Financial Network and Contagion

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Balance Sheet: An Example of Bank

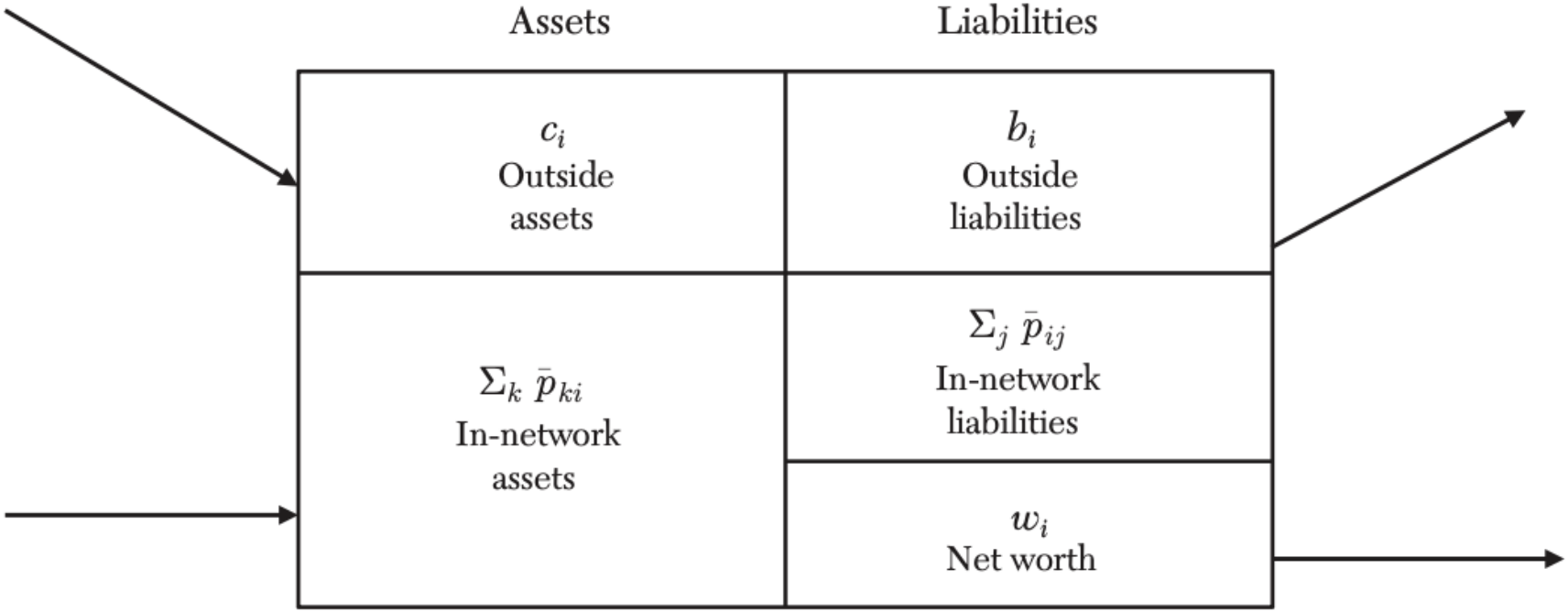


Figure 2. A Stylized Balance Sheet for Bank i

Balance Sheet: An Example of Bank

- Assets:
  - Outside assets: claims on nonfinancial entities, e.g. mortgages and commercial loans.
  - In-network assets: claims on other banks, including interbank loans and exposures through derivatives.
- Liabilities:
  - Outside liabilities: to non-financial entities, e.g. depositors
  - In-network liabilities: to other banks
- Net worth: assets - liabilities
- Links between balance sheets define a network.

Contagion in Financial Network

- A shock to bank i's assets: $c_i \downarrow$
  - e.g. a drop in the value of real estate or in an industrial sector with bank loans
- A mild drop in $c_i \Rightarrow$ net worth $w_i \downarrow$
- A large drop in $c_i \Rightarrow$ net worth $w_i = 0 \Rightarrow \bar{p}_{ij} \downarrow$ (default)
  - Bank j may default, and so on
- Spill-over of initial shock can create a cascade of defaults.

