Two Models of Firm Dynamics

Hopenhayn (1992)
Cooley and Quadrini (2001)

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Stylized Facts about Firm Dynamics

- size
  - size distribution of firms is skewed to the right and
  - the skewness of a cohorts size distribution declines with age
- investment
  - investment growth decreases with size and age, both unconditionally and conditionally
- employment
  - employment growth decreases with size and age, both unconditionally and conditionally
- entry and exit
  - exit hazard rate declines with age
  - entry rate is procyclical
  - exit rate is countercyclical

Classic Models of Firm Dynamics

- *Hopenhayn (ECMA, 1992)
  - workhorse model of industry dynamics
  - endogenous stationary distribution with entry-and-exit
  - no aggregate uncertainty
  - frictionless (except a fixed operation cost)
- Gomes (AER, 2001)
  - adding capital
  - financial friction
- *Cooley and Quadrini (AER, 2001)
  - financial market friction
  - persistent shock
- Clementi and Palazzo (AEJ: Macro, 2016)
  - real friction
  - aggregate shock
- Begenau and Salomao (RFS, 2018)
  - financial friction
  - business cycle

Hopenhayn (1992)

- discrete and infinite time horizon
  - discount factor: β
- continuum of firms
  - law of large numbers holds
- homogenous product
  - exogenous aggregate demand for output
  - single input: labor
  - exogenous aggregate supply of input
- entry and exit
  - potential entrants are *ex ante* identical
  - incumbents are heterogeneous in idiosyncratic productivity

Timeline


realization of idiosyncratic prod : a
labor demand & scale of output
wage bill paid & fixed cost paid
exit or continue
realization of idiosyncratic prod : a′
period t
period t+1

Firm's Problem

- production technology:

$$f(a,n) = ay(n) = an^{\alpha}$$

  - a: idiosyncratic productivity, Markov process: a $\rightarrow$ a'
  - labor input
  - $\alpha < 1$: decreasing return to scale $\rightarrow$ optimal size
- role of fixed cost: c^f
  - generating endogenous exit
- operating profit:

$$\pi(a,p,w) = \max_n pf(a,n) - wn - c^f$$

  - optimal output denoted as $q^* := f(a, n^*)$
  - optimal input denoted as $n^* := n(a, p.w)$

Incumbent's Problem

- two decisions:
  - size of employment: one-to-one mapping from productivity (a)
  - exit
- exit decision:
  - if exit: 0
  - if not exit: *expected* operating profit
- value function:

$$v_t(a;\mu) = \pi(a,p,w) + \beta max\{0, \int v_{t+1}(a';\mu')F(da'|a)\}$$

  - μ : aggregate state (i.e., distribution, thus prices)
  - exit cut-off value a^*:

$$0 = \int v_{t+1}(a';\mu')F(da'|a^*) \qquad \text{or}$$
$$a^* = inf\{a \in A : \int v_{t+1}(a';\mu')F(da'|a^*) \geq 0\}$$

Entrant's Problem

- size of potential entrants: M_t
- one decision:
  - entry, after paying a sunk entry cost c^e
- entry decision:
  - enter if

$$\int v_t(a,\mu)g(da) \geq c^e$$

  - free entry:

$$\int v_t(a,\mu)g(da) = c^e \text{ if } M_t > 0$$

Distribution

Law of Motion:

$$\mu_{t+1}([0,a']) = \underbrace{\int_{a \geq a^*} F(a'|a)\mu_t(da)}_{\text{Continuing Incumbent}} + \underbrace{M_{t+1}G(a')}_{Entrants} \tag{1}$$

Define

$$\hat{P}_t = \begin{cases} \int_{a \in A} F(a'|a) & if \quad a \geq a^* \\ 0 \quad otherwise \end{cases}$$

$\Rightarrow$ Law of Motion:

$$\mu_{t+1} = \hat{P}_t \mu_t + M_{t+1} g \tag{2}$$

Equilibrium

- aggregate supply (endogenous)

$$Q^s(\mu_t) = \int q_t(a, \mu)\mu_t(da)$$

- aggregate demand (exogenous)

$$Q^s$$

- aggregate labor demand (endogenous)

$$N^d(\mu_t) = \int n_t(a, \mu)\mu_t(da)$$

- aggregate labor supply (exogenous)

$$N^s$$

- both markets clear at equilibrium
- focus on stationary equilibrium
  - constant distribution over time

Distribution

- Stationary Distribution:

$$\mu^* = \hat{P}\mu^* + M^* g \tag{3}$$

$$\Rightarrow \quad \mu^* = M^*(I - \hat{P})^{-1} g \tag{4}$$

  - stationary distribution is linearly homogeneous in m (scalar)
  - stationary distribution can be found by simulation as well. (appendix)

