A Note on Martin (2008)

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Contents

1 Introduction

1.1 Contribution

- large literature: financial markets and macroeconomic fluctuations
  - financial system as amplifier of exogenous shocks
  - lax credit and rapid expansion of output as the seeds of a future downturn ?
- this research: financial market as a source of macroeconomic fluctuations
  - exhibition of fluctuations absent of exogenous shock
  - endogenous boom-bust cycles

1.2 Highlight

- adverse selection
  - borrowers with private information: good or bad
  - credit contract under asymmetric information
  - endogenous change of lending standards as source of fluctuations
- net worth and lending standard
  - higher net worth $\Leftrightarrow$ more investment
  - low net worth $\Rightarrow$ costly separation $\Rightarrow$ pooling contract
  - high net worth $\Rightarrow$ easier separation $\Rightarrow$ separating contract
- regime switch and fluctuation
  - low net worth $\Rightarrow$ pooling contract $\Rightarrow$ higher investment $\Rightarrow$ higher net worth $\Rightarrow$ separating contract $\Rightarrow$ lower investment $\Rightarrow$ low net worth

1.3 Empirical

- procyclical net worth and endogenous reversion into recession
- lending standard over the business cycles

2 Model

2.1 Set-up

The model features overlapping generations that live two periods: young and old. A new generation of measure one is born at every period. The utility goal of the young is to maximize expected old-age consumption of final goods. The young is endowed with one unit of labor and supply it inelastically. Their save their labor income in the production of capital goods. The old own the capital stock and live off their capital income. In each period, the labor supplied by the young is combined with capital owned by the old to produce final consumption goods according to a constant-return-to-scale technology. Capital fully depreciates after utilization.

- production technology of final product:

$$y_t = \theta g(k_{t-1}, 1) \tag{1}$$

- wage received by the young:

$$w_t(k_{t-1}) = \theta[g(k_{t-1}) - k_{t-1}g'(k_{t-1})] \tag{2}$$

- capital gain received by the old

$$q_t(k_{t-1}) = \theta g'(k_{t-1}) \tag{3}$$

2.2 Capital

Only a fraction of the young population, referred to as entrepreneurs, have the technology to produce capital. Entrepreneurs invest in production of capital when they are young, and consume capital gain when they are old. There are two types of entrepreneurs, a measure of λ^G being good (G) and λ^B being bad (B). We assume $\lambda^G + \lambda^B < 1$, and measure $1 - \lambda^G - \lambda^B$ are households. The technology is heterogeneous in the following way:

- The investment made by the young entrepreneur can either succeed or fail in subsequent period. The probability of success is p^j, and we assume $p^G > p^B$.
- When the investment succeeds, an entrepreneur who invests I unit of consumption good receive $\alpha^j f(I)$ units of capital [1]. We assume $\alpha^G < \alpha^B$.
- When the investment fails, an entrepreneur receives nothing. We assume $p^G\alpha^G > p^B\alpha^B$.

[1] The function $f()$ is an increasing, concave, and satisfies Inada condition.

2.3 Credit Market and Financial Contract

There is natural demand for credit market in this economy: while entrepreneurs can invest their wage income directly into production of capital, household needs to save their income. At the meantime, entrepreneurs may need external source to finance their investment.

- financial intermediary: competitive and risk neutral banks who take deposit with promised gross interest r_t.
- loan contract is characterized by (I_t, R_t, c_t), where
  - I_t: amount of consumption goods lent to borrower
  - R_t: gross interest rate on the loan
  - c_t: percentage of the loan that entrepreneur save as collateral using their own wealth.
- state-contingent repayment and default outcomes
  - success: entrepreneur repays $R_t I_t$ and claims residual value of project
  - failure: bank takes collateral plus interest rate and take residual value (=0)
- expected profit of entrepreneur j:

$$\pi^j(I_t, R_t, c_t) = r_t w_t + p^j[q^e_{t+1}\alpha^j f(I_t) - R_t I_t] - (1-p^j)r_t c_t I_t \tag{4}$$

