Endogenous Leverage and Belief Heterogeneity

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Leverage and Price Cycles

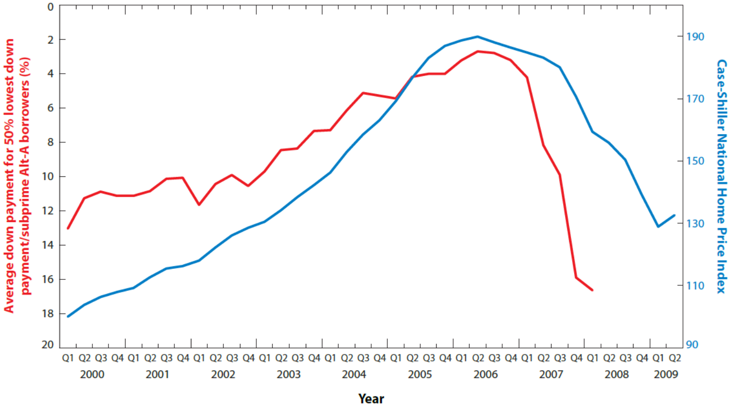


Figure: Pro-cyclical Asset Price and Leverage Dynamics

Geanakoplos, J. (2010). The leverage cycle. NBER macroeconomics annual, 24(1), 1-66.

Highlight

- leverage cycle arising from belief heterogeneity
  - optimistic agents: leverage to invest in asset (collateral)
  - pessimistic agents: lend (value collateral less)
  - $\rightarrow$ endogenous constraint: leverage and price
  - equilibrium leverage too high in boom and too low in recession
- two-period model:
  - endogenous loan contract
  - asset price rises with leverage
  - equilibrium repayment ensures no default
- three-period model:
  - a maturity mismatch problem
  - leverage cycle

Two-period Model

- time: discrete, two period t= 0,1, with two states in last period: good(G) or bad(B).
- asset: a risky asset, with
  - no payoff in period 0;
  - payoff 1 at G state
  - payoff 0.2 at B state
  - risky-free interest rate is zero
- investors: risk-neutral with heterogeneous belief, indexed by h $\in (0, 1)$
  - each endowed with 1 unit of cash and 1 unit of asset
  - can trade their endowed asset at period 0
  - h thinks probability of good is h.
  - h follows a uniform distribution over $(0, 1)$

Natural Buyer

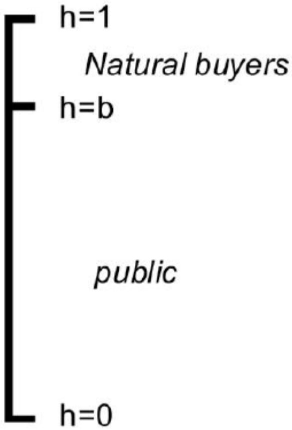


Figure: Natural buyers at period 0

Scenario I: Financial Autarky

- price of asset at period 0 as p.
- optimistic agents with belief above a threshold $\hat{h}$ will buy and others will sell.

$$h * 1 + (1 - h) * 0.2 > p \quad or \quad h > \frac{p - 0.2}{0.8} \equiv \hat{h}$$

- market clearing condition, $(1 - \hat{h}) * 1 = \hat{h}p$

$$1 - \frac{p - 0.2}{0.8} = \frac{p - 0.2}{0.8}p$$

- equilibrium price of $p = 2/3$ and $\hat{h} = 0.6$.

Scenario II: Exogenous Leverage

non-contingent contract: promises $= \varphi$ in both states.

- repayment under two states are

$$\min\{\varphi, 1\} \quad \text{if state is good}$$

$$\min\{\varphi, 0.2\} \quad \text{if state is bad}$$

- no default: a natural limit by setting $\varphi = 0.2$
- marginal buyer as $\tilde{h}$:

$$\underbrace{\tilde{h} * (1 - 0.2) + (1 - \tilde{h}) * 0}_{\text{expected pay-off}} = \underbrace{p - 0.2}_{\text{equity}}, \quad \Leftrightarrow \tilde{h} = \frac{p - 0.2}{0.8},$$

- market clearing condition that $(1 - \tilde{h}) * 1 + \varphi = \tilde{h}p$

$$p = \frac{(1 - \tilde{h}) * 1 + 0.2}{\tilde{h}}$$

- equilibrium price of $p = 0.75$ and $\tilde{h} = 0.69$.

