; Gargiulo et al., 2024), although less is known about the role of inflation disagreements. In a closely related work, Falck et al. (2021) study the implications of inflation disagreements on the transmission of monetary policy, focusing on the responses of inflation and inflation expectations to contemporaneous monetary policy shocks.[3] Our study has a different focus and a different model mechanism. We focus on the interactions of heterogeneous beliefs about the central bank's inflation target with borrowing constraints in generating inflation disagreements and thereby dampening the power of forward guidance. We find that our model's mechanism and predictions are supported by empirical evidence. To our knowledge, our model's heterogeneous-belief channel of monetary policy transmission is novel to the literature.

II. Model

II.1. **Forward Guidance Puzzle.** Assume that the monetary policy rule follows

$$R_{ft} = R_0 \Pi_t^* \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\varphi} \exp(\xi_t), \qquad \varphi > 1, \tag{1}$$

where R_0 is constant equal to the natural rate of real interest rate, Π_t^* is the inflation target of monetary authority, and ξ_t is monetary policy shock. We assume the true process of targeted inflation is

$$\Pi_{t+1}^* = \Pi_t^* \exp(\varepsilon_{t+1}), \tag{2}$$

where ε_{t+1} is a constant of 0.

Standard monetary policy models imply that news about future real interest rates at any horizon—however far in the future—has the same effect on current consumption as an equally-sized change to the current interest rate. To see this, we consider the standard Euler equation derived from textbook New Keynesian model with log-utility:

$$\frac{1}{C_t} = \beta R_{ft} E_t \frac{1}{C_{t+1}} \frac{1}{\Pi_{t+1}},$$

where C_t denotes real aggregate consumption at period t, R_{ft} denotes the risk-free nominal interest rate, $E_t \Pi_{t+1}$ denotes the expected one-period inflation rate, and β is the discount

[3]Barbera et al. (2023) further documents that the conditional effects of inflation disagreement on monetary policy transmission depend on the term-structure: Disagreement about the short-term inflation seems to attenuate monetary policy efficacy while disagreement about the long-term inflation does not.

factor. The log-linearized Euler equation is given by

$$\hat{C}_t = E_t\hat{C}_{t+1} - (\hat{R}_{ft} - E_t\hat{\Pi}_{t+1}),$$

where $\hat{C}_t$ denotes the log deviation of C_t from its steady state. Iterating the Euler equation forward, we obtain

$$\hat{C}_t = -\sum_{j=0}^{\infty} E_t(\hat{R}_{ft+j} - E_t\hat{\Pi}_{t+j+1}). \tag{3}$$

Note that there is no discount on the right-hand of equation (3), implying that (far) future policy rate change will have an implausibly large effect on current consumption.[4]

One way to fix this issue is to introduce a time-varying discounting factor. Now suppose that β_t is endogenous and depends on C_t, such that

$$\hat{C}_t = -\hat{\beta}_t + E_t\hat{C}_{t+1} - (\hat{R}_{ft} - E_t\hat{\Pi}_{t+1})$$

and

$$\hat{\beta}_t \equiv \frac{1-\rho}{\rho}\hat{C}_t, \tag{4}$$

where $\rho \in (0, 1)$. The 'discounted Euler equation' becomes

$$\hat{C}_t = \rho E_t\hat{C}_{t+1} - \rho(\hat{R}_{ft} - E_t\hat{\Pi}_{t+1}), \tag{5}$$

which implies that a future interest rate change has a smaller effect on current consumption compared to a current interest rate change of the same magnitude, i.e.

$$\hat{C}_t = -\rho\sum_{j=0}^{\infty} E_t\rho^j(\hat{R}_{ft+j} - E_t\hat{\Pi}_{t+j+1}), \quad \rho \in (0, 1)$$

We now provide a micro-foundation for this discounted Euler equation with a model featuring heterogeneous inflation expectations.

II.2. **A heterogeneous-agent model.** The model features a household family, consisting of a large number of members with heterogeneous beliefs about ε_{t+1}, such that

$$E_t^j\frac{\Pi^*_{t+1}}{\Pi^*_t} = e_{jt}, \tag{6}$$

where e_{jt} is drawn from a time-varying distribution with C.D.F. $G_t(e)$. In the beginning of each period t, all members of the family supply a homogeneous amount of labor and receive a homogeneous lump-sum transfer of net worth A_t. Afterwards, family members disperse to make individual consumption-saving decisions. As we shall see, household members with higher inflation expectations will choose to consume more today, potentially by borrowing, which would bring them closer to the liquidity constraint. By contrast, household members

[4]When inflation target is stochastic, an equivalent Euler equation can be derived from a rational expectation economic system scaled by π^*_t. We prove this in the Appendix V.1.

with lower inflation expectations will prefer to save more today and consume more in the future. The household's welfare is characterized by the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \log C_{jt} dj - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

where C_{jt} is consumption by agent j, and N_t is homogeneous labor supply. The family is subject to a budget constraint

$$A_t \leq \frac{\int_0^1 B_{jt} dj}{P_t} + \frac{W_t}{P_t} N_t + D_t, \tag{7}$$

where A_t is the homogeneous transfer to each family member, B_{jt} is agent j's net savings maturing at period t, P_t is the aggregate price level at period t, W_t is the wage rate, and D_t is the aggregate profit from production sector. Each individual receives the transfer from the household and chooses their consumption and savings based on their inflation expectation, subject to a budget constraint

$$C_{jt} + \frac{B_{jt+1}/R_{ft}}{P_t} \leq A_t, \tag{8}$$

and a liquidity constraint

$$\frac{B_{jt+1}/R_{ft}}{P_t} \geq -\bar{B} \tag{9}$$

where $\bar{B}$ is exogenous and cannot exceed A_t.

The first order condition with respect to the aggregate labor supply is given by

$$\Lambda_t \frac{W_t}{P_t} = \psi N_t^{\gamma}, \quad where \quad \Lambda_t = \int_0^1 \Lambda_{jt} dj = \int_0^1 \frac{1}{C_{jt}} dj, \tag{10}$$

and Λ_{jt} is the multiplier associated with the budget constraint (8) and equals the marginal utility from consumption.

The first-order condition with respect to nominal savings is

$$\frac{\Lambda_{jt}/R_{ft}}{P_t} = \beta \mathbb{E}_t^j \frac{\Lambda_{t+1}}{P_{t+1}} + \Omega_{jt} \frac{1/R_{ft}}{P_t}$$

where Ω_{jt} is the multiplier associated with the credit constraint (9). Multiplying both sides with $R_{ft} P_t$, we obtain that

$$\Lambda_{jt} = \beta R_{ft} \mathbb{E}_t^j \frac{\Lambda_{t+1}}{\Pi_{t+1}} + \Omega_{jt} \quad \forall j \tag{11}$$

Define $r_{ft} = R_{ft}/\Pi_t^*$ and $\pi_t = \Pi_t/\Pi_t^*$. The Euler equation of individual j, who believes that $\frac{\Pi_{t+1}^*}{\Pi_t^*} = e_{jt}$, can be written as

$$\Lambda_{jt} = \beta r_{ft} \mathbb{E}_t^j \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \frac{\Pi_t^*}{\Pi_{t+1}^*} \right] + \Omega_{jt} \tag{12}$$

Motivated by the equilibrium with a stochastic inflation target under rational expectations characterized in section V.1, in equilibrium $\mathbb{E}_t^j \frac{\Lambda_{t+1}}{\pi_{t+1}} = \mathbb{E}_t \frac{\Lambda_{t+1}}{\pi_{t+1}}$ for all j[5], which allows us to rewrite the Euler equation as

$$\Lambda_{jt} = \beta \frac{1}{e_{jt}} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] + \Omega_{jt}. \tag{13}$$

Define the belief of the marginal consumer (saver) as e_t^*, such that

$$\frac{1}{\bar{C}_t} = \frac{\beta}{e_t^*} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] \tag{14}$$

where $\bar{C}_t = A_t + \bar{B}$ is the maximum consumption level. It is then clear that $\Omega_{jt} > 0$ if and only if $e_{jt} > e_t^*$. Namely if the agent j believes that inflation target is higher in the next period, she would borrow to the maximum limit to consume.

For family members with $e_{jt} > e_t^*$, their Euler equation is

$$\Lambda_{jt} \equiv \frac{1}{C_{jt}} = \frac{\beta}{e_{jt}} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] + \Omega_{jt}, \quad \Omega_{jt} > 0 \quad \forall e_{jt} > e_t^*$$

In the presence of borrowing constraints, they are limited to the same consumption level:

$$C_{jt} = A_t + \bar{B} \equiv \bar{C}_t, \qquad \forall e_{jt} > e_t^*.$$

For family members with $e_{jt} \leq e_t^*$, we have $\Omega_{jt} = 0$, and their Euler equation is

$$\Lambda_{jt} \equiv \frac{1}{C_{jt}} = \frac{\beta}{e_{jt}} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right], \qquad \forall e_{jt} < e_t^*,$$

which implies that

$$C_{jt} = \frac{e_{jt}}{e_t^*} \bar{C}_t, \qquad \forall e_{jt} < e_t^*. \tag{15}$$

Note we can rewrite

$$\begin{aligned} \Lambda_t \quad &\equiv \quad \int_0^1 \Lambda_{jt} dj = \int_{e_t^*} \frac{1}{\bar{C}_t} dG(e) + \int^{e_t^*} \frac{e_t^*}{e} \frac{1}{\bar{C}_t} dG(e) \\ &= \quad \frac{1}{\bar{C}_t} [1 - G(e_t^*) + \int^{e_t^*} \frac{e_t^*}{e} dG(e)] \end{aligned} \tag{16}$$

and

$$\Lambda_{t+1} \equiv \frac{1}{\bar{C}_{t+1}} e_{t+1}^* F(e_{t+1}^*), \tag{17}$$

where $F(e_{t+1}^*)$ is defined as

$$F(e_{t+1}^*) = \left[\frac{1 - G(e_{t+1}^*)}{e_{t+1}^*} + \int_{e_{\min}}^{e_{t+1}^*} \frac{1}{e} dG(e) \right] \tag{18}$$

[5]The conjecture is proved in Appendix V.2.

Aggregating Eq. (8) implies that $A_t = C_t$ or $\bar{C}_t \equiv C_t + \bar{B}$. Equation (14) can be written as

$$\frac{1}{C_t + \bar{B}} = \frac{\beta}{e_t^*} r_{ft} \mathbb{E}_t \frac{1}{C_{t+1} + \bar{B}} \frac{1}{\pi_{t+1}} e_{t+1}^* F(e_{t+1}^*). \tag{19}$$

Log-linearizing equation (19) obtains

$$-\frac{C}{C+\bar{B}} \hat{C}_t = -\hat{e}_t^* + \hat{r}_{ft} - \mathbf{E}_t \hat{\pi}_{t+1} - \frac{C}{C+\bar{B}} E_t \hat{C}_{t+1} + E_t[1-\theta]\hat{e}_{t+1}^*,$$

where θ measures the (inverse) elasticity of $F()$ w.r.t. e^*, such that

$$\theta \equiv -\frac{F'(e^*)e^*}{F(e^*)} = \frac{1 - G(e^*)}{1 - G(e^*) + e^* \int_{e_{\min}}^{e^*} \frac{1}{e} dG(e)} \in [0, 1), \tag{20}$$

and $\theta = 0$ if and only if inflation expectation is homogeneous. After rearrangement, we have

$$\hat{C}_t - \frac{C+\bar{B}}{C} \hat{e}_t^* = E_t \hat{C}_{t+1} - \frac{C+\bar{B}}{C} E_t[1-\theta]\hat{e}_{t+1}^* - \frac{C+\bar{B}}{C} \left(\hat{r}_{ft} - \mathbf{E}_t \hat{\pi}_{t+1}\right). \tag{21}$$

Finally, we replace $\hat{e}_t^*$ with $\hat{C}_t$ using aggregate consumption condition:

$$C_t = (C_t + \bar{B}) \left[1 - G(e_t^*) + \int_{e_{\min}}^{e_t^*} \frac{e}{e_t^*} dG(e)\right],$$

or equivalently,

$$\frac{C_t}{C_t + \bar{B}} \equiv \Phi(e_t^*), \tag{22}$$

where $\Phi(e_t^*)$ is defined as

$$\Phi(e_t^*) \equiv \left[1 - G(e_t^*) + \frac{\int_{e_{\min}}^{e_t^*} e dG(e)}{e_t^*}\right].$$

Denote the (inverse) elasticity of $\Phi()$ to e^* as μ, such that

$$\mu \equiv -\frac{\Phi'(e^*)e^*}{\Phi(e^*)} = \frac{\int_{e_{\min}}^{e^*} e dG(e)}{[1 - G(e^*)]e^* + \int_{e_{\min}}^{e^*} e dG(e)} \in (0, 1]. \tag{23}$$

We have that $\mu = 1$ if and only if inflation expectation is homogeneous. We can derive $\hat{e}_t^*$ as a function of $\hat{C}_t$

$$\frac{\bar{B}}{C+\bar{B}} \hat{C}_t = -\mu \hat{e}_t^* \tag{24}$$

Plugging equation (24) into the Euler equation (21), we have

$$\hat{C}_t \left(1 + \frac{\bar{B}}{\mu C}\right) = E_t \hat{C}_{t+1}[1 + (1-\theta)\frac{\bar{B}}{\mu C}] - \frac{C+\bar{B}}{C} \left(\hat{r}_{ft} - \mathbf{E}_t \hat{\pi}_{t+1}\right)$$