- expected profit of bank from the contract:

$$\pi^b(I_t, R_t, c_t) = p^j R_t I_t + (1-p^j)r_t c_t I_t - r_t I_t \tag{5}$$

2.4 Full Information

Here in this section we discuss main property of loan contract in a partial equilibrium setting, where deposit interest rate r and expected rental price of capital q^e are taken as given. Under full information, the equilibrium $\{I^{j*}_t, R^{j*}_t, c^{j*}_t\}$ is straightforward:

- optimal size of funding I^{j*}_t:

$$f'(I^{j*}_t) = \frac{r}{q^e \alpha^j p^j} \quad \text{for j=G,B} \tag{6}$$

- collateral required by banks c^{j*}_t and gross interest rate R^{j*}_t:

$$p^j R^{j*}_t + (1-p^j)c^{j*}_t r = r \quad \text{for j=G,B} \tag{7}$$

Thus under full information, 1) good entrepreneurs invest more than bad entrepreneurs; 2) banks break even, and 3) investment is independent of entrepreneurs' wealth w_t.

- i.e., gross interest rate R_t^{j*} (not unique) if $w_t = 0$:

$$R_t^{j*} = \frac{r}{p^j} \quad \text{for j=G,B} \tag{8}$$

2.5 Asymmetric Information

Now consider the case of asymmetric information, where *ex ante* banks are unable to distinguish among different types of borrowers. Following Hellwig (1987), contract at credit market is modelled in three stages:

- 1st stage: banks design contract;
- 2nd stage: entrepreneurs apply for these contract;
- 3rd stage: banks accept or reject applications.

We assume exclusivity and no cross-subsidization that 1) entrepreneurs can apply to no more than one contract; 2) banks are not allowed to offer contracts that lose money in expectation. The following equilibrium contracts are characterized for an economy indexed by $\{r, q^e, w_t\}$.

2.5.1 separating equilibrium: $C^{SEP}(r, q^e, w_t)$

Definition 1: Given $\{r, q^e, w_t\}$, a separating equilibrium is characterized by contracts $\{(I_t^G, R_t^G, c_t^G), (I_t^B, R_t^B, c_t^B)\}$ that satisfy the following constraints:

- feasibility:

$$c_t^j \in [0, \frac{w_t}{I_t^j}] \quad \text{for j=G,B} \tag{9}$$

- incentive compatibility:

$$\pi^j(I_t^j, R_t^j, c_t^j) \geq \pi^j(I_t^i, R_t^i, c_t^i) \quad \text{for } i \neq j \text{ and } i, j \in \{G, B\} \tag{10}$$

- break-even condition for banks:

$$p^j R_t^j + (1 - p^j) c_t^j r = r \quad \text{for j=G,B} \tag{11}$$

- no deviation for banks.

Proposition 1: Given $\{r, q^e, w_t\}$, a separating equilibrium is characterized by contracts $\{(I_t^G, R_t^G, c_t^G), (I_t^B, R_t^B, c_t^B)\}$ that satisfy:

- contract chosen by the bad-type is not distorted:

$$(I_t^B, R_t^B, c_t^B) = (I_t^{B*}, R_t^{B*}, 0) \tag{12}$$

- contract chosen by the good-type is distorted[2]:

$$c_t^G = \frac{[q^e p^B \alpha^B f(I_t^G) - \frac{p^B}{p^G} I_t^G r] - [q^e p^B \alpha^B f(I_t^B) - I_t^B r]}{(1 - \frac{p^B}{p^G}) I^B r} \leq 1 \tag{13}$$

$$q^e \alpha^G p^G f'(I_t^G) > r \quad \Rightarrow \quad c_t^G = \frac{w_t}{I_t^G} \tag{14}$$

Proposition 1 implies that cost of separation is undertaken by good-type entrepreneurs who either provide higher level of collateral or choose lower level of investment, aka lower leverage.