Parametrization

- entry cost parameter: c^e $\uparrow$
    - expected discounted profits: $\uparrow$
    - exit threshold a^*: $\downarrow$
    - entrants mass m^*: $\downarrow$
    - output price p^*: $\uparrow$
    - entry rate/exit rate m^*/μ^*: $\downarrow$
    - firm-size distribution: ambiguous
        - price effect: incumbents increase output q^* and employment n^*
        - selection effect: more incumbent firms are relatively-low productivity firms
        - selection effect: entrants are of better productivity

Results

- size effect
  - size of output $\leftrightarrow$ size of employment $\leftrightarrow$ productivity draw
  - unconditionally large firms have lower growth rate on average
- age effect
  - unconditionally old firms have lower growth rate on average
  - firms age as they survive in the market over time
  - no conditional age effect
- frictionless environment
  - model: young firms are small because they have lower draw on productivity
  - data: young firms are small not because they are inefficient
- Next Step: adding frictions to Hopenhayn (1992)

Cooley and Quadrini (2001)

- persistent shock + financial constraint $\rightarrow$ size + age effect
    - conditional on age, the dynamics of firms are negatively related to the size of firms
    - conditional on size, the dynamics of firms are negatively related to the age of firms
- capture the features of the financial behavior of firm
    - small and younger firms pay fewer dividends, take on more debt, and invest more
    - small firms have higher values of Tobin's q
    - investment of small firms is more sensitive to cash flows
- financial frictions
    - equity: cost or premium associated with increasing equity
    - debt: costly default
    - trade-off theory

Intuition: firm's problem

- a stylized and simplified model
- decreasing return to scale production technology:

$$y = af(k + b)$$

  - a: idiosyncratic productivity, i.i.d
  - k: owned capital (equity), no depreciation
  - b: borrowed capital (financed with debt)

- borrowing constraint:

$$b \leq k$$

  - interest rate: r

- value function:

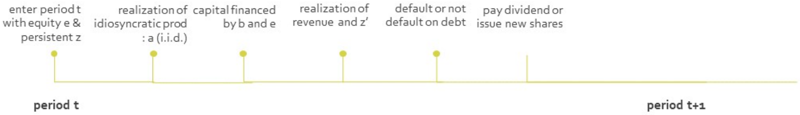
$$v(k, b) = \max_{k', b'} af(k + b) - br - (k' - k) + \beta \int v(z', k')F(dz')$$

  - efficient size: $E\{af'(k^* + b^*)\} = r$

Intuition: constrained firms

- optimal borrowing:
$$b' = k'$$
- capital accumulation:
$$k' = af(2k) - rk + k$$
- growth rate:
$$\frac{k' - k}{k} = \frac{af(2k)}{k} - r$$
    - decreasing in k
- financial constraint impedes firms to jump directly to their efficient size.

Full Model



- depreciation
- inter-temporal debt
- compound idiosyncratic shocks: persistent + transitory
- financial market frictions

Firm's problem

- net worth end-of-period:

$$\pi(e, b, z + a) = (1 - \phi)(e + b) + (z + a)f(e + b) - (1 + \tilde{r})b$$

  - a: transitory productivity (accidents), i.i.d, unexpected
  - z: persistent productivity (technology), Markov process: z $\rightarrow$ z', revealed 1 period in advance
  - e: equity (asset)
  - ϕ: cost of capital (depreciation)
  - $\tilde{r}$: interest rate charged by intermediary

- endogenous default: threshold i.i.d shock $\underline{a}$ implicitly defined by

$$\underbrace{(1 - \phi)(e + b) + (z + \underline{a})f(e + b) - (1 + \tilde{r})b}_{=\pi(e,b,z+\underline{a})} = \underline{e}(z')$$

  - default if value of continuation is less than zero
  - threshold net worth of default: $\underline{e}(z')$
  - $e(z') < \underline{e}(z') \Rightarrow$ liability renegotiated until $e(z') = \underline{e}(z')$

External Finance: Debt

- interest rate:

$$(1+r)b = (1+\tilde{r})b \int_{\underline{a}}^{\infty} g(da) + \int_{-\infty}^{\underline{a}} [(1-\phi)(e+b)+(z+a)f(e+b)-\xi]g(da)$$

  - r: risk-free interest rate
  - ξ: default loss
- $\Rightarrow$ threshold i.i.d. shock $\underline{a} = \underline{a}(z, e, b, z')$:

$$(1+r)b+\underline{e}(z') \int_{\underline{a}}^{\infty} g(da)+\xi \int_{-\infty}^{\underline{a}} g(da) = (1-\phi)(e+b)+h(\underline{a})F(e+b) \tag{5}$$

  - where $h(\underline{a}) = z + \underline{a} \int_{\underline{a}}^{\infty} g(da) + \int_{-\infty}^{\underline{a}} ag(da)$