Denoting the steady state loan-to-value ratio as $\kappa \equiv \frac{\bar{B}}{A} = \frac{\bar{B}}{C} \in (0, 1)$, we derive a *discounted Euler equation* as

$$\hat{C}_t = \underbrace{\frac{\mu + (1-\theta)\kappa}{\mu + \kappa}}_{\equiv \beta_1} \mathbb{E}_t \hat{C}_{t+1} - \underbrace{\frac{(1+\kappa)\mu}{\mu + \kappa}}_{\equiv \beta_2} \left(\hat{r}_{ft} - \mathbf{E}_t \hat{\pi}_{t+1}\right) \tag{25}$$

Ceteris paribus, a higher θ or lower μ will reduce the responsiveness of current aggregate consumption to future interest rates and future wealth changes. Lower μ will also weaken the effect of contemporaneous interest rate changes on consumption. Intuitively, aggregate consumption is less responsive to shocks when there is a larger mass of constrained household members, who do not adjust sufficiently to changes in wealth (i.e., changes in expected future consumption) or changes in the real interest rate.[6]

Labor market clearing yields

$$\Lambda_t W_t = \psi N_t^{\gamma}, \qquad \text{where} \quad \Lambda_t = \frac{1}{C_t + \bar{B}} e_t^* F(e_t^*),$$

which implies

$$\hat{W}_t = \frac{\mu + \kappa(1-\theta)}{(1+\kappa)\,\mu} \hat{C}_t + \gamma \hat{N}_t. \tag{26}$$

Aggregate production function and goods market clearing yield

$$\hat{C}_t = \hat{N}_t + \hat{Z}_t \tag{27}$$

The producer's optimal pricing condition yields the following Phillips curve equation[7]

$$\hat{\pi}_t = \varphi_y[\hat{W}_t - \hat{Z}_t] + \beta \mathbb{E}_t \hat{\pi}_{t+1}. \tag{28}$$

The monetary policy rule in equation (1) implies

$$\hat{r}_{ft} = \varphi \hat{\pi}_t + \xi_t \tag{29}$$

II.2.1. *Equilibrium.* Eq., (25)-(29) fully characterize a linearized system of $\{C_t, N_t, W_t, r_{ft}, \pi_t\}$, given initial interest rate R_0, aggregate productivity shock $\hat{Z}_t$ and monetary policy shock ξ_t.

II.2.2. *Results.* Now we prove that inflation disagreement can generate discount Euler equation in this simple framework. We first show that

Proposition II.1. *(Discounted Euler equation) The effect of interest rates change in the future on current consumption is discounted by a factor less than one.*

[6]Aggregate response of consumption to current and future interest rate change also depends on the mass of constrained household thus on degree of financial friction. In our model, conventional monetary policy and forward guidance are less effective to affect current consumption if κ is large, i.e. β_1 and β_2 are decreasing functions of κ. Intuitively when κ is large, debt capacity and consumption are concentrated at a small share of households with unrealistically high inflation expectation, who are insensitive to interest rate changes. This echoes empirical evidence that consumption response to monetary policy is large when there are a relatively large share of indebted household (Cumming and Hubert, 2023).

[7]The derivation is provided in Appendix V.3.

Proof. We prove this by showing $\beta_1 \equiv \frac{\mu+(1-\theta)\kappa}{\mu+\kappa} < 1$ in (25). Since we have shown that $\theta, \mu \in (0,1)$ from Eq. (20) and (23), it's immediate that

$$\beta_1 \equiv \frac{\mu + (1-\theta)\kappa}{\mu + \kappa} = 1 - \frac{\theta\kappa}{\mu + k} \in (0,1) \tag{30}$$

since $\kappa \in (0,1)$. □

Recall that in the standard New Keynesian framework *a la* section II.1, the intertemporal discount factor in linearized equation (3) equals 1. In our framework with heterogeneous expectation about future inflation, by contrast, the coefficient β_1 is less than 1. More generally, we can prove that endogenous discount factor is a decreasing function of dispersion in inflation expectation. To do so, we assume that the idiosyncratic beliefs of households follow a Pareto distribution, such that

$$G(e) = \begin{cases} 1 - (\frac{e_{min}}{e})^{\alpha} & \text{if } e \geq e_{min} \\ 0 & \text{if } e < e_{min} \end{cases} \tag{31}$$

We fix the mean $E(e) = 1$ by setting $e_{min} \equiv \frac{\alpha-1}{\alpha}$, such that the average expectation is rational.

Proposition II.2. *The effect of forward guidance on current consumption decreases with inflation disagreement. In other words, the endogenous discount factor β_1 in eq. (25) decreases with a mean-preserving dispersion of inflation expectations.*

Proof. In Appendix V.4, we prove in Lemma V.2 and V.3 that θ is a decreasing function of α and that μ is an increasing function of α. It's immediate that

$$\beta_1 \equiv 1 - \frac{\theta\kappa}{\mu + k}, \qquad \kappa \in (0,1)$$

is an increasing function of α, or that the endogenous discount factor in Eq. (25) decreases with the dispersion of inflation beliefs. □

In a representative-agent framework *a la* section II.1, the coefficient associated with contemporaneous interest rate surprises on consumption in Eq.(3) equals 1. By contrast, in our framework with heterogeneous beliefs about future inflation, the coefficient $\beta_2 \equiv \frac{(1+\kappa)\mu}{\mu+\kappa}$ is less than 1 since $\kappa, \mu \in (0,1)$. This suggests that inflation disagreement may reduce the effectiveness of contemporaneous interest rate surprises. This insight is formalized in the following proposition:

Proposition II.3. *The effect of a contemporaneous interest rate surprise on consumption decreases with inflation disagreement. In other words, β_2 in Eq. (25) decreases with a mean-preserving dispersion of inflation expectations.*

Proof. In Appendix V.4 we prove that μ is an increasing function of α. Since that

$$\beta_2 = \frac{(1+\kappa)(\mu+\kappa-\kappa)}{\mu+\kappa} = 1+\kappa-\frac{(1+\kappa)\kappa}{\mu+\kappa}, \qquad \kappa\in(0,1),$$

we have β_2 as an increasing function of α, or that effect of contemporaneous interest rate surprise on consumption decreases with inflation disagreement. □

The last proposition also implies that higher inflation disagreement weaken the effect of other demand shocks, e.g., a shock to the natural real interest rate.[8]

Finally, we evaluate the effectiveness of monetary policy in stabilizing inflation.

Proposition II.4. *The effectiveness of contemporaneous monetary surprise on inflation decreases with inflation disagreement.*

Proof. Inflation response to monetary policy shock (assuming that $\hat{Z}_t = 0$) can be derived as

$$\begin{aligned}
\hat{\pi}_t &= \varphi_y \hat{W}_t + \beta E_t\hat{\pi}_{t+1} \\
&= \varphi_y\left[\frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu}\hat{C}_t + \gamma\hat{N}_t\right] + \beta E_t\hat{\pi}_{t+1} \\
&= \varphi_y\left[\frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu}\hat{C}_t + \gamma\hat{C}_t\right] + \beta E_t\hat{\pi}_{t+1} \\
&= \varphi_y\left[(\gamma+\frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu})[\beta_1 E_t\hat{C}_{t+1} - \beta_2(\hat{r}_{ft} - \beta E_t\hat{\pi}_{t+1})]\right] + \beta E_t\hat{\pi}_{t+1} \\
&= \varphi_y\left[(\gamma+\frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu})[\beta_1 E_t\hat{C}_{t+1} - \beta_2(\varphi\hat{\pi}_t + \xi_t - \beta E_t\hat{\pi}_{t+1})]\right] + \beta E_t\hat{\pi}_{t+1} \\
&\equiv \varphi_y\left[\beta_3[\beta_1 E_t\hat{C}_{t+1} - \beta_2(\varphi\hat{\pi}_t + \xi_t - \beta E_t\hat{\pi}_{t+1})]\right] + \beta E_t\hat{\pi}_{t+1} \qquad (33)
\end{aligned}$$

where $\beta_3 \equiv \gamma + \frac{\mu+\kappa(1-\theta)}{(1+\kappa)\mu}$. Thus, we can write

$$\hat{\pi}_t = -\frac{\varphi_y\beta_3\beta_2}{(1+\varphi_y\beta_3\beta_2\varphi)}\xi_t + \frac{\varphi_y\beta_3\beta_1}{(1+\varphi_y\beta_3\beta_2\varphi)}E_t\hat{C}_{t+1} + \frac{\beta(1+\varphi_y\beta_3\beta_2)}{(1+\varphi_y\beta_3\beta_2\varphi)}E_t\hat{\pi}_{t+1}. \qquad (34)$$

It's sufficient to prove that $\frac{\varphi_y\beta_3\beta_2}{1+\varphi_y\beta_3\beta_2\varphi}$ increases with α, or that $\beta_3\beta_2$ increases with α.

$$\beta_3\beta_2 = \gamma\beta_2 + \frac{\mu+\kappa(1-\theta)}{\mu+\kappa} = \gamma\beta_2 + 1 - \frac{\kappa\theta}{\mu+\kappa}$$

---

[8] Introducing stochastic natural real interest rate (denoted as r_t^n) into the model obtains

$$\hat{C}_t = \underbrace{\frac{\mu+(1-\theta)\kappa}{\mu+\kappa}}_{\equiv\beta_1}\mathbb{E}_t\hat{C}_{t+1} - \underbrace{\frac{(1+\kappa)\mu}{\mu+\kappa}}_{\equiv\beta_2}(\hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1} - \hat{r}_t^n) \qquad (32)$$

where $\hat{r}_t^n$ denotes deviation of natural real interest rate from steady state. According to Prop. II.3 that β_2 is negative, it is implied that higher inflation disagreements weaken the effects of shocks to r_t^n.

We have proved that β_2 increases with α (Prop. II.3), that θ is a decreasing function of α (Lemma V.2), and that μ is an increasing function of α (Lemma V.3). It's immediate then that $\beta_3\beta_2$ increases with α. $\square$

III. Supporting Evidence for the Derived Discounted Euler Equation

III.1. **Inflation Disagreement and Effectiveness of Forward Guidance.** The first testable prediction derived from our discounted Euler equation is that the effectiveness of forward guidance shocks on current economic activity and inflation decreases with inflation disagreement. For this purpose, we estimate the impulse responses of monthly unemployment, industrial production and inflation to identified forward guidance shocks using the local projections approach of Jordà (2005) under the following empirical specification

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \sum_{i=0,1,2,3} \alpha^h_{1,i} FG_{t-i} + \sum_{i=1,2,3} \alpha^h_{i,2} IQR^{\pi}_{t-i} + \alpha^h_3 IQR^{\pi}_{t-1} * FG_t$$
$$+ \sum_{j=1,2,3} \sum_{i=1,2,3} \alpha^h_{4,i} \Delta \log(y^j_{t-i}) + \sum_{i=0,1,2,3} \alpha^h_{5,i} SFFR_{t-i} + \varepsilon_{t+h} \quad (35)$$

with $h = 0, 1, 2, ..., 36$. In this specification, $\log(y^j_{t+h}) - \log(y^j_{t-1})$ denotes cumulative log changes in the unemployment rate ($j = 1$), industrial production ($j = 2$) or the CPI price level ($j = 3$) from period $t-1$ to period $t+h$. FG_t is the forward guidance shock constructed from intraday interest rate changes around FOMC announcements following Swanson (2021), aggregated to the monthly frequency following time weighting method of Gertler and Karadi (2011). IQR^{π}_{t-i} is the demeaned interquartile range of inflation forecasts over a one-year horizon from the Michigan Survey of Consumers at month $t - i$. We use lagged inflation forecast dispersion to avoid potential endogenous effects of monetary policy on inflation forecast dispersion, though this seems less a concern according to Coibion et al. (2020). $SFFR_t$ denotes the shadow federal funds rate constructed by Wu and Xia (2016) and is included to control for the effects from actual interest rate changes. ε_{t+h} is the error term. The sample covers the period from July 1991 to December 2022.

The coefficient α_1 captures the average effects of forward guidance shocks on macroeconomic variables of interest. The coefficient α_3 captures the marginal effect of high inflation expectation dispersion on monetary policy transmission. If α_1 and α_3 have opposite signs, it suggests that high inflation disagreement may weaken or even overturn the effect of forward guidance.