Scenario III: Endogenous Leverage

- loan contract, characterized by a pair of (promise, collateral)
  - e.g. $(\varphi_j = 0.1, 0.5)$ promised to repay 0.1 at each state, otherwise lender seizes 0.5 unit of asset (collateral)
  - risk-less
- focus on contract backed by collateral of one unit of asset
  - homogeneity of degree one
  - e.g. $(\varphi_j = 0.1, 0.5) \sim (\varphi_j = 0.2, 1)$
- state-dependent (actual) repayment, each unit of loan contract can be traded at price of π_j,
  - e.g., one unit of loan contract $\varphi_j \leq 0.2$ is simply priced at one over risk-free rate

Scenario III: Endogenous Leverage

- optimal contract: one with promise = 0.2.
- off-equilibrium path (non-traded contract):
  - if $\varphi_j \leq 0.2$, $\pi_j = 1$
  - if $\varphi_j \in (0.2, 1)$, $\pi_j = \tilde{h}\varphi_j + (1 - \tilde{h})0.2$
  - if $\varphi_j \geq 1$, $\pi_j = \tilde{h}1 + (1 - \tilde{h})0.2$
- why only the contract $\varphi_j = 0.2$ is chosen
  - benefit of higher φ_j: get more funding (lenders' belief)
  - cost: repaying more (borrowers' belief).
  - e.g., by increasing φ_j from 0.2 to 0.3, the borrowers $h_j > \tilde{h}$ get $0.1\tilde{h}$ more at the beginning, but have to repay $0.1h_j$ more in his expectation.

Three-period Model

- time: discrete, three period t= 0,1,2, with two states in period 1 and 2: good(G) or bad(B).
- asset: a risky asset, with
  - no payoff in period 0 and 1
  - possible realization at period 2: GG, GB, BG, BB
  - payoff at state GG, GB, BG: 1
  - payoff at state BB: 0.2
  - risky-free interest rate is zero
- investors: risk-neutral with heterogeneous belief, indexed by h $\in (0, 1)$
  - each endowed with 1 unit of cash and 1 unit of asset
  - can trade their endowed asset at period 0 and 1
  - h thinks probability of good is h, i.i.d. across states
  - h follows a uniform distribution over $(0, 1)$

Equilibrium

- loan contracts: one-period loans.
- repayment to period- 0 debt:

$$\min\{\varphi_0, p_G\} \quad \text{if state is good}$$

$$\min\{\varphi_0, p_B\} \quad \text{if state is bad}$$

- equilibrium contract: bears no default at each period/state.
    - at period 0, $\varphi_0 = p_B$
    - at period 1 if realized state is G, $\varphi_G = 1$
    - at period 1 if realized state is B, $\varphi_B = 0.2$
- task: solve the allocation $\{p_0, \hat{h}_0, p_B, \hat{h}_B\}$
    - $\hat{h}_0$ and $\hat{h}_B$:marginal buyer at period 0 and state B at period 1.
    - p_0 and p_B: asset price at period 0 and state B at period 1.

Marginal Buyer

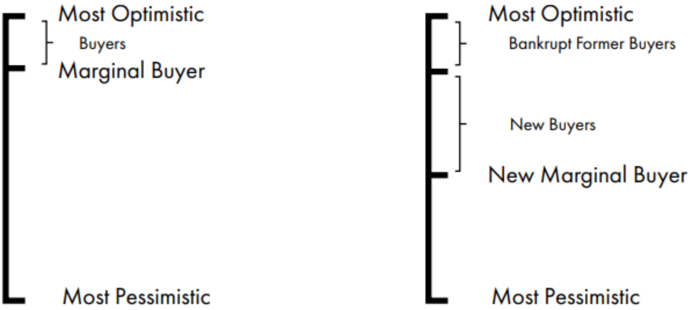


Figure: Marginal buyers at period 0 and period 1 (state B)

Equilibrium

The following equilibrium conditions solves $\{p_0, \hat{h}_0, p_B, \hat{h}_B\}$:

- Euler equation at date 1: The marginal buyer in state B at period 1 must be indifferent b/w buying or not:

$$\underbrace{\hat{h}_B * (1 - \varphi_B) + (1 - \hat{h}_B) * 0}_{\text{expected return}} = \underbrace{p_B - \varphi_B}_{equity}$$

- Euler equation: The marginal buyer at period 0 must be indifferent b/w buying or not:

$$\underbrace{\frac{\hat{h}_0 * (1 - \varphi_0) + (1 - \hat{h}_0) * 0}{p_0 - \varphi_0}}_{\text{expected return of buying}} = \underbrace{\hat{h}_0 * 1}_{G-state} + \underbrace{(1 - \hat{h}_0) \frac{\hat{h}_0 (1 - \varphi_B)}{p_B - \varphi_B}}_{B-state}$$

Equilibrium

- market clearing condition at period 0:

$$\underbrace{(1-\hat{h}_0)*1}_{\text{internal fund}} + \underbrace{\varphi_0}_{debt} = \underbrace{\hat{h}_0 p_0}_{asset} \quad \rightarrow \quad p_0 = \frac{(1-\hat{h}_0)*1 + p_B}{\hat{h}_0}$$