External Finance: Debt

- sequence of decisions: (a). default $\rightarrow$ (b). equity issuance/ dividend payment $\rightarrow$ (c). next period debt
- default does not lead to exit of the firm
- debt is re-negotiated after default
  - if $\pi(e, b, z + a) < \underline{e}(z')$, intermediary loss $= \underline{e}(z') - \pi(e, b, z + a)$
- end-of-period net worth

$$q(e, b, z + a, z') = \begin{cases} \underline{e} + (a - \underline{a})f(e + b), & \text{if} \quad a \geq \underline{a}(z, e, b, z') \\ \underline{e}, \quad \text{if} \quad a \leq \underline{a}(z, e, b, z') \end{cases} \tag{6}$$

External Finance: Equity

- sequence of decisions: (a). default $\rightarrow$ (b). equity issuance/ dividend payment $\rightarrow$ (c). next period debt
- equity finance:

$$d(x, e') = \begin{cases} x - e', & if \quad x \geq e' \\ (x - e')(1 + \lambda), & if \quad x \leq e' \end{cases} \tag{7}$$

  - where x: end-of-period equity of the firm before (b)
  - if $d(x, e')$ is positive, firm pays dividend;
  - if $d(x, e')$ is negative, firm issues equity, at cost λ;

Firm's Problem

- sequence of decisions: (a). default $\rightarrow$ (b). equity issuance/ dividend payment $\rightarrow$ (c). next period debt
- value of the firm at the end of the period after (b) but before (c):

$$\Omega(z,e) = \max_b\{\beta \sum_{z'} \int_{\underline{a}} \tilde{\Omega}(z', q((e,b,z+a,z'))\Gamma(z'|z)f(da)\} \quad (8)$$

s.t. equation (5) and (6)

- where $\tilde{\Omega}(z,e)$: end-of-period value after (a) but before (b) s.t.

$$\tilde{\Omega}(z', \underline{e}) = 0$$

s.t.

$$\tilde{\Omega}(z', x) = \max_{e'}\{d(x,e') + \Omega(z', e')\}$$

s.t. equation (7)

Proposition 3

PROPOSITION 3: There exists a unique function $\Omega^*(z, e)$ that satisfies the functional equation (8). In addition, if for a_1 and a_2 sufficiently small, $g(a) < a_1$ for all $a < -a_2$, then

- the firm's solution is unique, and the policy rule b(z, e) is continuous in e;
- the input of capital k = e + b(z, e) is increasing in e;
- there exist functions $\underline{e}(z) < \hat{e}(z) < \bar{e}(z)$, $z \in Z$, for which the firm renegotiates the loan if the end-of-period resources are smaller than $\underline{e}(z)$, will issue new shares if they are smaller than $\hat{e}(z)$, and distribute dividends if they are bigger than $\bar{e}(z)$;
- the value function $\Omega^*(z, e)$, is strictly increasing and strictly concave in $[\underline{e}, \bar{e}]$.

Proposition 3: Comment

There exist functions $\underline{e}(z) < \hat{e}(z) < \bar{e}(z)$:

- if $e < \hat{e}(z)$: the firm issues new shares to increase equity level to $\hat{e}(z)$, as marginal increase in value w.r.t. $e > 1 + \lambda$
- if $\hat{e}(z) < e < \bar{e}(z)$: the firm will not issue new shares, as marginal increase in value w.r.t. $e < 1 + \lambda$
- if $\bar{e}(z) < e$: the firm distribute dividends,as marginal increase in value w.r.t. $e < 1$
- who issues equity?
  - with relatively lower net worth
  - with improvement in technology

Entrants

- new firms are created with an initial value of equity raised by issuing new shares to an optimal size: $\hat{e}(z)$
- the cost of creating a new firm with initial productivity z:

$$\kappa + (1+\lambda)\hat{e}(z)$$

- surplus of entry:

$$\Omega(z, \hat{e}(z)) - \kappa - (1+\lambda)\hat{e}(z)$$

- free entry (general equilibrium property)

$$\Omega(z_N, \hat{e}(z_N)) = \kappa + (1+\lambda)\hat{e}(z_N)$$

- invariant measure of firms μ^* exists.

i.i.d shock: role of financial friction

- z takes only two values: an absorbing shock $z_0 = 0$ and z_1
- conditional on surviving, the shock is i.i.d.
- isolate the financial mechanisms from persistence mechanism
- key properties of the financial behavior of firm detail
  - small firms take on more debt (higher leverage).
  - small firms face higher probability of default.
  - small firms have higher rates of profits.
  - small firms issue more shares and pay fewer dividends.
- key properties of firm dynamics detail
  - small firms grow faster and experience higher volatility of growth.
  - small firms face higher probability of default.
  - small firms experience higher rates of job reallocation.
  - *without conditioning on size*, young firms experience higher rates of growth, default, and job reallocation.