Research Questions

- Role of financial network
  - Diversifying firms' risk exposures
  - Creating channels of contagion
  - Literature: Allen and Gale (2000); Gai, Haldane, and Kapadia (2011)
- Central questions
  - 1. What are the reasons for the growing inter-connectedness of the financial system?
  - 2. Do more connections tend to amplify or dampen systemic shocks?
  - 3. Does the structure of the network matter?
  - 4. What structural features are relevant for setting policy?
- This paper:
  - Focus on Q2, taken network structure as given

Set Up

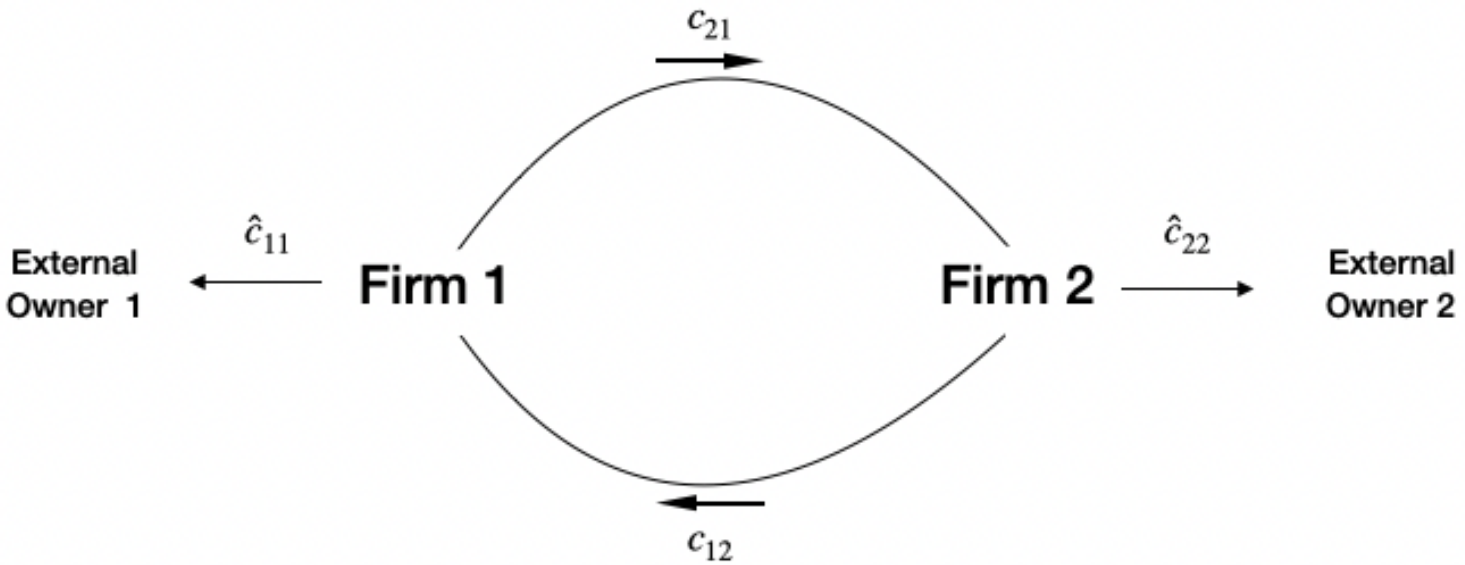


Figure: Ownership Structure (Example)

Ownership Structure

- Cross-holding: matrix C
  - C_{ij}: fraction of firm j owned by firm i
  - $C_{ii} = 0$: a firm does not 'cross' own itself
  - diagonal entries of the matrix C defined to be 0
- Private-ownership: matrix $\hat{C}$
  - $\hat{C}_{ii} = 1 - \sum_j C_{ji}$: fraction of firm i owned by outsiders
  - off-diagonal entries of the matrix $\hat{C}$ defined to be 0
- Asset (business):
  - p_k: present value of asset k
  - D_{ik}: fraction of asset k 'directly' owned by firm i