The upper panel of Figure 2 shows that an identified forward guidance shock is followed by a rise in the unemployment rate ($\alpha^h_1 > 0$), but the effect is mitigated if the current state is characterized by high inflation disagreement ($\alpha^h_3 < 0$). Similar results are obtained from regressions for industrial production (lower panel), indicating that a positive forward

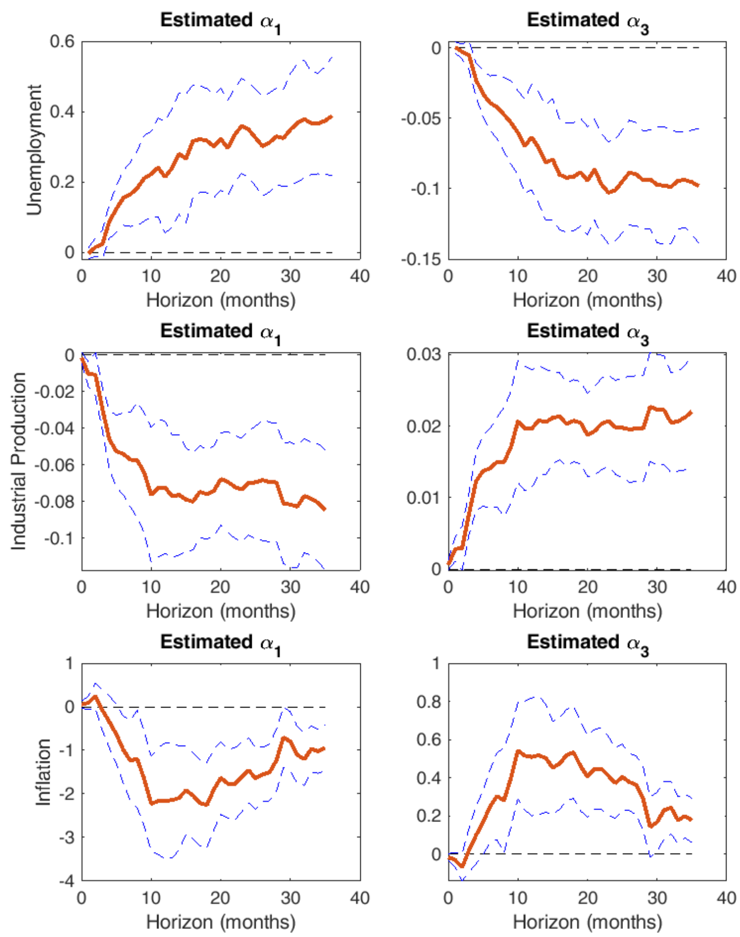

FIGURE 2. Estimated response to forward guidance shocks

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to forward guidance shock from the local projections model (35). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

guidance shock predicts a decline in output ($\alpha_1^h < 0$), but the effect is again mitigated in states with high inflation disagreement ($\alpha_3^h > 0$). For example, a one-standard-deviation higher inflation forecast disagreement will reduce the effects of forward guidance shocks on 1-year ahead unemployment by 29.6% and 27% respectively[9].

Our model also predicts that forward guidance policy is less effective in stabilizing inflation when it is carried out during times of high disagreement in inflation expectations *a la* Prop. II.4. The bottom panel of Figure 2 supports this prediction: news about future monetary

[9]The demeaned time series of inflation disagreement has a standard deviation of 0.983.

tightening helps stabilize the price level ($\alpha_1^h < 0$), but the effect is mitigated if the current economy features high inflation disagreement (i.e. $\alpha_3^h > 0$).

For robustness, we control for a number of other variables, including consumers' perceived uncertainty concerning vehicle purchases, business uncertainty, consumer sentiment, and the level of inflation expectation, all from the same survey. We add these variables and their interactions with forward guidance shocks to Eq. (35), one variable at a time. We also utilize Blue Chip Professional Forecasts data to control for effects from disagreements about real GDP and interest rate paths. Table 1 reports the estimated coefficients for industrial production and inflation one year after the shock; the full response paths are reported in Appendix V.5. Overall, the inclusion of those additional variables has little effect on our findings.

III.2. **Inflation Disagreement and the Effectiveness of Conventional Monetary Policy.** This subsection provides supporting evidence for our second prediction on the role of inflation disagreement in the transmission of conventional monetary policy (i.e. Proposition II.3). For this purpose, we estimate the impulse responses of cumulative changes in the unemployment rate, industrial production and the price level to identified interest rate shocks.

We consider the empirical specification:

$$\begin{aligned}\log(y^j_{t+h}) - \log(y^j_{t-1}) = \beta_0^h &+ \sum_{i=0,1,2,3} \alpha^h_{1,i} MP_{t-i} + \sum_{i=1,2,3} \alpha^h_{i,2} IQR^\pi_{t-i} + \alpha^h_3 IQR^\pi_{t-1} * MP_t \\ &+ \sum_{j=1,2,3} \sum_{i=1,2,3} \alpha^h_{4,i} \Delta \log(y^j_{t-i}) + \sum_{i=0,1,2,3} \alpha^h_{5,i} SFFR_{t-i} + \varepsilon_{t+h} \quad (36)\end{aligned}$$

with $h = 0, 1, 2, ..., 36$. In this specification, $\log(y^j_{t+h}) - \log(y^j_{t-1})$ denotes cumulative unemployment ($j = 1$), industrial production ($j = 2$) and inflation ($j = 3$) growth from period t-1 to period t+h, MP_t are the monetary policy rate target shocks constructed using the method of Swanson (2021), aggregated to the monthly frequency following time weighting method of Gertler and Karadi (2011).[10]

The upper panel of Figure 3 shows that a positive policy rate shock predicts a rise in the unemployment rate ($\alpha_1^h > 0$), but the effect is mitigated in a state with high inflation disagreement ($\alpha_3^h < 0$). Similar results are obtained from regression on output (middle panel): a positive policy rate shock predicts a decline in industrial production ($\alpha_1^h < 0$), but the effect is mitigated in state with high inflation disagreement ($\alpha_3^h > 0$). For example, a one-standard-deviation increase in inflation forecast disagreement would reduce the effects

[10]The estimated results are robust to replacing the shadow federal fund rate by the 2-year Treasury yield.

Table 1. Impulse responses to forward guidance shocks

$h = 12$	Industrial Production					Inflation				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
FG_t	-0.072	-0.110	-0.106	-0.073	-0.107	-1.921	-3.000	-2.496	-2.182	-2.930
	(0.036)	(0.060)	(0.052)	(0.038)	(0.052)	(1.161)	(1.481)	(1.226)	(1.205)	(1.226)
IQR^{π}_{t-1}	0.002	0.002	0.004	0.002	0.004	0.409	0.348	0.378	0.431	0.443
	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)	(0.253)	(0.261)	(0.179)	(0.247)	(0.196)
$IQR^{\pi}_{t-1} * FG_t$	0.020	0.033	0.020	0.020	0.020	0.483	0.859	0.474	0.526	0.486
	(0.008)	(0.017)	(0.008)	(0.008)	(0.008)	(0.282)	(0.423)	(0.247)	(0.276)	(0.247)
EX^{π}_{t-1}		-0.000					0.044			
		(0.001)					(0.041)			
$EX^{\pi}_{t-1} * FG_t$		-0.004					-0.121			
		(0.004)					(0.084)			
UNC^{c}_{t-1}			0.001					-0.262		
			(0.008)					(0.191)		
$UNC^{c}_{t-1} * FG_t$			0.010					0.167		
			(0.008)					(0.196)		
UNC^{b}_{t-1}				-0.001					-0.010	
				(0.002)					(0.053)	
$UNC^{b}_{t-1} * FG_t$				0.000					0.022	
				(0.001)					(0.034)	
IQR^{gdp}_{t-1}					0.001					-0.006
					(0.008)					(0.200)
$IQR^{gdp}_{t-1} * FG_t$					0.010					0.291
					(0.008)					(0.199)

Other Control Variables		
$\sum_{i=1,2,3} FG_{t-i}$	Yes	Yes
$\sum_{i=2,3} IQR_{t-i}$	Yes	Yes
$\sum_{j=0,1,2,3} SFFR_{t-i}$	Yes	Yes
$\sum_{i,j=1,2,3} \Delta y^{j}_{t-i}$	Yes	Yes
Constant	Yes	Yes

Note: This table shows the responses of industrial production and inflation one year after the forward guidance shocks estimated under the baseline specification (Eq. (35)) and alternative ones from appendix V.5. FG_t is forward guidance shock identified using the method of Swanson (2021), aggregated to monthly frequency following time weighting method of Gertler and Karadi (2011). IQR^{π}_{t-i} denotes inflation forecast dispersion over one-year horizon from Michigan Survey of Consumers. UNC^{c}_{t},EX^{π}_{t}, and UNC^{b}_{t} denote consumer uncertainty, mean of one-year ahead inflation expectation, and business uncertainty from the same survey, and IQR^{gdp} denotes professional forecast dispersion of real GDP from Blue Chip Financial Forecasts. All regressions cover the sample period from July 1991 to December 2022. The Newey-West standard errors are shown in the parentheses.

of monetary policy rate shocks on 1-year ahead unemployment and industrial production by 27.5% and 29.3% respectively.

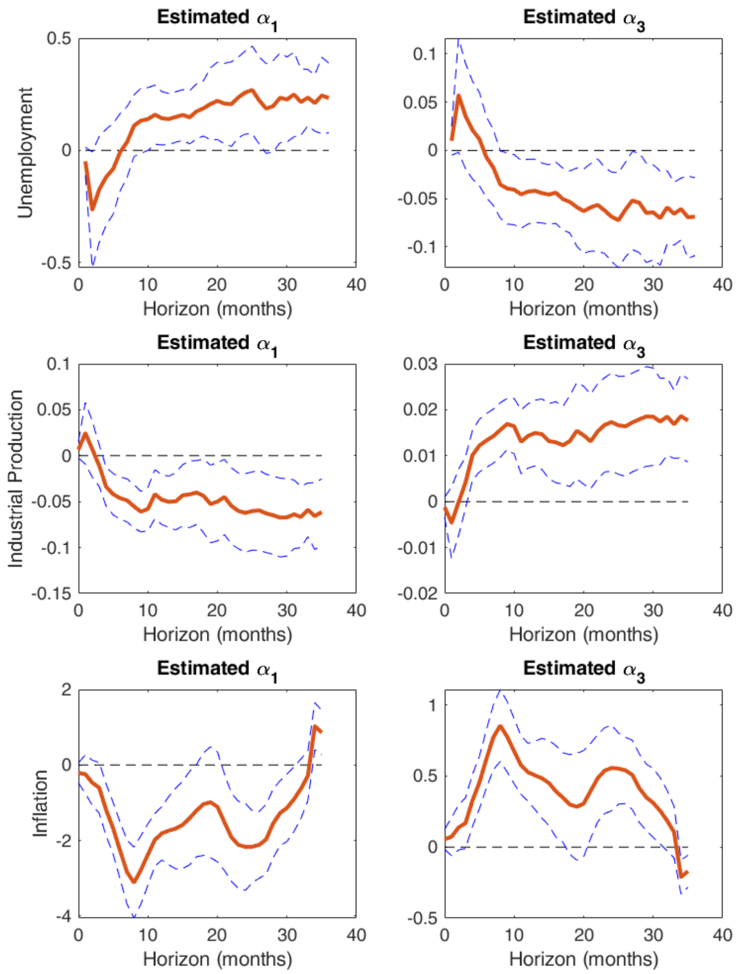

FIGURE 3. Estimated response to federal fund rate shocks

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shock from the local projections model (36). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

For robustness, we control for a number of other variables, including consumer uncertainty, business uncertainty, inflation expectation, as well as professional forecast disagreements on real GDP and interest rate paths. Table 2 reports the estimated coefficients for industrial production and inflation one year after the shock; the full response paths are reported in Appendix V.6. Again, our findings are robust to these alternative specifications.

Table 2. Impulse responses to federal fund rate shocks

$h = 12$	Industrial Production					Inflation				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
MP_t	-0.048	-0.063	-0.074	-0.046	-0.074	-1.809	-2.659	-1.338	-1.957	-2.734
	(0.027)	(0.030)	(0.027)	(0.028)	(0.027)	(0.706)	(0.811)	(0.946)	(0.766)	(0.782)
IQR^{π}_{t-1}	0.002	0.001	0.002	0.002	0.002	0.439	0.376	0.430	0.466	0.473
	(0.006)	(0.006)	(0.004)	(0.006)	(0.004)	(0.231)	(0.249)	(0.191)	(0.233)	(0.179)
$IQR^{\pi}_{t-1} * MP_t$	0.014	0.021	0.013	0.014	0.013	0.527	0.877	0.230	0.567	0.528
	(0.007)	(0.009)	(0.007)	(0.007)	(0.007)	(0.203)	(0.286)	(0.193)	(0.189)	(0.153)
EX^{π}_{t-1}		-0.000					0.073			
		(0.001)					(0.043)			
$EX^{\pi}_{t-1} * MP_t$		-0.004					-0.138			
		(0.004)					(0.080)			
UNC^{c}_{t-1}			0.001					00.247		
			(0.001)					(0.186)		
$UNC^{c}_{t-1} * MP_t$			-0.003					0.163		
			(0.003)					(0.143)		
UNC^{b}_{t-1}				-0.001					-0.009	
				(0.001)					(0.050)	
$UNC^{b}_{t-1} * MP_t$				0.000					0.002	
				(0.002)					(0.061)	
IQR^{gdp}_{t-1}					0.004					0.027
					(0.008)					(0.213)
$IQR^{gdp}_{t-1} * MP_t$					0.008					0.264
					(0.008)					(0.126)
Other Control Variables										
$\sum_{i=1,2,3} MP_{t-i}$	Yes					Yes				
$\sum_{i=2,3} IQR_{t-i}$	Yes					Yes				
$\sum_{j=0,1,2,3} SFFR_{t-i}$	Yes					Yes				
$\sum_{i,j=1,2,3} \Delta y^{j}_{t-i}$	Yes					Yes				
Constant	Yes					Yes				

Note: This table shows the responses of industrial production and inflation one year after the federal fund rate shocks estimated under the baseline specification (Eq. (36)) and alternative ones from appendix V.6. MP_t is federal fund rate shock identified using the method of Swanson (2021), aggregated to monthly frequency following time weighting method of Gertler and Karadi (2011). IQR^{π}_{t-i} denotes inflation forecast dispersion over one-year horizon from Michigan Survey of Consumers. UNC^{c}_{t},EX^{π}_{t}, and UNC^{b}_{t} denote consumer uncertainty, mean of one-year ahead inflation expectation, and business uncertainty from the same survey, and IQR^{gdp} denotes professional forecast dispersion of real GDP from Blue Chip Financial Forecasts. All regressions cover the sample period from July 1991 to December 2022. The Newey-West standard errors are shown in the parentheses.