- market clearing condition in state B at period 1:

$$\underbrace{(\hat{h}_0 - \hat{h}_B)*\frac{1}{\hat{h}_0}}_{\text{internal fund}} + \underbrace{\varphi_B}_{debt} = \underbrace{\frac{\hat{h}_B}{\hat{h}_0} p_B}_{asset} \quad \rightarrow \quad p_B = \frac{\hat{h}_0(1+\varphi_B) - \hat{h}_B}{\hat{h}_B}$$

Price and Leverage Cycles

- price and allocation:
  - period 0: $\hat{h}_0 = 0.87$ and $p_0 = 0.95$;
  - state G at period 1, $\hat{h}_G = 1$ and $p_G = 1$.
  - state B at period 1, $\hat{h}_B = 0.61$ and $p_B = 0.69$.
- three forces accounting for the crash in state B at period 1.
  - fundamental: the realization of bad news.
  - loss of natural buyers: leveraged buyers at period 0 go bankrupt
  - deleveraging process: the margin increases from 0.27 to 0.71; the leverage decreases from 3.6 to 1.4.

Simsek, A. (2013). Belief disagreements and collateral constraints. Econometrica, 81(1), 1-53.

Highlight

- Geanakoplos (2010): belief disagreement $\leftrightarrow$ debt contract
  - optimistic agents: leverage to invest in asset (collateral)
  - pessimistic agents: lend (but value collateral less)
  - endogenous debt contract: leverage and price
  - equilibrium leverage too high in boom and too low in recession
- nature of debt contract: asymmetric payoff
  - default only in bad states
  - more sensitive to probability of bad states
- this paper: nature of belief disagreement $\leftrightarrow$ debt contract
  - "what investors disagree about matters"
  - disagreement on 'bad state' is disciplined
  - disagreement on 'good state' is not

What's New

- Geanakoplos (2010):
  - disagreement is concentrated on bad states ("BB")
  - belief disagreement $\uparrow$ margin (asset price $\downarrow$)
  - key assumption: two continuation states
  - equilibrium loan contract is risk-less
- this paper:
  - nature of belief disagreement matters for asset price
  - e.g. belief disagreement about bad states $\uparrow$ margin ($\downarrow$ asset price)
  - e.g. belief disagreement about good states $\uparrow$ asset price
  - more than two states
  - equilibrium loan contract can be risky

Two-period Model

- time: discrete, two period t= 0,1, with a continuum of states in last period: $s \in [s^{min}, s^{max}]$
- asset: a risky asset (owned by outsider initially), with
  - no payoff in period 0;
  - final payoff: s dollar in state s
  - risky-free interest rate $= 0$
- investors: risk-neutral with heterogeneous belief,

$$E_1[s] > E_0[s]$$

  optimists with belief F_1 and cash n_1
  pessimists with belief F_0 and cash n_0

- optimists want to borrow cash and buy asset

Borrowing Contract

- loan contract, characterized by (promise, collateral)

$$\beta \equiv \left(\underbrace{\varphi(s)}_{promise}, \underbrace{\alpha}_{asset}, \underbrace{\gamma}_{cash} \right)$$

- repayment

$$\min\{\varphi(s), \alpha s + \gamma\}$$

- focus on simple debt contract:

$$B \equiv (\varphi(s) = \varphi, \alpha = 1, \gamma = 0)$$

- price of one unit of contract:

$$q(B) = E_0[\min(s, \varphi)]$$

Principle-Agent Problem

- optimist solves

$$\max_{(a_1,\varphi)\in\mathbb{R}^2_+} a_1 E_1[s] - a_1 E_1[\min(s,\varphi)]$$

s.t. participation constraint + budget constraint

$$a_1 p = n_1 + a_1 E_0[\min(s,\varphi)]$$

and some regular conditions ensuring $p \in (E_0[s], E_1[s])$.

- trade-offs on higher φ:
  - higher loan size: $\varphi \uparrow \to E_0[\min(s,\varphi)] \uparrow$
  - higher risk: $\varphi \uparrow \to$ default threshold $\uparrow$

Principle-Agent Problem (transformed)

- optimist solves

$$\max_{(\varphi)\in\mathbb{R}^2_+} n_1 R_1^L(\varphi)$$

where $R_1^L(\varphi)$ is expected return rate on equity

$$R_1^L(\varphi) \equiv \frac{E_1[s] - E_1[\min(s,\varphi)]}{p - E_0[\min(s,\varphi)]} \tag{1}$$