Claim

- Book value:

$$V_i = \sum_k D_{ik} p_i + \sum_j C_{ij} V_j \tag{1}$$

or in matrix form:

$$V = (I - C)^{-1} Dp \tag{2}$$

where I: identity matrix; C: matrix of cross-holding; p: vector of asset value; D: matrix of direct ownership

- Adjustment for double-counting, value to private owner

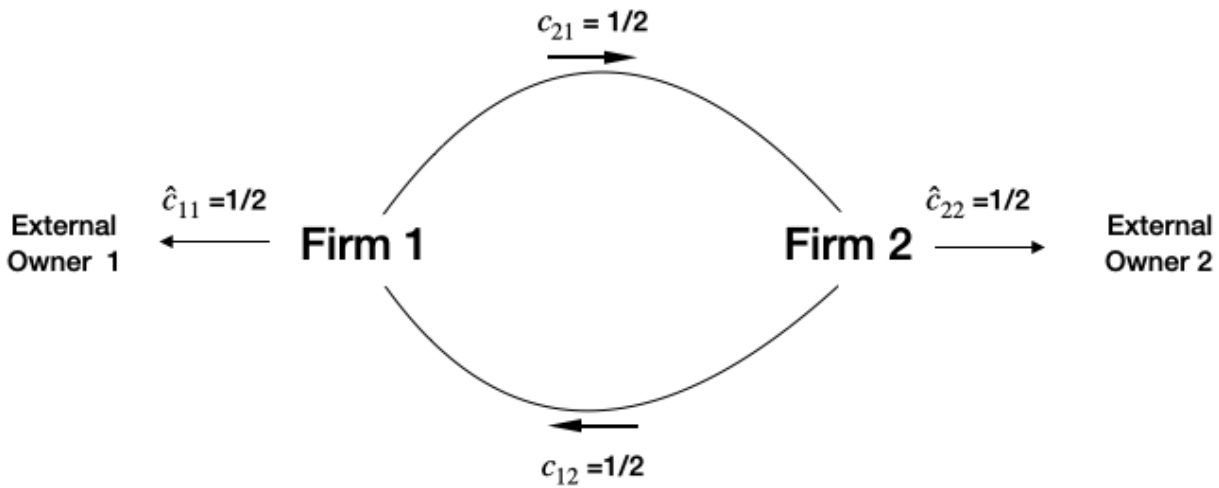
$$v_i = \hat{C}_{ii} V_i \tag{3}$$

or in matrix form:

$$v = \underbrace{\hat{C}(I - C)^{-1}}_{\equiv A} Dp \tag{4}$$

Example

- Cross-holding: $C = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix}$
- External-holding: $\hat{C} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$
- Dependency matrix: $A \equiv \hat{C}(I - C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$
- Figure:



Example

- What happens when project operated by firm 1 generate $ 1 ?
- Round 1:
  - External owner of firm 1 gets 0.5
  - Firm 2 gets 0.5
- Round 2:
  - External owner of firm 2 gets 0.5*0.5 = 0.25
  - Firm 1 gets 0.5*0.5 = 0.25
- Round 3:
  - External owner of firm 1 gets 0.5*0.5*0.5 = 0.125
  - Firm 2 gets 0.5*0.5*0.5 = 0.125
- ...
- Eventually,
  - External owner of firm 1 gets 2/3
  - External owner of firm 2 gets 1/3

Amplification: Discontinuity

- Failure cost

$$b_i(v_i) = \begin{cases} \beta_i(p), & \text{if} \quad v_i < \underline{v}_i \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

- With failures,

$$v = A[Dp - b(v)] \tag{6}$$

- Example: proportional liquidation cost

$$b_i(v_i) = \begin{cases} \lambda_i p_i, & \text{if} \quad v_i < \underline{v}_i \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

- Equilibrium multiplicity
  - Standard story of self-fulfilling bank runs
  - Interdependence of the values of the organizations