IV. Conclusion

Survey data on consumers show significant time variations in the cross-sectional dispersion of inflation forecasts. This paper studies the implication of this heterogeneity in inflation expectation in the transmission of conventional and unconventional monetary policy shocks. In a tractable general equilibrium framework, we introduce heterogeneity in household beliefs about the central bank's inflation target into an otherwise standard New Keynesian model, and derive a micro-founded discounted Euler equation. In this model, higher inflation disagreement not only weakens the potency of contemporaneous monetary policy rate shocks on inflation, real output and employment, but also mitigates the responses of those variables to the forward guidance type of unconventional monetary policy. We provides direct empirical support for model predictions that positive surprises about current and future monetary policy rates raise unemployment and reduce output and inflation, but the effects are greatly mitigated if the policy is conducted during periods of high inflation disagreement.

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V. Appendix

V.1. **Representative-agent rational-expectation equilibrium with a stochastic inflation target.** Assume that the true process of the central bank's inflation target is

$$\Pi_{t+1}^{*} = \Pi_{t}^{*} \exp(\varepsilon_{t+1}), \qquad \varepsilon_{t+1} \sim N(0, \sigma) \tag{37}$$

and that the monetary policy follows the rule

$$R_{ft} = R_0 \Pi_t^{*} \left(\frac{\Pi_t}{\Pi_t^{*}} \right)^{\varphi} \exp(\xi_t), \qquad \varphi > 1.$$

We can rewrite the Euler equation as

$$\frac{1}{C_t} = \beta R_0 \pi_t^{\varphi} \exp(\xi_t) E_t \left[\frac{1}{C_{t+1}} \frac{1}{\pi_{t+1}} \frac{\Pi_t^{*}}{\Pi_{t+1}^{*}} \right].$$

The log-linearized Euler equation is given by

$$\hat{C}_t = E_t \hat{C}_{t+1} + E_t \hat{\pi}_{t+1} + E_t \varepsilon_{t+1} - \varphi \hat{\pi}_t - \xi_t,$$

where $E_t \varepsilon_{t+1} = 0$. Iterating the Euler equation forward, we obtain

$$\hat{C}_t = -\sum_{j=0}^{\infty} E_t [\xi_{t+j} - (\varphi - 1) E_t \hat{\pi}_{t+j+1}] - \varphi \hat{\pi}_t, \tag{38}$$

which echoes Eq. (3) in implying that (far) future policy rate change will have an implausibly large effect on current consumption.

V.2. **Proof on that $\mathbb{E}_t^j \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] = \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right], \forall j$.**

Proof. The Euler equation of individual j, who believes that $\frac{\Pi_{t+1}^{*}}{\Pi_t^{*}} = e_{jt}$, can be written as

$$\Lambda_{jt} = \beta \frac{1}{e_{jt}} r_{ft} \mathbb{E}_t^j \frac{\Lambda_{t+1}}{\pi_{t+1}} + \Omega_{jt}, \quad \forall j \tag{39}$$

Define $X_{t+1} \equiv \frac{\Lambda_{t+1}}{\pi_{t+1}}$, we now prove that

$$\mathbb{E}_t^j \left[X_{t+1} \right] = \mathbb{E}_t \left[X_{t+1} \right], \qquad \forall j$$

We prove by contradiction. Assuming that X_{t+1} depends on ε_{t+1} or that $\mathbb{E}_t^j \left[X_{t+1} \right]$ depends on idiosyncratic belief e_{jt}, we can write

$$\mathbb{E}_t^j \left[X_{t+1} \right] \equiv \mathbb{E}_t \left[X_{t+1} \right] + H_t(S_t, e_{jt})$$

where $\mathbb{E}_t$ denotes rational expectation operator and $H_t(S_t, e_{jt}) \equiv \mathbb{E}_t^j \left[X_{t+1} \right] - \mathbb{E}_t \left[X_{t+1} \right]$ denotes forecast deviation from rational expectation as a function of aggregate fundamental state excluding ε_t (summarized as S_t, common to all individuals) and idiosyncratic belief on ε_{t+1} (i.e. e_{jt}).

The individual Euler equation becomes

$$\Lambda_{jt} = \beta \frac{1}{e_{jt}} r_{ft} \mathbb{E}_t \left[X_{t+1}(S_{t+1}) | S_t \right] \tilde{H}_t(S_t, e_{jt}) + \Omega_{jt}, \qquad \forall j \tag{40}$$

where $\tilde{H}_t(S_t, e_{jt}) \equiv 1 + \frac{H_t(S_t, e_{jt})}{\mathbb{E}_t[X_{t+1}(S_{t+1})|S_t]}$.

There exists a marginal consumer (saver) with belief e_t^*, such that

$$\Lambda_{jt}(e_t^*) = \beta \frac{1}{e_t^*} r_{ft} \mathbb{E}_t \left[X_{t+1}(S_{t+1}) | S_t \right] \tilde{H}_t(S_t, e_{jt}^*) \tag{41}$$

In other wise, marginal consumer is indifferent between consuming and saving one additional unit of goods such that $\Omega_{jt}(e_t^*) = 0$. For any stochastic processes of fundamental states S_t, the equation above implies that e_t^* can be solved as an implicit function of:

$$e_t^* \equiv e_t^*(S_t) \tag{42}$$

For family members with $e_{jt} \geq e_t^*$, they are constrained to the same consumption level:

$$C_{jt} = A_t + \bar{B} \equiv \bar{C}_t, \qquad \forall e_{jt} > e_t^*.$$

For family members with $e_{jt} \leq e_t^*$, we have $\Omega_{jt}(e_{jt}) = 0$, and their Euler equations satisfy

$$\Lambda_{jt}(e_{jt}) = \beta \frac{1}{e_{jt}} r_{ft} \mathbb{E}_t \left[X_{t+1}(S_{t+1}) | S_t \right] \tilde{H}_t(S_t, e_{jt}), \qquad \forall j \tag{43}$$

which solves consumption as a function of aggregate state and idiosyncratic belief:

$$C_{jt} \equiv C_{jt}(S_t, e_{jt}), \qquad \forall e_{jt} < e_t^*. \tag{44}$$

We obtain that

$$\Lambda_t = \int_{e_t^*(S_t)} \frac{1}{\bar{C}_t} dG(e) + \int^{e_t^*(S_t)} \frac{1}{C_{jt}(S_t, e_{jt})} dG(e_{jt}) \equiv \Lambda_t(S_t) \tag{45}$$

which is independent of ε_t. In addition, π_t derived from producers' problem, as well as r_{ft} derived from monetary rule, is also independent of ε_t. Thus, we prove that $\frac{\Lambda_t}{\pi_t}$ is independent of ε_t or that $\mathbb{E}_t^j \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] = \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right], \forall j$. □

V.3. **Producers' Problem.** We now consider the Phillips curve. The aggregate production take the form

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}},$$

which leads to

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} Y_t, \tag{46}$$

The intermediate goods producers face an adjustment cost a la Rotemberg (1982).

$$\frac{\chi_P}{2} \left[\frac{P_t(i)}{\Pi_t^* P_{t-1}(i)} - 1 \right]^2 Y_t$$

Define $\Phi_{t,t+\tau} = \Pi^*_{t+1} \times \cdots \times \Pi^*_{t+\tau}$, for $\tau \geq 1$. We can normalize the price as $\tilde{P}_t(i) = \frac{P_t(i)}{\Phi_{0,t}}$, and the cost becomes

$$\frac{\chi_P}{2}\left[\frac{\tilde{P}_t(i)}{\tilde{P}_{t-1}(i)} - 1\right]^2 Y_t$$

In a symmetrical equilibrium,

$$\frac{P_t(i)}{\Pi^*_t P_{t-1}(i)} = \frac{P_t}{\Pi^*_t P_{t-1}} = \frac{\Pi_t}{\Pi^*_t} \equiv \pi_t,$$

and we have that the firm solves

$$\begin{aligned}
& E_t \sum \beta^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\left(\frac{P_{jt+\tau}}{P_{t+\tau}}\right)^{1-\sigma} Y_t - \frac{W_t/P_t}{Z_t}\left(\frac{P_{jt+\tau}}{P_{t+\tau}}\right)^{-\sigma} Y_t - \frac{\chi_P}{2}\left[\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{jt+\tau-1}} - 1\right]^2 Y_t\right\} \\
= \quad & E_t \sum \beta^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\left(\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{t+\tau}}\right)^{1-\sigma} Y_t - \frac{w_t}{Z_t}\left(\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{t+\tau}}\right)^{-\sigma} Y_t - \frac{\chi_P}{2}\left[\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{jt+\tau-1}} - 1\right]^2 Y_t\right\}
\end{aligned}$$

$$\begin{aligned}
& (1-\sigma)\left(\frac{\tilde{P}_{jt}}{\tilde{P}_t}\right)^{-\sigma}\frac{Y_t}{\tilde{P}_t} + \sigma\frac{W_t/P_t}{Z_t}\left(\frac{\tilde{P}_{jt}}{\tilde{P}_t}\right)^{-\sigma-1}\frac{Y_t}{\tilde{P}_t} - \chi_P\left[\frac{\tilde{P}_{jt}}{\tilde{P}_{jt-1}} - 1\right]\frac{Y_t}{\tilde{P}_{jt-1}} \\
& + \chi_P \beta \frac{\Lambda_{t+1}}{\Lambda_t} E_t\left[\frac{\tilde{P}_{jt+1}}{\tilde{P}_{jt}} - 1\right]\frac{\tilde{P}_{jt+1}}{\left(\tilde{P}_{jt}\right)^2} Y_{t+1} = 0
\end{aligned}$$

or we have

$$\chi_P\left[\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right]\frac{\tilde{P}_t}{\tilde{P}_{t-1}} = \sigma\frac{W_t/P_t}{Z_t} + (1-\sigma) + \chi_P \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{\tilde{P}_{t+1}}{\tilde{P}_t} - 1\right]\frac{\tilde{P}_{t+1}}{\tilde{P}_t}\frac{Y_{t+1}}{Y_t}$$

This generates the linearized Phillips curve as

$$\hat{\pi}_t = \varphi_y[\hat{W}_t - \hat{Z}_t] + \beta \mathbb{E}_t \hat{\pi}_{t+1} \tag{47}$$

V.4. **Belief under a Pareto Distribution.** Suppose that the idiosyncratic beliefs of households follow a Pareto distribution, such that

$$G(e) = \begin{cases} 1 - (\frac{e}{e_{min}})^{-\alpha} & \text{if } e \geq e_{min} \\ 0 & \text{if } e < e_{min} \end{cases} \tag{48}$$

We fix $E(e) = 1$ by setting $e_{min} = \frac{\alpha-1}{\alpha}$. The variance of inflation expectation is a decreasing function of α:

$$Var(e) = \frac{\alpha}{\alpha-2} \cdot \left(\frac{e_{min}}{\alpha-1}\right)^2 = \frac{1}{\alpha(\alpha-2)}, \quad \alpha > 2. \tag{49}$$

We can prove the following Lemmas:

Lemma V.1. *e^*_t is an increasing function of α.*

Proof. Incorporating the distribution function of inflation expectations (e) and the assumption that $e_{min} \equiv \frac{\alpha-1}{\alpha}$ into Eq. (24), we obtain

$$\begin{aligned} \frac{1}{1+\kappa} &= (1 - G(e_t^*)) + \frac{\int_{e_{\min}}^{e_t^*} eg(e)de}{e_t^*} = (1 - (1 - (\frac{e_{min}}{e_t^*})^{\alpha})) + \frac{\int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^{\alpha}}{e^{\alpha+1}})de}{e_t^*} \\ &= -\frac{1}{\alpha-1} \cdot \left(\frac{e_{min}}{e_t^*}\right)^{-\alpha} + \frac{\alpha}{\alpha-1} \cdot \frac{e_{min}}{e_t^*} = -\frac{1}{\alpha-1} \cdot \left(\frac{e_{min}}{e_t^*}\right)^{-\alpha} + \frac{1}{e_t^*}, \end{aligned} \tag{50}$$

which implies that e_t^* is an increasing function of α, or a *decreasing* function of inflation disagreement. □