- collateral: a costless way of screening / separating entrepreneurs.
  - good-type entrepreneurs are willing to increase c^G to lower R^G
  - bad-type entrepreneurs are worse off
  - separation in this way becomes very costly when w_t is low [3]
  - increase in w_t enhances the probability of separation via collateralization.
  - for sufficiently high w_t, first-best can be achieved: $I_t^G = I_t^{G*}$

The equilibrium doesn't always entail separation. A pooling equilibrium may exist whenever it Parato dominates the separating contract of *Proposition 1*.

[2]The good type solves the following problem:

$$\max_{I^G, R^G, c^G} \pi^G \equiv rw + p^G[q^e \alpha^G f(I^G) - R^G I^G] - (1 - p^G) c^G I^G r$$

s.t.

$$p^G R^G + (1 - p^G) c^G r = r = p^B R^B$$
$$p^B[q^e \alpha^B f(I^B) - R^B I^B] = p^B[q^e \alpha^B f(I^G) - R^G I^G] - (1 - p^B) c^G I^G r$$
$$c^G \in [0, \frac{w}{I^G}]$$

[3]It can be shown that when $w_t = 0$, $I_t^G < I_t^B$ (inefficiency).

2.5.2 pooling equilibrium: $C^{POOL}(r, q^e, w_t)$

Definition 2: Given $\{r, q^e, w_t\}$, a pooling equilibrium is characterized by contracts $\{(\bar{I}_t, \bar{R}_t, \bar{c}_t)\}$ that satisfy the following constraints:

- feasibility:

$$\bar{c}_t \in [0, \frac{w_t}{\bar{I}_t}] \tag{15}$$

- break-even condition for banks:

$$E_j[p^j \bar{R}_t + (1 - p^j)\bar{c}_t r] = r \tag{16}$$

- no deviation for banks.

Proposition 2: Given $\{r, q^e, w_t\}$, a pooling equilibrium $\{(\bar{I}_t, \bar{R}_t, \bar{c}_t)\}$ satisfies [4]:

- gross interest rate

$$\bar{R}_t = r\frac{1 - (1 - \bar{p})\bar{c}_t}{\bar{p}} \tag{17}$$

- collateral requirement

$$\bar{c}_t = \frac{w_t}{\bar{I}_t} \tag{18}$$

- investment size

$$p^G \alpha^G f'(\bar{I}_t) = \frac{p^G}{\bar{p}}\frac{r}{q^e} \tag{19}$$

Proposition 2 implies the following properties of pooling equilibrium:

- investment size is independent of wealth w_t
- collateral constraint is binding and is increasing with wealth w_t
- degree of cross-subsidization is decreasing with wealth w_t

[4] At pooling equilibrium good-type entrepreneurs solve the following problem:

$$\max_{\bar{I},\bar{c}} \pi^G \equiv rw + p^G[q^e \alpha^G f(\bar{I}) - \bar{R}\bar{I}] - (1 - p^G)r\bar{c}\bar{I}$$

s.t.

$$\bar{p}\bar{R} + (1 - \bar{p})\bar{c}r = r$$
$$0 \leq \bar{c}$$
$$\bar{c} \leq \frac{w}{\bar{I}}$$

2.5.3 equilibrium contract: $C^{EQ}(r, q^e, w_t)$

- Separating or pooling contract?
  - Depend on the level of wealth w_t
  - when w_t is low, separation is costly: i.e., for $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, the equilibrium is always pooling when $w_t = 0$.
  - when w_t is increased, the separating equilibrium emerges.
  - cut-off for regime switch: $w^*(r, q^e)$
- what's the impact of regime switch on aggregate investment?
  - investment drops as long as $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$
  - when good-type is abundant, i.e. $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, pooling equilibrium represents mostly technology of the good-type: $\bar{I}_t(r, q^e) > I_t^B(r, q^e)$
  - switch to separation contracts investment made by bad-type (obvious).
  - switch to separation contracts investment made by good-type at the margin [5]
  - aggregate investment is discontinuous at the switching point:

$$\bar{I}_t(r, q^e) > I_t^G(r, q^e, w^*) > w^* > I_t^B(r, q^e, w^*)$$

2.6 Endogenous Cycles

- timeline
  - investment project undertaken by the old yields capital stock of the economy;
  - production of final goods takes place using capital and labor supplied by the young
  - the old repay their debt; the young save their labor income and invest.
- assumptions
  - unique, stable steady state at full information
  - parameter: $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$
  - exogenous interest rate: r

[5] *Prima facie* this result is surprising. The intuition is as follows: Good-type entrepreneurs are indifferent between pooling and separating equilibrium at the switching point. Given that pooling contract provides fund at a higher cost due to cross-subsidization, it must entail higher level of investment compared to separating equilibrium. In other words, $\bar{I}_t(r, q^e) > I_t^G(r, q^e, w^*)$.

Definition 3: *intertemporal equilibrium* of the asymmetric information economy is defined as a trajectory $\{k_t, w_t, q^e_{t+1}, r_t, C^{EQ}(w_t, q^e_{t+1}) : t \geq 0\}$ that satisfies

- contract $C^{EQ}(w_t, q^e_{t+1})$ as characterized before
- labor and capital market clears: w_t and q_t
- perfect foresight: $q^e_{t+1} = q_{t+1}$

2.6.1 full information: no dynamics

The equilibrium under full information is trivial as in section 2.4, where

- optimal size of funding I^{j*}_t independent of state variables:

$$\alpha^j p^j f'(I^{j*}_t) = \frac{r}{q^e_{t+1}} \quad \text{for j=G,B}$$

- perfect foresight:

$$q^e_{t+1} = q_{t+1} = \theta g'[k^*_t(r_t, q^e_{t+1})] \tag{20}$$

- capital stock $k^*_t(r_t, q^e_{t+1})$ independent of state variables:

$$k^*_t(r_t, q^e_{t+1}) = \lambda^G \alpha^G p^G f[I^{G*}_t(r_t, q^e_{t+1})] + \lambda^B \alpha^B p^B f[I^{B*}_t(r_t, q^e_{t+1})] \tag{21}$$

Applying assumption 1 regarding existence and uniqueness of steady state, the economy always converges to a unique equilibrium denoted as $\{k^*, w^*, q^*\}$.

2.6.2 pooling regime: no dynamics

The consideration of pooling regime under asymmetric information resembles that under full information, where

- optimal size of funding I^{j*}_t independent of state variables:

$$\alpha^j p^j f'(\bar{I}_t) = \frac{r}{q^e_{t+1}} \frac{p^G}{\bar{p}}$$

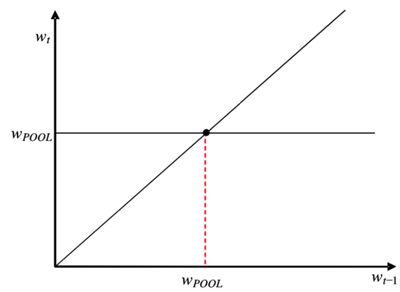
- perfect foresight:

$$q^e_{t+1} = q_{t+1} = \theta g'[k^{POOL}_t(r_t, q^e_{t+1})] \tag{22}$$

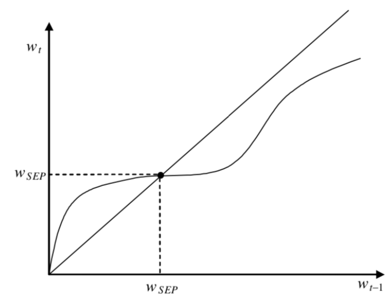
- capital stock $k^{POOL}_t(r_t, q^e_{t+1})$ independent of state variables:

$$k^{POOL}_t(r_t, q^e_{t+1}) = [\lambda^G \alpha^G p^G + \lambda^B \alpha^B p^B] f[\bar{I}_t(r_t, q^e_{t+1})] \tag{23}$$