Aside: Equilibria refinement

- Best-case equilibrium
  - Focus on equilibrium with fewest failing firms
  - Not consider multiplicity due to coordination failures
  - Consider multiplicity due to interdependency between firms
- Algorithm to find best-case equilibrium
  - Step 1: assume no firm fails
    - see if an equilibrium exist
    - if not, go to step 2
  - Step 2: assume firm with lowest value in iteration step 1 fails
    - see if all other firms can survive
    - if not, go to step 3
  - Step 3: assume two firms with lowest value in iteration step 2 fail
    - see if all other firms can survive
    - if not, go to step 4
  - .....

Amplification: Three stages

- First failure
  - (Asset of) some firm hit by a shock and fails
- Local contagion
  - Some others firms exposed to this failing firms fail
- Wider propagation
  - Network propagates the effect

Amplification: Two Ingredients

- Diversification
  - How many other firms a firm hold
  - Random network G:

$$Pr(G_{ij}) = d/(n-1)$$

  - d = expected level of diversification
- Integration
  - How much of a firm is cross-held by other firms
  - Fraction c is evenly split among cross-holders
  - Fraction 1-c is held by outsider (private-investor)
  - c = level of integration
- Calibration:

$$C_{ij} = \frac{cG_{ij}}{d_j} \tag{8}$$

where d_j is realized level of diversification of a firm

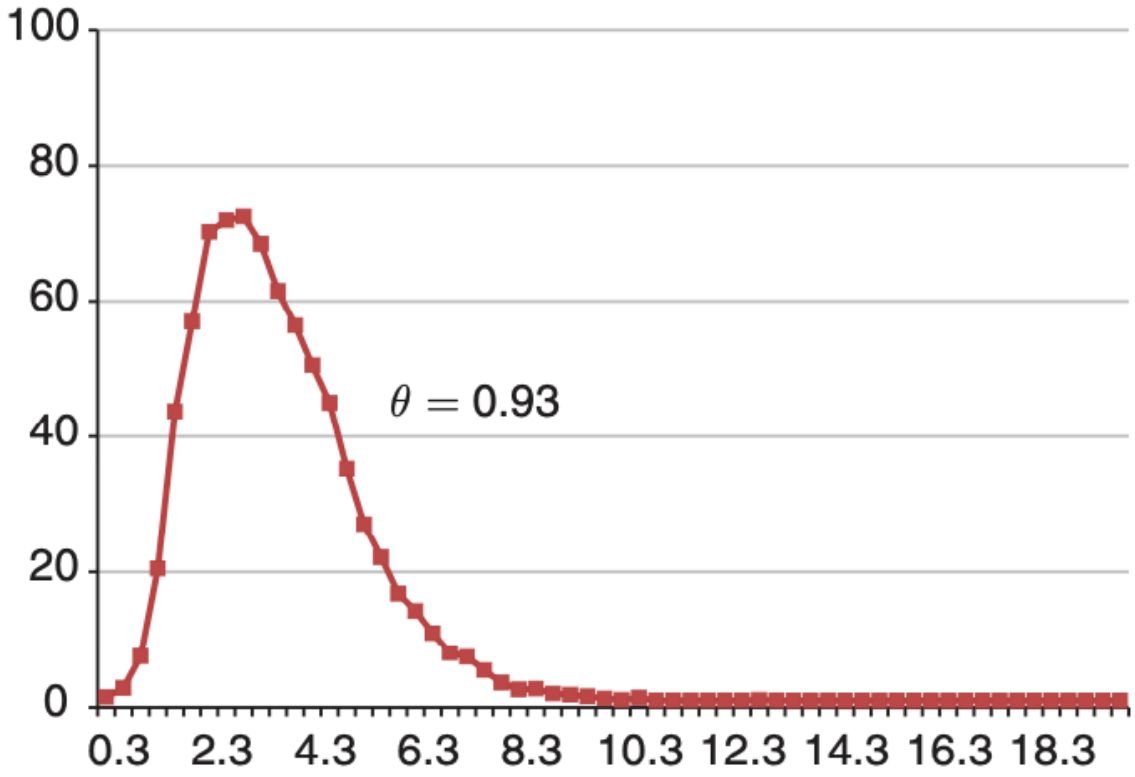
$$\hat{C}_{ii} = 1 - c \tag{9}$$

Example

- Asset
  - One asset associated with one firm: D = identity matrix
  - $p = 1$ for all firms initially
  - Value of firms: $v = Ap = A1$
- Value
  - Threshold value: $\underline{v}_i = \theta v_i$
  - Failure cost = losing all the value
- Shock to economy:
  - $p_i = 0$ for one i

Effect of Diversification

Panel A. Effects of diversification: the percentage of organizations failing as a function of expected degree for $\theta = 0.93$ ($c = 0.5$, $n = 100$)



Effect of Diversification

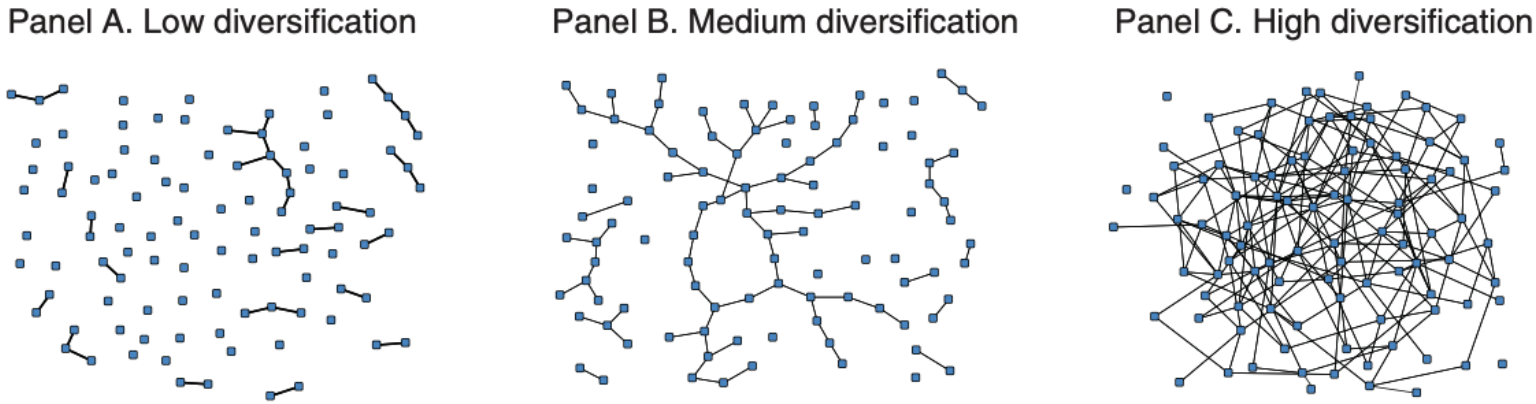
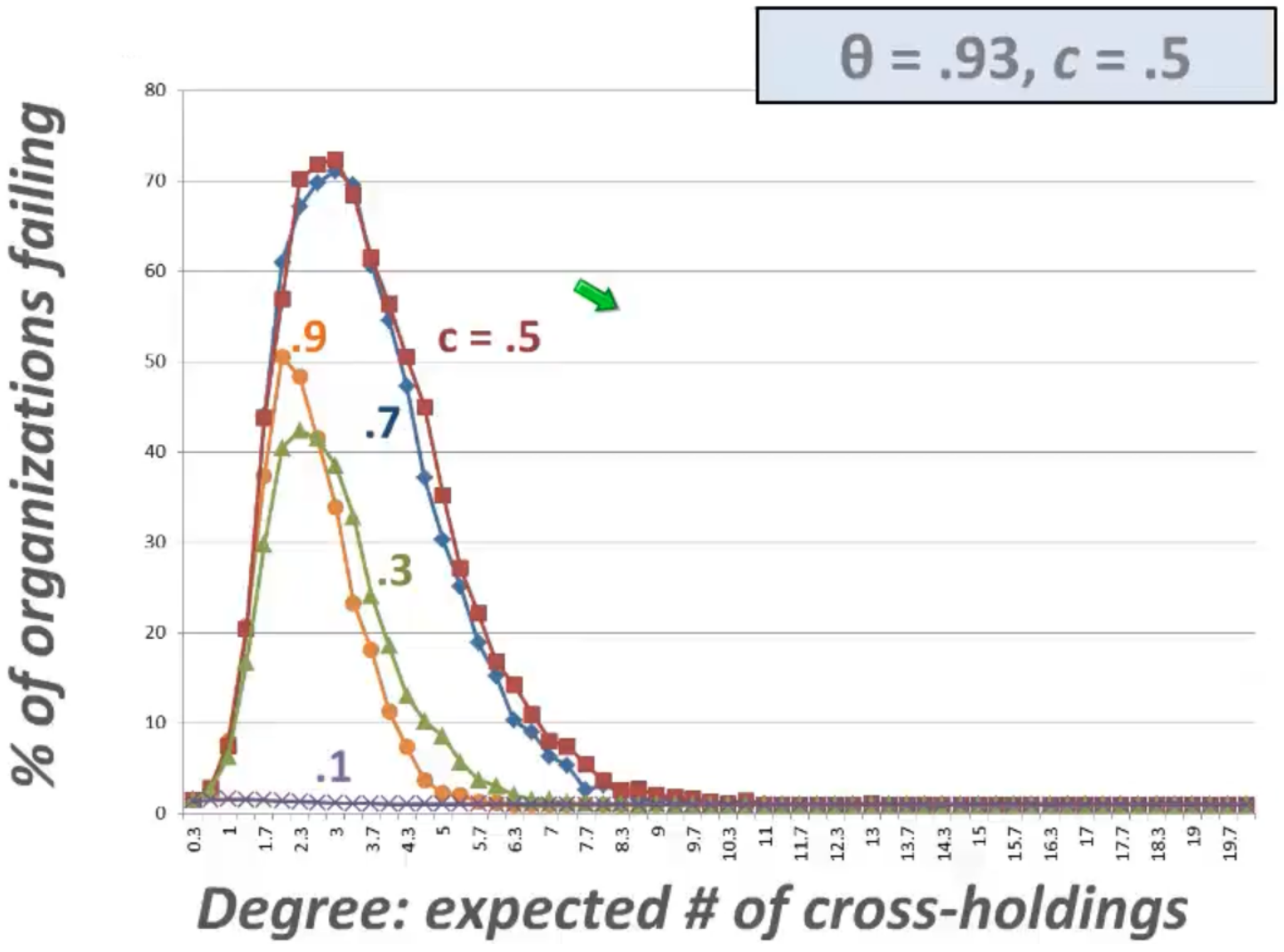


FIGURE 3. EXAMPLE RANDOM NETWORKS (*Plotted here with undirected edges*) FOR DIFFERENT LEVELS OF DIVERSIFICATION

- Low d
  - disconnected majority survive
  - shocked firm and its holders fail
- High d
  - connectedness of the network lowers the chance of contagion
  - cross-holders of shocked firm can survive

Effect of Integration



Trade-offs: Diversification and Integration

Three Stages	Integration	Diversification
Initial Failure	$\downarrow$	
Local Contagion	$\uparrow$	$\downarrow$
Wider Propagation		$\uparrow$

Summary

- Financial network + discontinuity in values $\rightarrow$ cascading defaults
  - Simple framework of cross-holdings: Eisenberg and Noe (2011, MS), w/o bankruptcy cost
- Double-edged nature of connectivity: risk sharing vs. spread of shocks
  - Allen and Gale (2000): stylized network structures with analytical analysis (complete vs. pairs vs. cycles)
  - Gai, Haldane, and Kapadia (2011): richer variety of network structures with numerical analysis
- Review literature: Glasserman and Young (2016, JEL) etc.