Lemma V.2. *θ is a decreasing function of α.*

Proof. Use α, $e_{min} \equiv \frac{\alpha-1}{\alpha}$, and e_t^* to solve for θ from Eq. (20):

$$\begin{aligned} \theta = &\frac{1 - G(e^*)}{1 - G(e^*) + e^* \int_{e_{\min}}^{e^*} \frac{1}{e} g(e)de} = \frac{(\frac{e_{min}}{e_t^*})^{\alpha}}{(\frac{e_{min}}{e_t^*})^{\alpha} + e_t^* \int_{e_{min}}^{e_t^*} \frac{1}{e} \frac{\alpha \cdot e_{min}^{\alpha}}{e^{\alpha+1}} de} \\ = &\frac{1}{\frac{1}{\alpha+1} + \frac{\alpha}{\alpha+1} \cdot \left(\frac{e_t^*}{e_{min}}\right)^{\alpha+1}}, \end{aligned} \tag{51}$$

which implies that θ is a decreasing function of α, or an *increasing* function of inflation disagreement. □

Lemma V.3. *μ is an increasing function of α.*

Proof: Use α, $e_{min} \equiv \frac{\alpha-1}{\alpha}$, e_t^* and θ to solve for μ from Eq. (23).

$$\begin{aligned} \mu = &\frac{\int_{e_{\min}}^{e^*} eg(e)de}{(1 - G(e^*))e^* + \int_{e_{\min}}^{e^*} eg(e)de} = \frac{\int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^{\alpha}}{e^{\alpha+1}})de}{(1 - (1 - (\frac{e_{min}}{e_t^*})^{\alpha}))e_t^* + \int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^{\alpha}}{e^{\alpha+1}})de} \\ = &\frac{\frac{\alpha e_{min}^{\alpha}}{-\alpha+1}(e_t^{*-\alpha+1} - e_{min}^{-\alpha+1})}{(\frac{e_{min}}{e_t^*})^{\alpha} e_t^* + \frac{\alpha e_{min}^{\alpha}}{-\alpha+1}(e_t^{*-\alpha+1} - e_{min}^{-\alpha+1})} = \frac{\frac{\alpha}{-\alpha+1}(1 - (\frac{e_{min}}{e_t^*})^{-\alpha+1})}{1 + \frac{\alpha}{-\alpha+1}(1 - (\frac{e_{min}}{e_t^*})^{-\alpha+1})} \end{aligned} \tag{52}$$

which implies that μ is an increasing function of α, or a *decreasing* function of inflation disagreement.

V.5. **Robustness: Inflation Disagreement and the Effectiveness of Forward Guidance.**

V.5.1. *Control for inflation expectation, consumer uncertainty, and business uncertainty.* We modify Eq. (35) by adding control for consumers' perceived uncertainty (concerning vehicle purchases),[11] denoted as UNC^c_t, and its interaction with forward guidance shocks:

$$\begin{aligned}\log(y^j_{t+h}) - \log(y^j_{t-1}) &= \alpha^h_0 + \sum_{i=0,1,2,3} \alpha^h_{1,i} FG_{t-i} + \sum_{i=1,2,3} \alpha^h_{i,2} IQR^{\pi}_{t-i} + \alpha^h_3 IQR^{\pi}_{t-1} * FG_t \\ &+ \sum_{j=1,2,3}\sum_{i=1,2,3} \alpha^h_{4,i} \Delta \log(y^j_{t-i}) + \sum_{i=0,1,2,3} \alpha^h_{5,i} SFFR_{t-i} \\ &+ \sum_{i=1,2,3} \alpha^h_{i,7} UNC^c_{t-i} + \alpha^h_8 UNC^c_{t-1} * FG_t + \varepsilon_{t+h} \qquad (53)\end{aligned}$$

In line with the baseline results, the upper two panels of Figure 4 shows that an identified forward guidance shock predicts a rise (decline) in unemployment rate (industrial production), but the effect is mitigated in current state with high inflation disagreement. The lower panel confirms that high inflation disagreement today also weakens the potency of forward guidance policy in stabilizing inflation.

For robustness, we replace UNC^c_t in Eq. (53) by the level of 1-year ahead inflation expectation (denoted as EX^{π}_t) and business uncertainty (denoted as UNC^b_t) from the same survey, and control for their interaction with forward guidance shocks. The estimation results remain robust to these alternative specifications as reported in Figures 5 and 6 respectively.

V.5.2. *Control for forecast disagreements of GDP and interest rate paths.* Expectation about future output and future interest rate can potential affect consumption response to monetary policy as well. To check the robustness of our baseline results, we utilize Blue Chip Financial Forecast Data to control for the effects from forecast disagreements about output and interest rate paths. In specific, we replace UNC^c_t in Eq. (53) by measures of forecast disagreements about real GDP (denoted as IQR^{gdp}_t), federal fund rate (denoted as IQR^{ffr}_t), and 2-year Treasury yield (denoted as IQR^{rt2y}_t), and control for their interaction with forward guidance shocks. The estimation results remain robust to these alternative specifications as reported in Figure 7 to Figure 9.

V.5.3. *Simplified specification.* We also estimate a simplified version of Eq. (35) by removing additional lagged terms:

$$\begin{aligned}\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 FG_t + \alpha^h_2 IQR^{\pi}_{t-1} + \alpha^h_3 IQR^{\pi}_{t-1} * FG_t \\ + \sum_{j=1,2,3} \alpha^h_4 \Delta \log(y^j_{t-1}) + \alpha^h_{5,i} SFFR_t + \varepsilon_{t+h} \qquad (54)\end{aligned}$$

[11]To ensure internal consistency, we measure consumer uncertainty based on Michigan Survey of Consumers following Leduc and Liu (2016). One question in the survey asks for reason why consumer thinks it is a good or bad time to buy a vehicle. The survey tallies the fraction of respondents who report that "uncertain future" is a reason, which we use to measure consumer uncertainty.

The estimation results remain robust to this alternative specification. The upper two panels of Figure 10 shows that an identified forward guidance shock predicts a rise (decline) in unemployment rate (industrial production), but the effect is mitigated in current state with high inflation disagreement. The lower panel confirms that high inflation disagreement today also weakens the potency of forward guidance policy in stabilizing inflation.

Replacing household's inflation forecast disagreement by that of professional forecasters obtains consistent results (see Figure 11 and Figure 12).

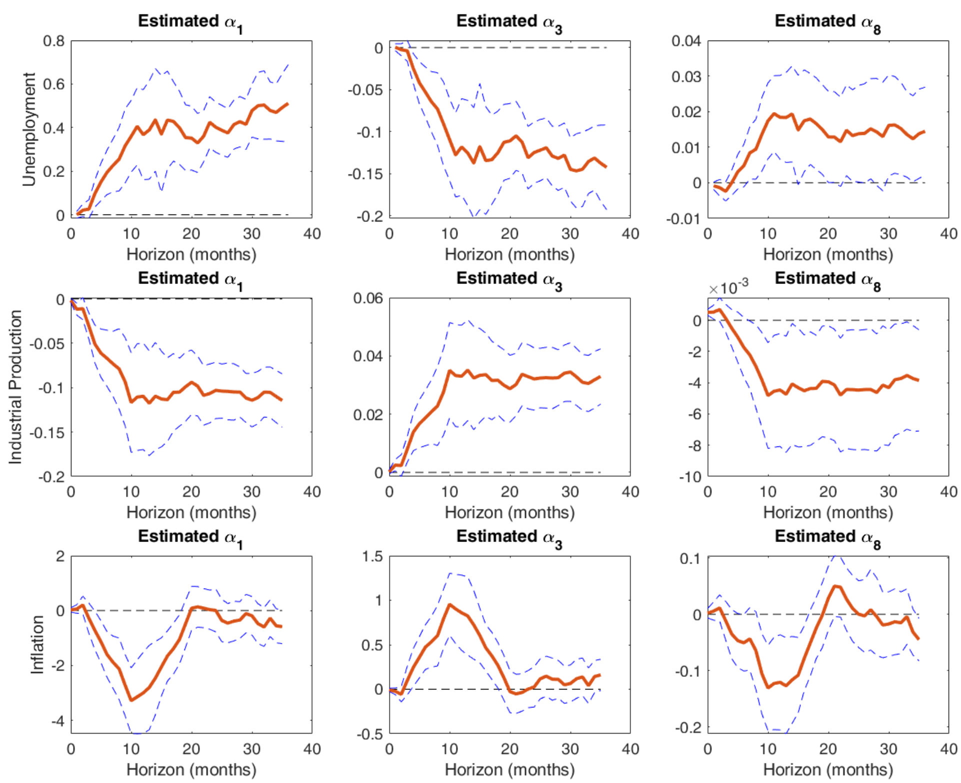

FIGURE 4. Estimated response to forward guidance shocks (controlling for consumer uncertainty)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (53). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

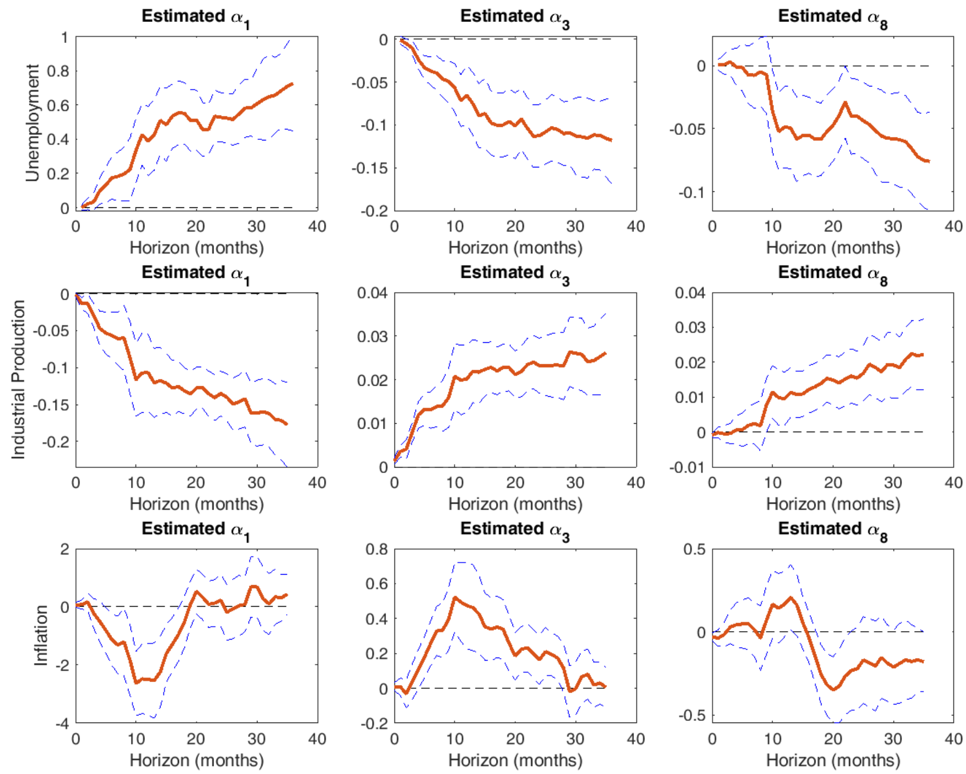

FIGURE 5. Estimated response to forward guidance shocks (controlling for inflation expectation)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (53), where controlled variable of consumer uncertainty is replaced by inflation expectation from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

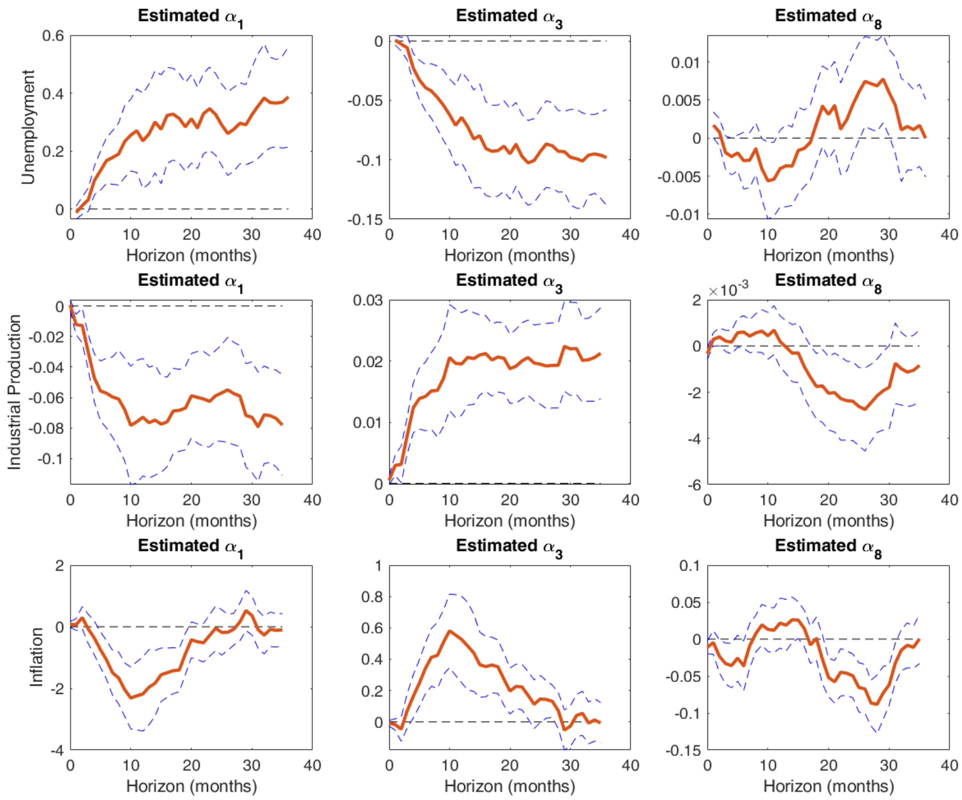

FIGURE 6. Estimated response to forward guidance shocks (controlling for business uncertainty)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (53), where controlled variable of consumer uncertainty is replaced by business uncertainty from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