Applying assumption 1 regarding existence and uniqueness of steady state, there is unique and stable steady state in the pooling regime as well. We denote this unique equilibrium as $\{k^{POOL}, w^{POOL}, q^{POOL}\}$. Similar to full information regime, for any w_{t-1} in the pooling equilibrium, $w_t = w^{POOL}$ so that the economy jumps to the steady state regardless of initial condition, i.e. there are no dynamics in the pooling regime.



Wage Dynamics under Pooling Contracts



Wage Dynamics under Separating Contracts

2.6.3 separating regime dynamics

Contrary to previous two regimes, there are dynamics in the separating regime: higher w_t $\Rightarrow$ higher investment $\Rightarrow$ higher w_{t+1}. In the separating regime,

- level of investment $I_t^{B,SEP}(r_t, q_{t+1}^e)$ independent of w_t:

$$\alpha^B p^B f'(I_t^{B,SEP}) = \frac{r}{q_{t+1}^e}$$

  $I_t^{G,SEP}(r_t, q_{t+1}^e, w_t)$ dependent on w_t

$$\frac{w_t}{I_t^{G,SEP}} = \frac{[q^e p^B \alpha^B f(I_t^{G,SEP}) - \frac{p^B}{p^G} I_t^{G,SEP} r] - [q^e p^B \alpha^B f(I_t^{B,SEP}) - I_t^{B,SEP} r]}{(1 - \frac{p^B}{p^G}) I^{B,SEP} r}$$

- perfect foresight:

$$q_{t+1}^e = q_{t+1} = \theta g'[k_t^{SEP}(r_t, q_{t+1}^e, w_t)] \tag{24}$$

- capital stock $k_t^{SEP}(r_t, q_{t+1}^e, w_t)$:

$$k_t^{SEP}(r_t, q_{t+1}^e, w_t) = \lambda^G \alpha^G p^G f[I_t^{G,SEP}(r_t, q_{t+1}^e, w_t)] + \lambda^B \alpha^B p^B f[I_t^{B,SEP}(r_t, q_{t+1}^e)] \tag{25}$$

The economy might display unique, stable steady state or multiple steady states. Here we restrict our attention to the former and denote the economy as $\{k^{SEP}, w^{SEP}, q^{SEP}\}$.

Assumption:

$$w^{SEP} < w^{POOL} \tag{26}$$

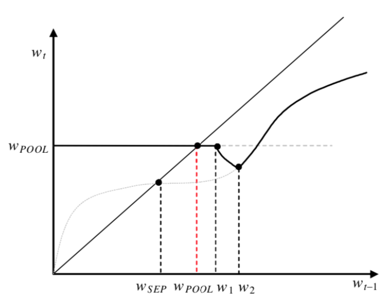
2.6.4 regime switching and cycles

Proposition 3: Assume an economy in which $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$. For wage $w_t \in [0, \bar{w}]$, there exists a unique pair of switching wages (w_1, w_2) such that:

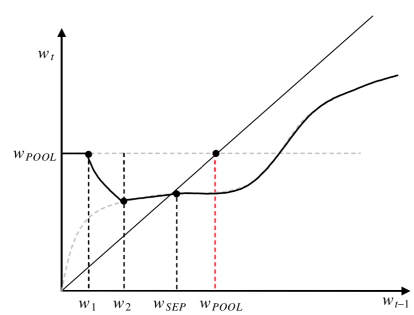
- if $w_t \leq w_1$, then the equilibrium loan contracts at time t are pooling;
- if $w_t \geq w_2$, then the equilibrium loan contracts at time t are separating;
- if $w_1 \leq w_t \leq w_2$, then the equilibrium loan contracts at time involve randomization between pooling and separating contracts.