- breaking down the return:
  - unleveraged return ($> 1 \rightarrow$ pushing $\varphi \uparrow$)

$$R_1^U = \frac{E_1(s)}{p}$$

  - perceived interest rate ($\frac{\partial r_1^{per}(\varphi)}{\partial \varphi} > 0 \rightarrow$ pushing $\varphi \downarrow$)

$$r_1^{per}(\varphi) = \frac{E_1[\min(s,\varphi)]}{E_0[\min(s,\varphi)]} - 1$$

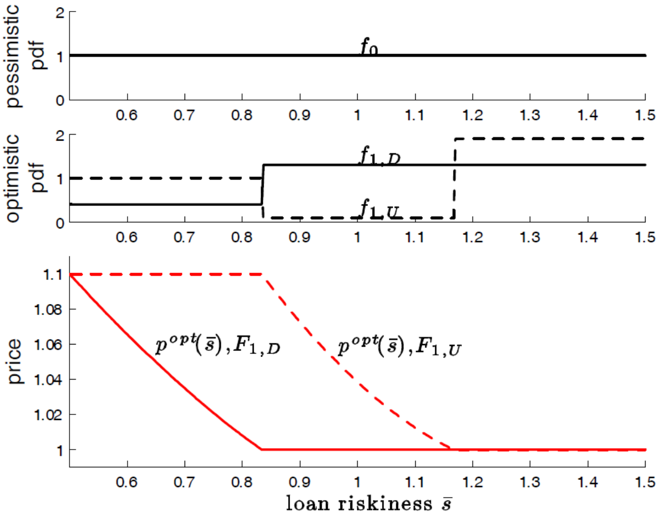
Optimal Loan Contract

- optimal loan contract $\varphi = \bar{s}$ given price p:

$$p^{opt}(\bar{s}) = F_0(\bar{s}) E_0[s|s < \bar{s}] + (1 - F_0(\bar{s})) E_1[s|s \geq \bar{s}] \quad (2)$$

- asset priced with a mixture of optimistic and pessimistic belief
  - pessimistic belief:
    - assess default probability ($F_0(\bar{s})$)
    - value of asset conditional on default ($E_0[s|s < \bar{s}]$)
  - optimistic belief:
    - value of asset conditional on no-default ($E_1[s|s \geq \bar{s}]$)
- asymmetric disciplining effect of optimism
  - optimism about prob of default states doesn't affect asset price
  - optimism about prob of non-default states increases asset price

Optimality Curve



Asset Market Clearing

- budget constraint implies a market clearing curve:

$$a_1 = \frac{n_1}{p^{mc}(\bar{s}) - E_0[\min(s, \bar{s})]} = 1$$

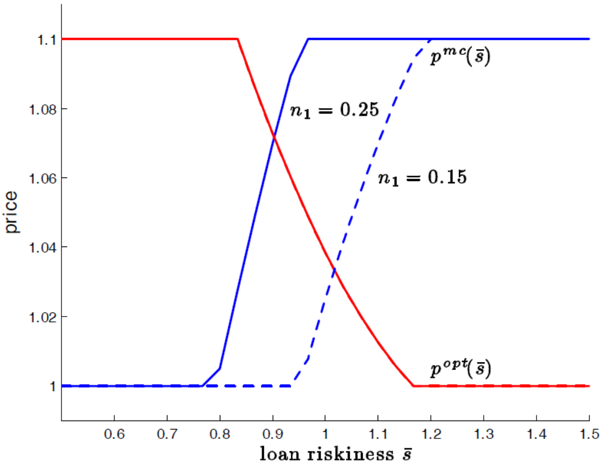
which is equivalent to

$$p^{mc}(\bar{s}) = n_1 + E_0[\min(s, \bar{s})] \tag{3}$$

- equilibrium contract and price $\{\bar{s}, p\}$ pinned down by

$$p^{mc}(\bar{s}) = p^{opt}(\bar{s})$$

Equilibrium



Nature of Belief Disagreement: Example

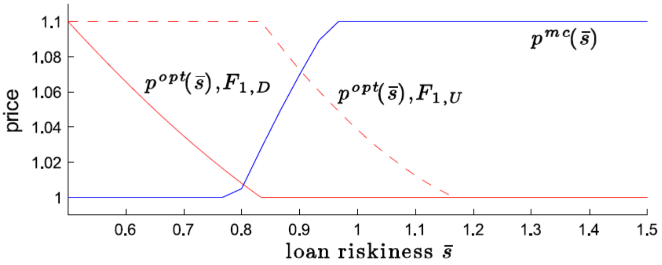


Figure: What investors disagree about matters for asset price.

Reference

- Geanakoplos, J. (2010). The leverage cycle. NBER macroeconomics annual, 24(1), 1-66.
- Simsek, A. (2013). Belief disagreements and collateral constraints. Econometrica, 81(1), 1-53.
- (review paper) Fostel, A., & Geanakoplos, J. (2014). Endogenous collateral constraints and the leverage cycle. Annu. Rev. Econ., 6(1), 771-799.
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- (review paper) Xiong, W. (2013). Bubbles, crises, and heterogeneous beliefs (No. w18905). National Bureau of Economic Research.