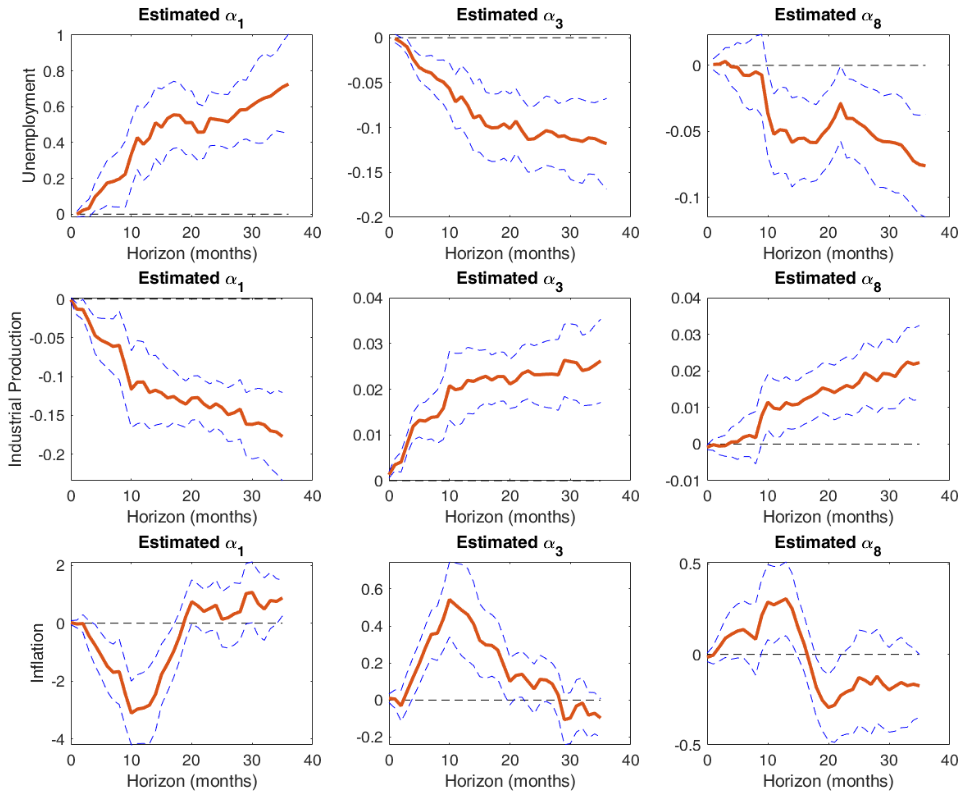

FIGURE 7. Estimated response to forward guidance shocks (controlling for forecast dispersion of GDP)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (53), where controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of real GDP from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

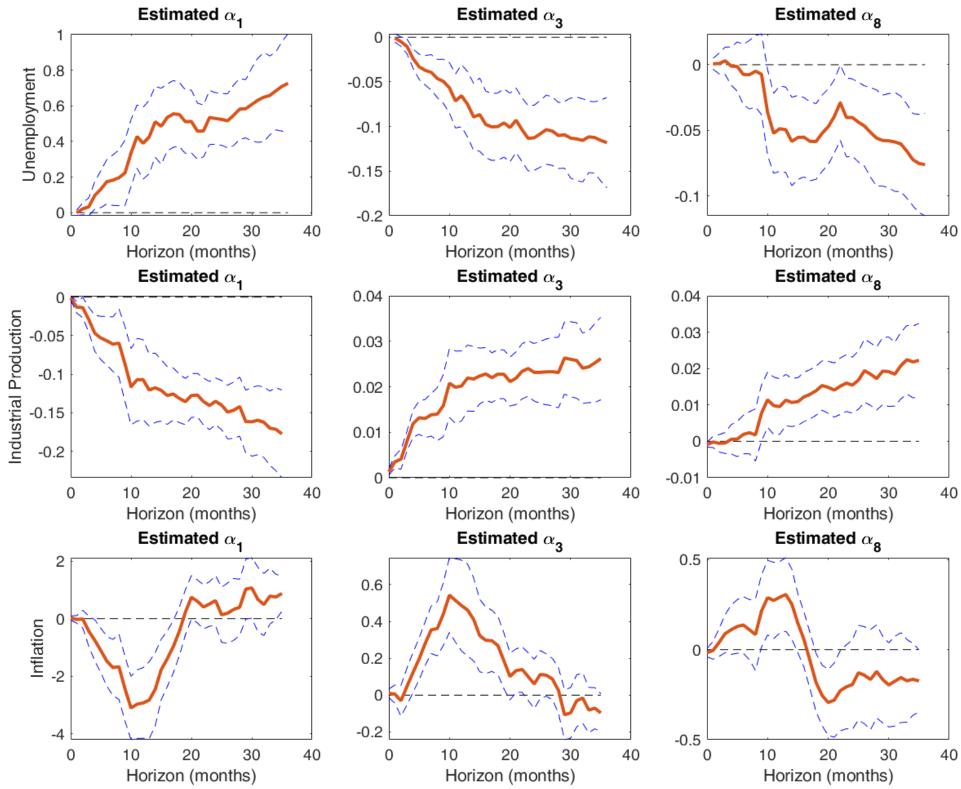

FIGURE 8. Estimated response to forward guidance shocks (controlling for forecast dispersion of federal funds rate)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (53), where controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of federal funds rate from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

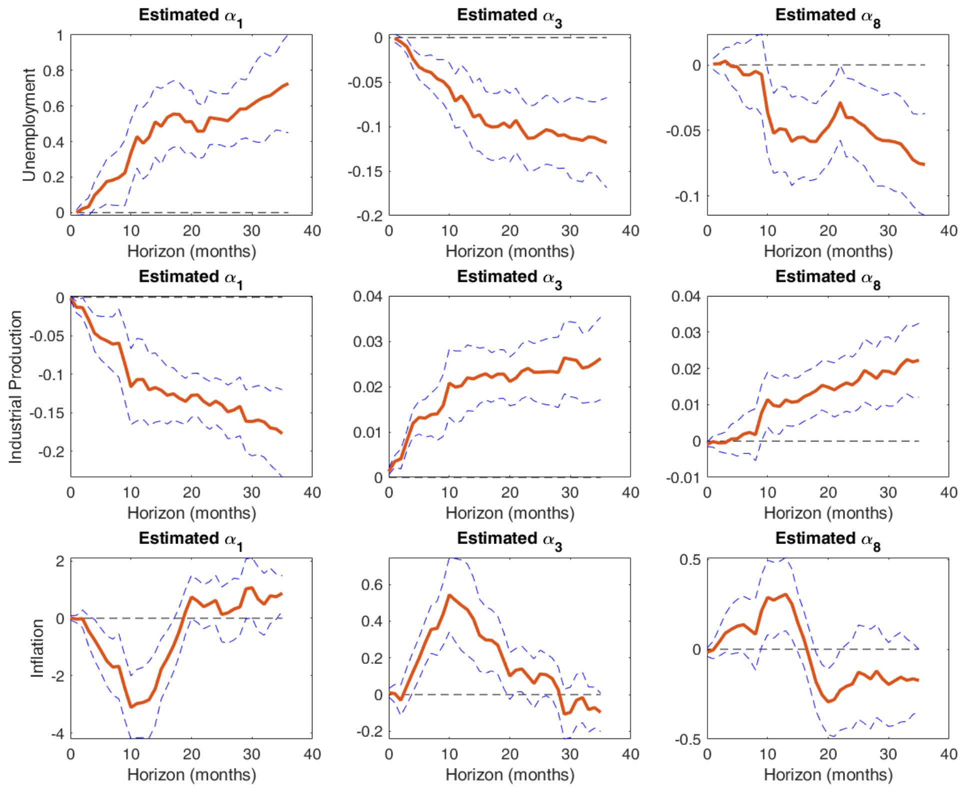

FIGURE 9. Estimated response to forward guidance shocks (controlling for forecast disagreement of 2-year Treasury yield)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (35), where controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of 2-year Treasury yield from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

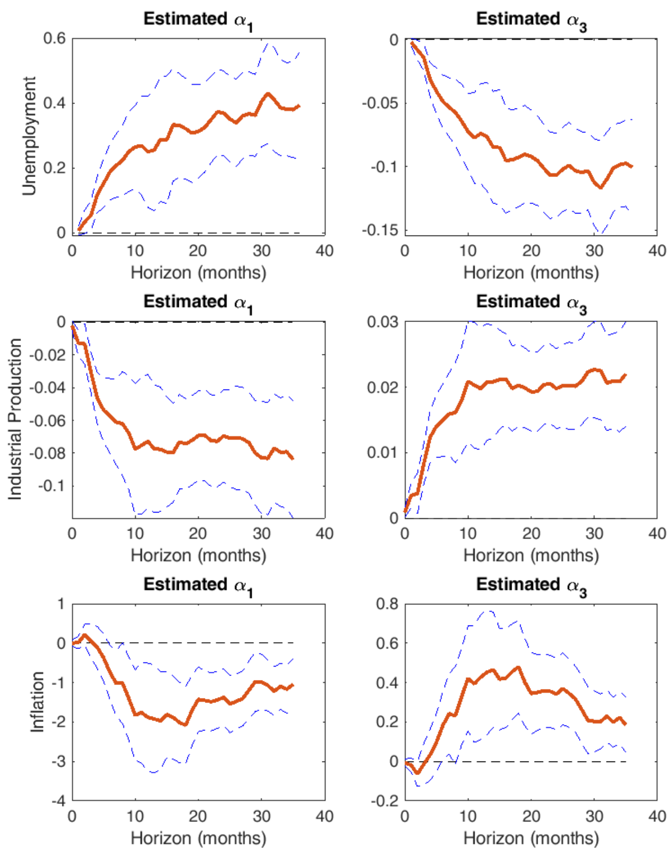

FIGURE 10. Estimated response to forward guidance shocks (based on a simplified specification)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (54). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

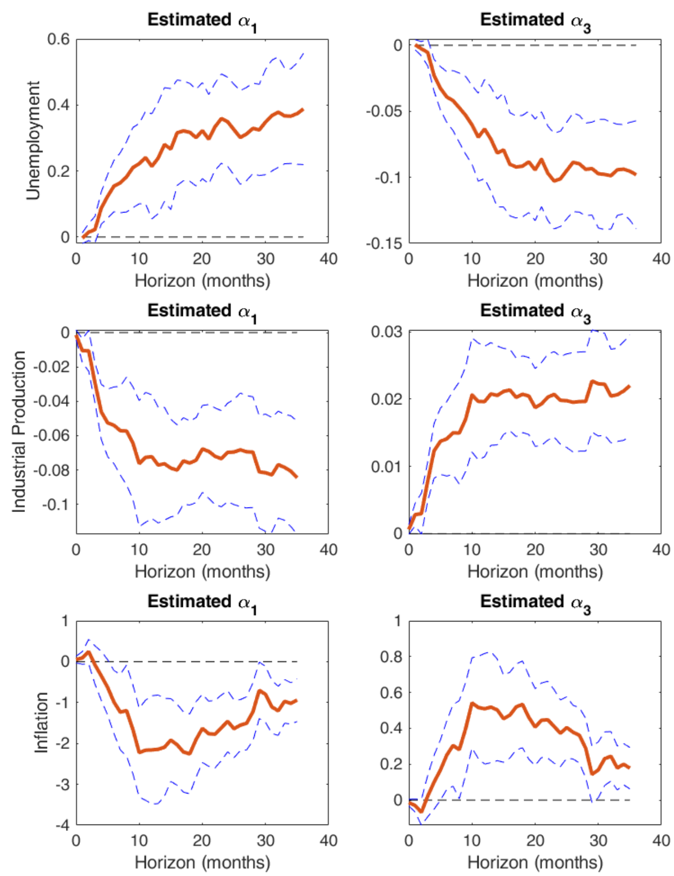

Figure 11. Estimated response to forward guidance shocks (based on professional forecast dispersion of inflation)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (35), where inflation forecast dispersion of household is replaced by that of professionals from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