Now we proceed to consider the following cases:

- Case 1: $w^{SEP} < w^{POOL} \leq w_1$:
  - unique, stable steady state at w^{POOL}
  - oscillatory convergence
  - monotonic convergence for initial $w_0 < w_1$
  - convergence with overshooting for some initial $w_0 > w_1$
- Case 2: $w_2 \leq w^{SEP} < w^{POOL}$:
  - unique, stable steady state at w^{SEP}
  - oscillatory convergence
  - monotonic convergence for initial $w_0 > w_2$
  - convergence with overshooting for some initial $w_0 < w_2$



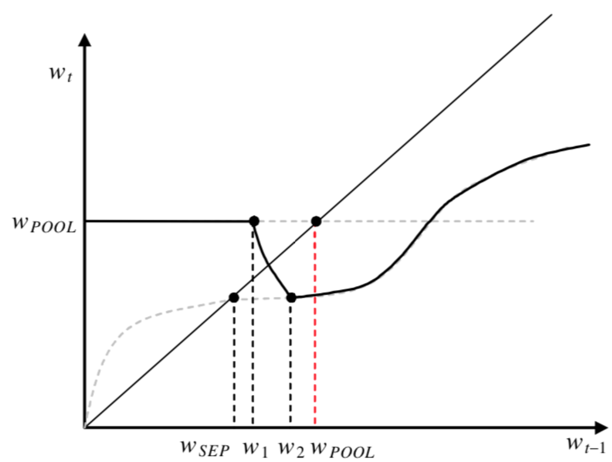
Wage Dynamics under Case 1: $w_{POOL} \leq w_1$



Wage Dynamics under Case 2: $w_{SEP} \geq w_2$

- Case 3: $w_1 < w^{POOL}; w^{SEP} < w_2$:
  - unique steady state at w^{SEP}
  - unstable steady state: permanent fluctuation
  - stable steady state: convergence with fluctuation

The last case is of particular interest: an economy with no dynamic under full information displays fluctuation in the presence of adverse selection [6]. The intuition is straightforward: For low level of w_t, separation is costly so that the economy is at pooling regime where investment and wages gradually build up. When the increase is wealth is sufficiently large, the economy switches to equilibrium with partial or complete separating contracts, and consequently, a fall in output. The decrease in output, in turn, decreases entrepreneurs' wealth and the economy goes back to pooling regime. In that sense, the economy features endogenous cycle without introduction of exogenous shock.



Wage Dynamics under Case 3: $w_{SEP} < w_2$ and $w_{POOL} > w_1$

[6] *Proposition 4* proves existence case 3 in any economy satisfying case 1 and 2.

3 Conclusion

- implication 1: financial friction
  - investment is increasing with net worth at separating regime
  - investment is independent of net worth at pooling regime
  - investment is more sensitive to net worth at recession (Bernanke, Gertler, & Gilchrist, 1999)
- implication 2: bank lending standard
  - changes in lending standards are determined by economy activity (wealth)
  - changes in lending standards are determinant of economy activity (investment)
  - procyclical loan size and countercyclical rates of collateralization
  - "lax" lending standard associated with low variance of interest rate (pooling)
  - "tight" lending standard associated with high variance of interest rate (separating)
- implication 3: positive productivity shock
  - net worth increases $\Rightarrow$ aggregate investment increases (amplification)
  - aggregate savings increase $\Rightarrow$ aggregate investment decrease (mitigation)
  - closed economy vs. open economy
  - financial liberalization and macroeconomic stability
- implication 4: sources of fluctuation
  - no aggregate shock
  - adverse selection $\Rightarrow$ changes in lending standard
  - perfect competition in credit market
- future directions:
  - OLG $\Rightarrow$ infinite horizon: endogenize interest rate r
  - liquidity and macroeconomy (Taddei, 2010)
  - endogenize distribution of different types: extensive margin problem (Hu, 2017) (Fishman, Parker, Straub, et al., 2019)

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