persistent shock: interaction with financial friction

- conditional on surviving, z follows a symmetric two-state Markov process
- firms differ over two dimensions: equity and productivity
- conditional on equity size, high productivity firms borrow more and implement larger production scales. trade-off:
  - a larger production scale allows higher expected profits
  - a larger production scale implies higher volatility of profits
- size dependence and age dependence in the dynamics of firm detail
  - unconditionally and conditionally
  - size dependence as before
  - age dependence derives from heterogeneous composition of firm types in each age class of firms
  - effect of entry: initial productivity of new firms
  - effect of persistent shock
  - age effect is more important for small firms; and it almost disappears for very large firms.

Conclusion

- Hopenhayn (1992)
  - frictionless environment (except fixed cost)
  - size dependence
  - no conditional age dependence
- *Jovanovic (1982)
  - learning model
  - age dependence
  - no conditional size dependence
- Cooley and Quadrini (2001)
  - financial friction and persistent effect
  - conditional size dependence
  - conditional age dependence
  - reconcile size-dependent financial features
- Next Step:
  - richer heterogeneity
  - business cycle

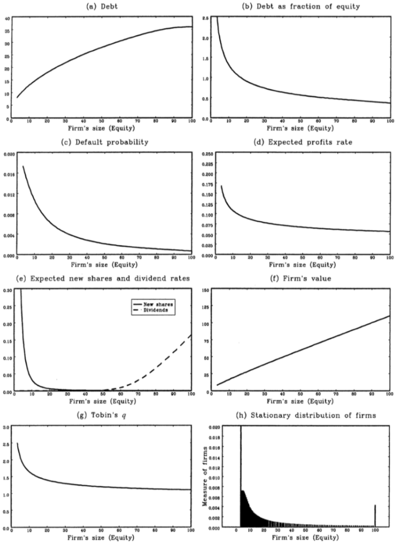


FIGURE 3. FINANCIAL BEHAVIOR WITH i.i.d. SHOCKS

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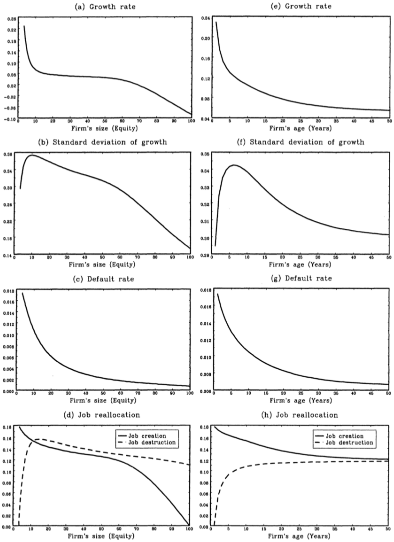

Figure 5. Industry Dynamics with i.i.d. Shocks

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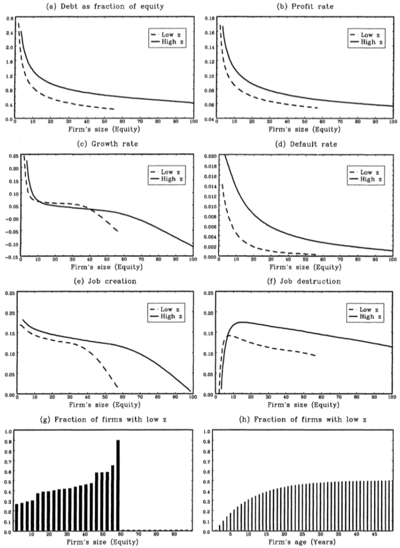

FIGURE 6. FINANCIAL BEHAVIOR AND INDUSTRY DYNAMICS FOR LOW AND HIGH PRODUCTIVITY FIRMS

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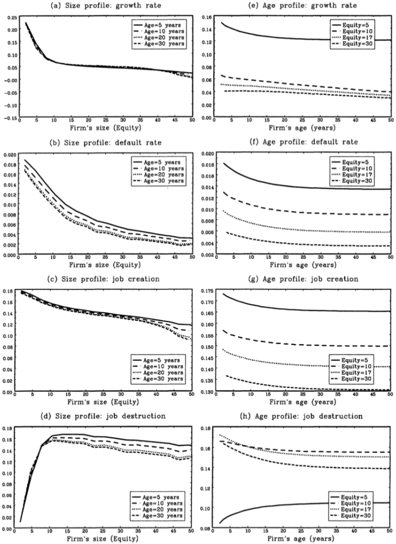

FIGURE 7. INDUSTRY DYNAMICS CONDITIONAL ON AGE AND SIZE

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