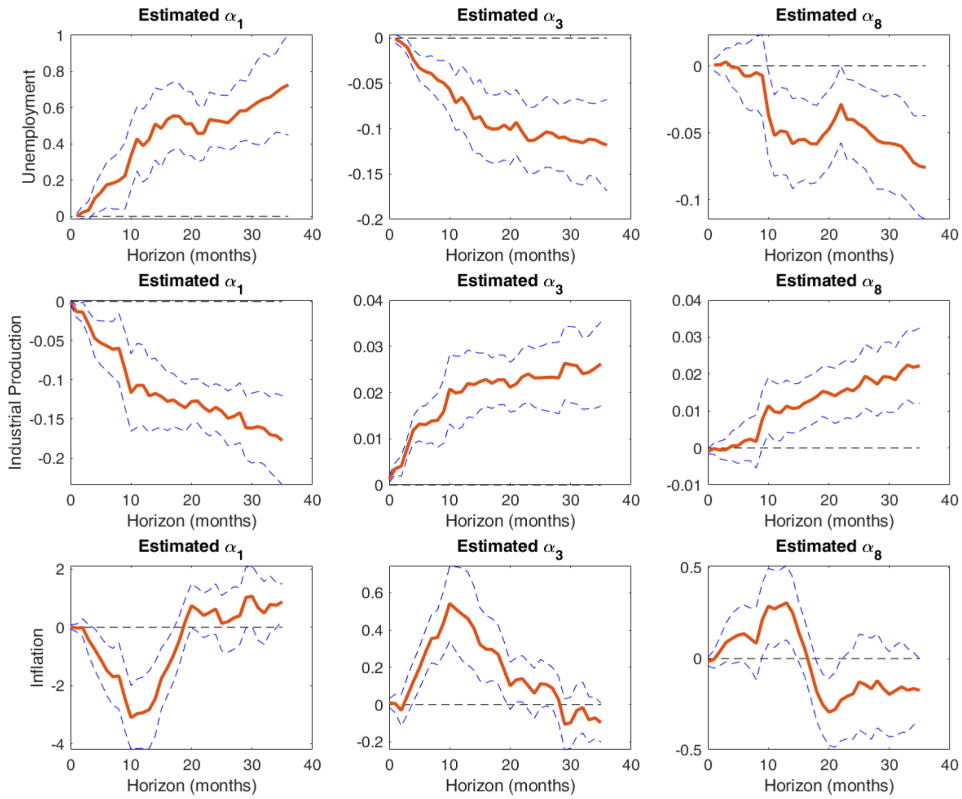

FIGURE 12. Estimated response to forward guidance shocks (based on professional forecast dispersion with additional controls)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to identified forward guidance shock from the local projections model (53), where inflation forecast dispersion of household is replaced by that of professionals, and controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of GDP from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

V.6. **Robustness: Inflation Disagreement and the Effectiveness of Federal Fund Rate Shock.**

$$\begin{aligned}\log(y^j_{t+h}) - \log(y^j_{t-1}) \;=\;& \alpha^h_0 + \sum_{i=0,1,2,3} \alpha^h_{1,i} MP_{t-i} + \sum_{i=1,2,3} \alpha^h_{i,2} IQR^{\pi}_{t-i} + \alpha^h_3 IQR^{\pi}_{t-1} * MP_t \\ & + \sum_{j=1,2,3} \sum_{i=1,2,3} \alpha^h_{4,i} \Delta \log(y^j_{t-i}) + \sum_{i=0,1,2,3} \alpha^h_{5,i} SFFR_{t-i} \\ & + \sum_{i=1,2,3} \alpha^h_{i,7} UNC^c_{t-i} + \alpha^h_8 UNC^c_{t-1} * MP_t + \varepsilon_{t+h} \qquad (55)\end{aligned}$$

In the same spirit of section V.5, we modify Eq. (36) by adding control for consumers' perceived uncertainty, 1-year ahead inflation expectation and business uncertainty from Michigan Survey of Consumers, as well as measures of forecast disagreement about real GDP, about federal fund rate, and about 2-year Treasury yield from Blue Chip Financial Forecasts database. We also control for their interaction with identified federal fund rate shocks. The estimation results remain robust to these alternative specifications as reported from Figures 13 and 21 respectively.

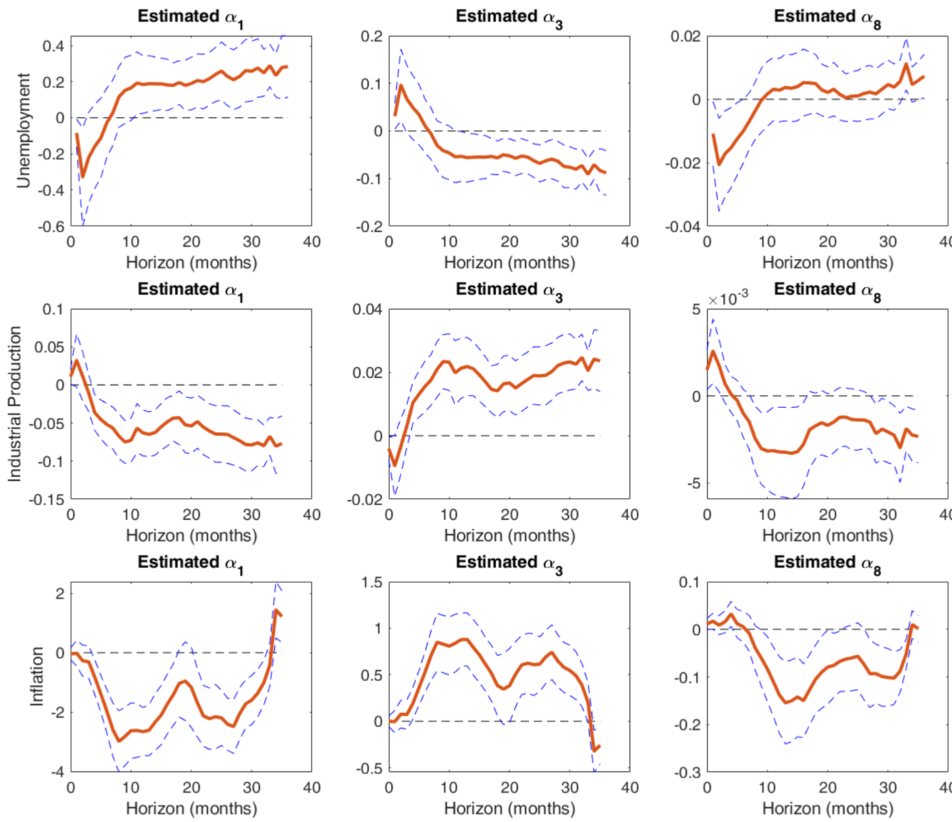

FIGURE 13. Estimated response to federal fund rate shocks (controlling for consumer uncertainty)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projection model (55). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

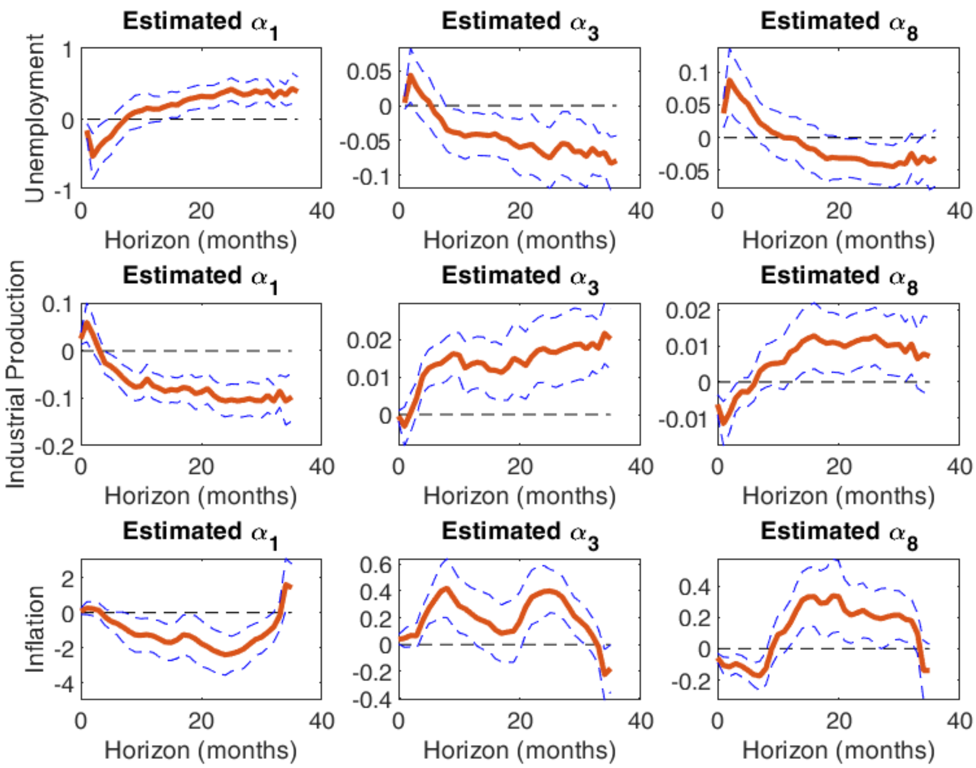

FIGURE 14. Estimated response to federal fund rate shocks (controlling for inflation expectation)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projection model (55), where controlled variable of consumer uncertainty is replaced by inflation expectation from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

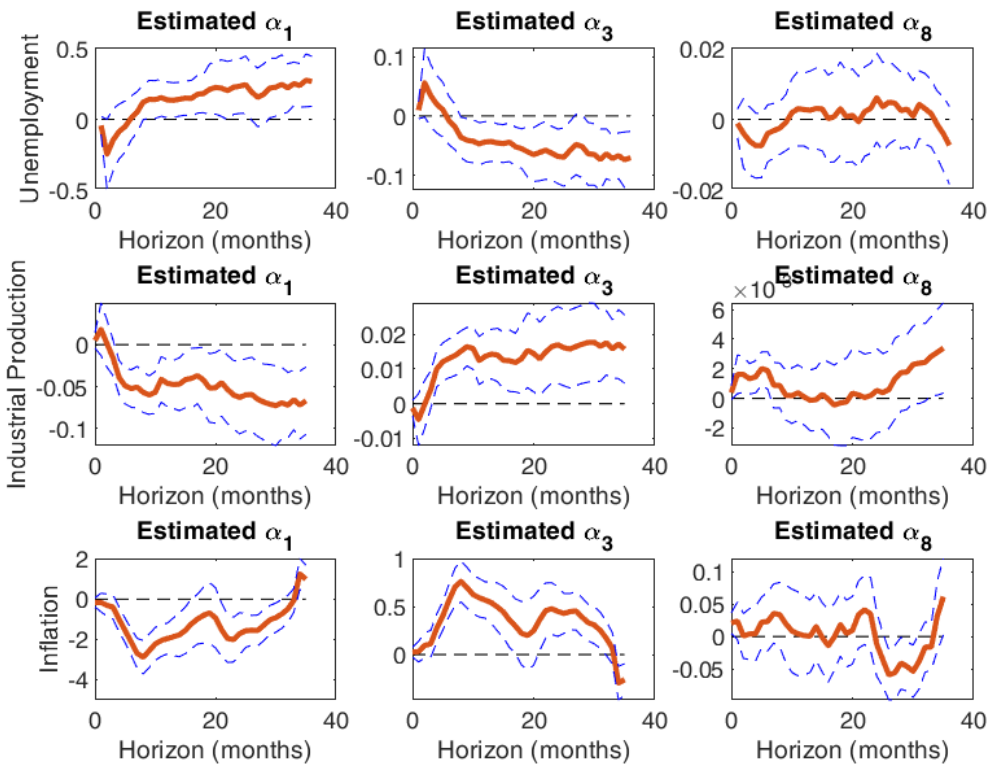

FIGURE 15. Estimated response to federal fund rate shocks (controlling for business uncertainty)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projections model (55), where controlled variable of consumer uncertainty is replaced by business uncertainty from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

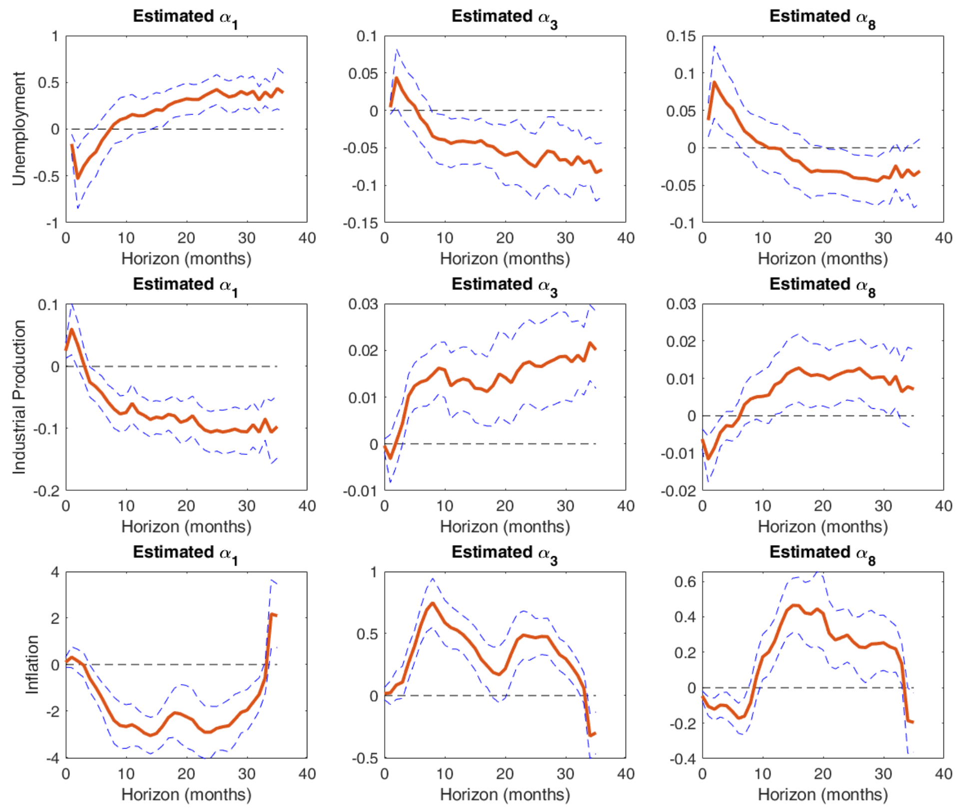

FIGURE 16. Estimated response to federal fund rate shocks (controlling for forecast dispersion of GDP)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projections model (55), where controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of real GDP from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

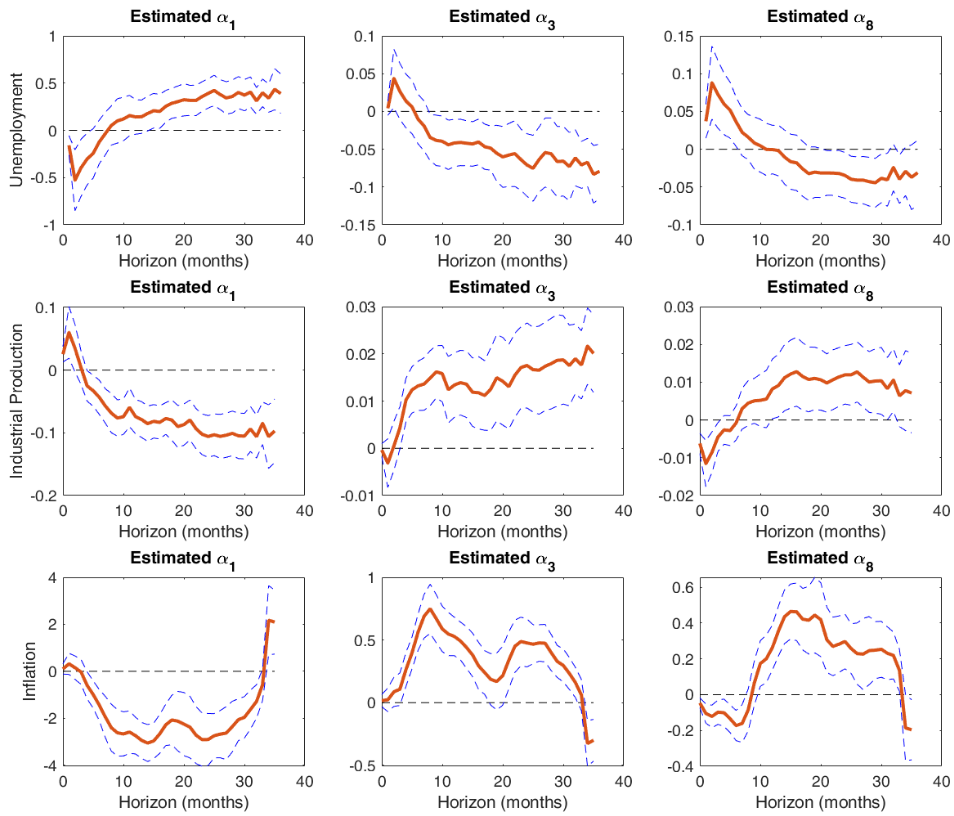

FIGURE 17. Estimated response to federal fund rate shocks (controlling for forecast dispersion of federal funds rate)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projections model (55), where controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of federal funds rate from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

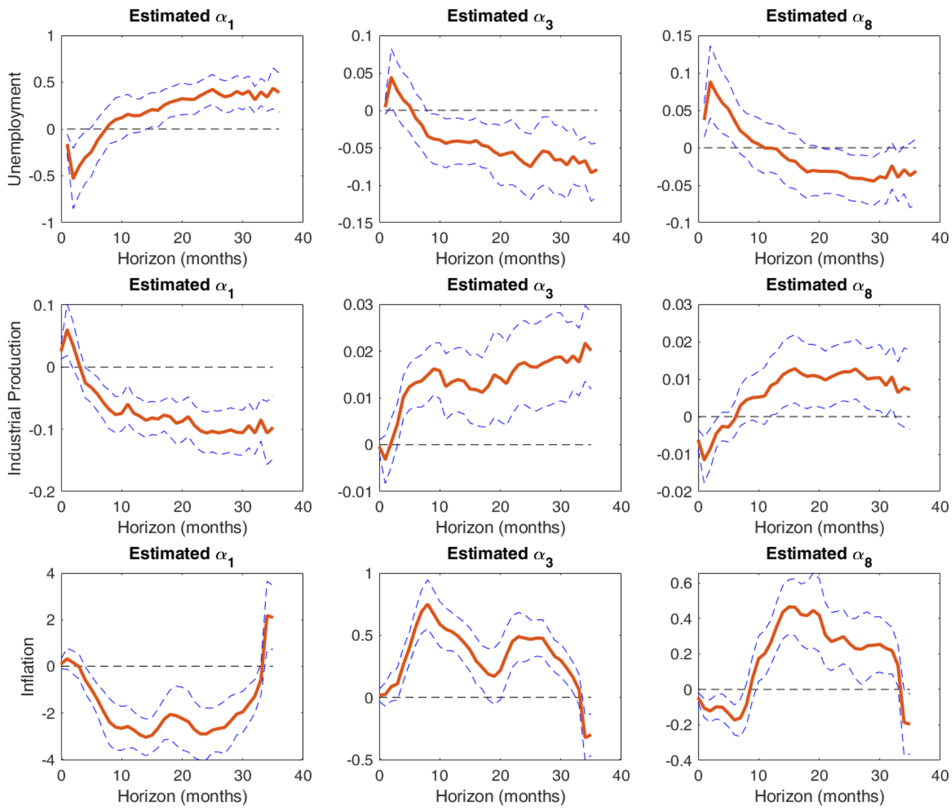

FIGURE 18. Estimated response to federal fund rate shocks (controlling for forecast disagreement of 2-year Treasury yield)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projections model (55), where controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of 2-year Treasury yield from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

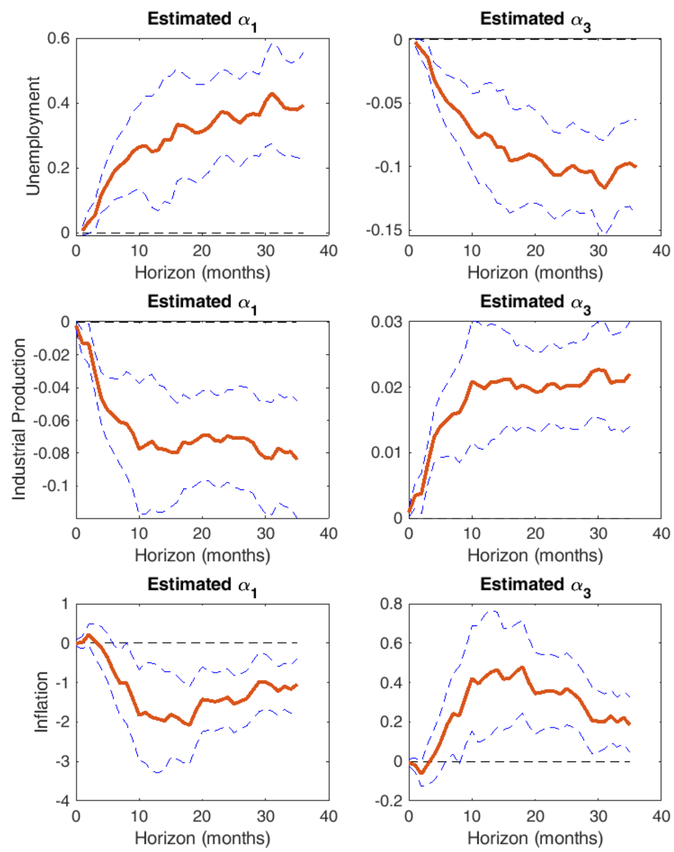

FIGURE 19. Estimated response to federal fund rate shocks (based on a simplified specification)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from a simplified local projections model. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

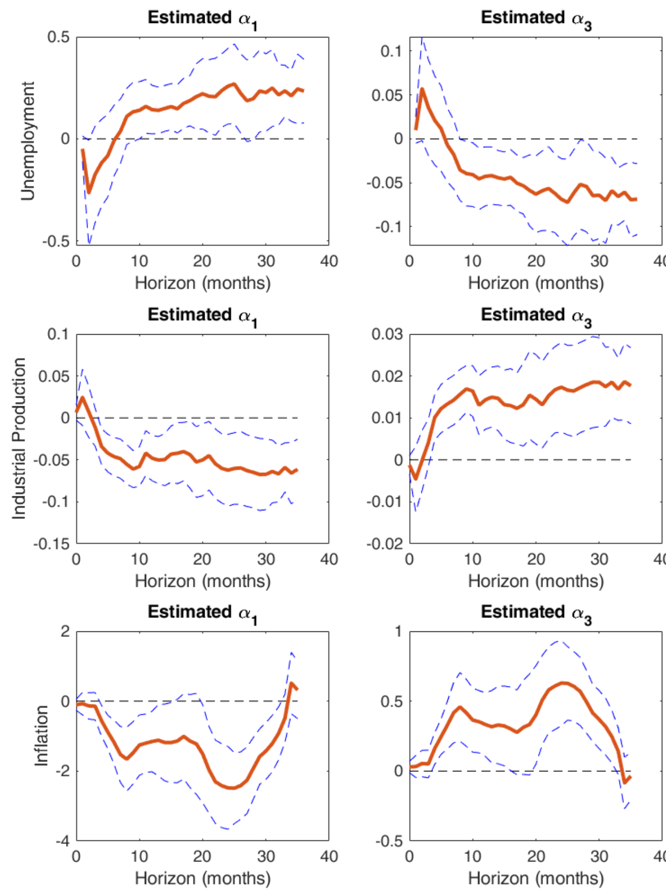

FIGURE 20. Estimated response to federal fund rate shocks (based on professional forecast dispersion of inflation)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projections model (35), where inflation forecast dispersion of household is replaced by that of professionals from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.

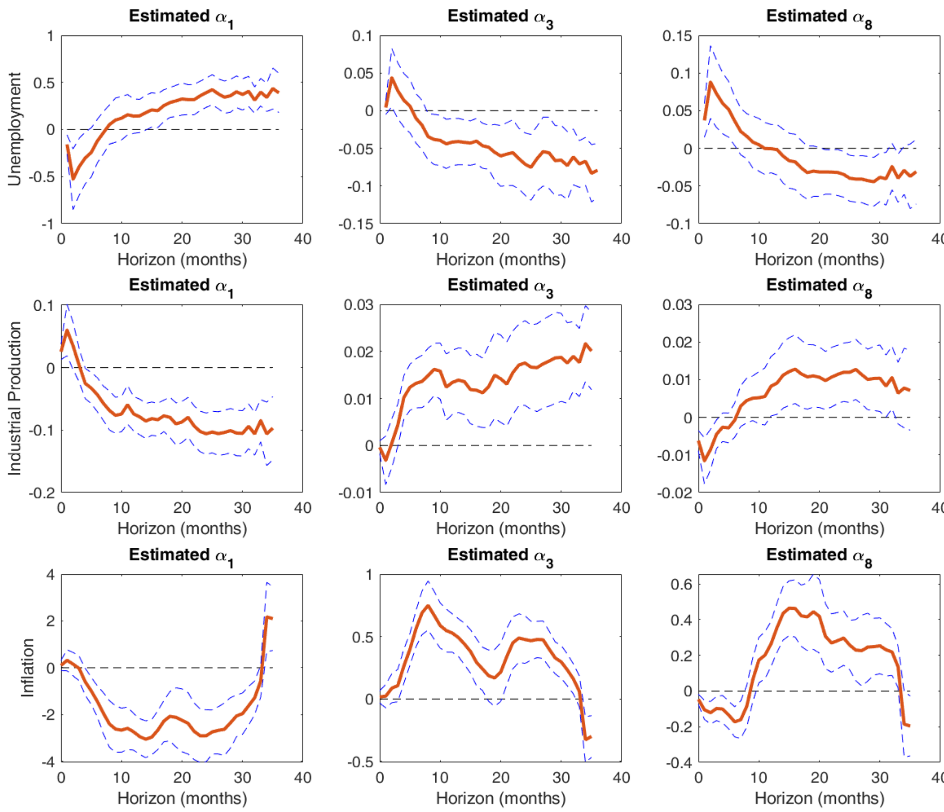

FIGURE 21. Estimated response to federal fund rate shocks (based on professional forecast dispersion with additional controls)

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and inflation to federal fund rate shocks from the local projections model (55), where inflation forecast dispersion of household is replaced by that of professionals, and controlled variable of consumer uncertainty is replaced by 1-year average forecast dispersion of GDP from Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals.