

TURBULENT BUSINESS CYCLES

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ABSTRACT. Recessions are associated with sharp increases in turbulence that reshuffle firms' productivity rankings. To study the business cycle implications of turbulence shocks, we use Compustat data to construct a measure of turbulence based on the (inverse of) Spearman correlations of firms' productivity rankings between adjacent years. We document evidence that turbulence rises in recessions, reallocating labor and capital from high- to low-productivity firms and reducing aggregate TFP and the stock market value of firms. A real business cycle model with heterogeneous firms and financial frictions can generate the observed macroeconomic and reallocation effects of turbulence. In the model, increased turbulence makes high-productivity firms less likely to remain productive, reducing their expected equity values and tightening their borrowing constraints relative to low-productivity firms. This leads to a reallocation that reduces aggregate TFP. Unlike uncertainty, turbulence changes both the conditional mean and the conditional variance of the firm productivity distribution, enabling a turbulence shock to generate a recession with synchronized declines in aggregate activities.

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I. INTRODUCTION

Recessions are characterized by declines in aggregate economic activity. They are also characterized by a sharp rise in micro-level turbulence with increased churning of firms' productivity rankings. This paper studies the macroeconomic and reallocation effects of turbulence shocks over business cycles.

We develop an empirical measure of turbulence using data from publicly traded U.S. firms listed in Compustat. We first construct a measure of firm-level total factor productivity (TFP) following the approach in the literature (Syverson, 2004; Foster et al., 2008; Bloom et al., 2018). We then sort the firm-level TFP in each year and estimate the Spearman rank correlations (denoted by ρ_t) between adjacent years. A low Spearman correlation indicates more churning of firm rankings in the productivity distribution: a high-productivity firm this year is less likely to maintain its productivity ranking next year; whereas a low-productivity firm now might become more productive in the future. Turbulence is inversely related to the Spearman correlation of firm-level TFP, and we measure it by $1 - \rho_t$.

Our measure of turbulence is countercyclical, rising sharply in recessions. An increase in turbulence reallocates labor and capital from high- to low-productivity firms, with the magnitude of the reallocation effects depending partly on financial frictions. Reflecting its reallocation effects, turbulence is negatively correlated with aggregate manufacturing TFP and the stock market value of firms. Turbulence is also associated with synchronized and persistent declines in real GDP, consumption, investment, and employment.

To understand the economic mechanism through which turbulence can drive macroeconomic fluctuations and cross-sectional reallocation as those observed in the data, we construct a real business cycle (RBC) model with heterogeneous firms and financial frictions. In the model, firms produce a homogeneous good using capital and labor, subject to idiosyncratic productivity shocks. Firms rely on external financing of working capital, with the borrowing capacity constrained by a fraction of the expected future equity value (Jermann and Quadrini, 2012; Liu and Wang, 2014; Lian and Ma, 2021). Firms also face idiosyncratic production distortions, reflecting differential policy interventions or government subsidies at the firm level (Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). At each given level of productivity, firms with sufficiently high levels of subsidies choose to operate, facing binding credit constraints while those with low levels of subsidies remain inactive. Given productivity, there is an endogenously determined threshold level of subsidy, at which a firm is indifferent between producing and staying inactive.

Under the stochastic process of the idiosyncratic productivity shocks, a firm can maintain its productivity from the current period to the next period with a time-varying probability ρ_t . With the complementary probability $1 - \rho_t$, the firm's productivity will be an independent

and identically distributed (i.i.d.) random variable. A lower value of ρ_t implies more frequent switching in firm productivity rankings between adjacent periods or, equivalently, greater turbulence.

The model predicts that a shock that increases turbulence leads to a recession. With greater turbulence, a high-productivity firm today would be less likely to remain productive in the future and a low-productivity firm today would be more likely to get a better productivity draw in the future. Thus, the expected equity value of a high-productivity firm falls relative to that of a low-productivity firm. Since firms' borrowing capacity depends on the expected equity value, turbulence disproportionately tightens the current-period credit constraints for high-productivity firms and reallocates labor and capital from high- to low-productivity firms. This reallocation reduces aggregate TFP. The endogenous decline in TFP is quantitatively important, enabling the model to generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. These model predictions are in line with empirical evidence.

Financial frictions are crucial for amplifying the macroeconomic effects of turbulence shocks. Since labor and capital are perfectly mobile across firms, competition for input factors from high-productivity firms bids up wages and capital rents. Absent credit constraints and production subsidies, resources would be concentrated in the most productive firms, and the equilibrium allocation would be efficient. Credit constraints and idiosyncratic production distortions restrict the borrowing capacity of high-productivity firms, allowing some low-productivity firms to stay active in production. Such financial frictions lead to steady-state misallocation and they also create room for between-firm reallocation following a turbulence shock. Such reallocation leads to procyclical TFP, enabling the model to generate business cycle comovements.

The presence of financial frictions implies that competitive equilibrium allocations are inefficient. Appropriate policy interventions can potentially mitigate credit constraints and improve allocative efficiency. Since financial frictions are the key transmission channel for turbulence, policy interventions that alleviate credit constraints might mitigate its recessionary effect.

We use our model framework to evaluate the effectiveness of two alternative policy interventions for stabilizing turbulence-driven recessions. The first policy is a borrowing subsidy that reduces the effective costs of hiring capital and labor, therefore reducing the amount of working capital that firms need to finance. The second policy is credit easing, under which the government injects liquidity to enhance the borrowing capacity of active firms. Each policy is transitory and unexpected, and it is triggered by the realization of a turbulence shock, with the same persistence as that of the shock.

Under our calibration, both types of policies are effective for mitigating the recessionary effects of turbulence relative to the *laissez-faire* benchmark economy with no policy interventions. However, the policies operate through different channels and therefore have different implications for reallocation.

Borrowing subsidies reduce the effective costs of hiring input factors for all firms, expanding the set of active firms at each level of productivity and boosting aggregate output. However, by enabling a larger fraction of low-productivity firms to stay active, the policy exacerbates misallocation, reducing aggregate TFP relative to the benchmark. The decline in TFP partly offsets the stimulus effects on aggregate output.

Credit easing expands the borrowing capacity for active firms. Competition for input factors from high-productivity firms pushes up equilibrium wages and capital rents, forcing some low-productivity firms to stay inactive. This reallocation improves aggregate TFP, contributing to increased output.¹

II. RELATED LITERATURE

Our work is closely related to the important contribution of Bloom et al. (2018), who study the macroeconomic implications of micro-level uncertainty. They show that, in a real business cycle model with capital and labor adjustment costs, an increase in micro-level uncertainty (i.e., an increase in the standard deviation of the firm-level TFP shocks) reduces net aggregate investment, net hiring, and aggregate output. However, in their model, aggregate consumption rises following an uncertainty shock. Generating a recession with aggregate comovements requires a simultaneous negative shock to the level of aggregate TFP.

Unlike uncertainty, which is a mean-preserving spread of the productivity distribution, turbulence changes not just the conditional variance but also the conditional mean of firm-level productivity. Following an increase in turbulence, firms with high productivity in the current period may not be as productive in the future. Thus, this turbulence-induced changes in conditional expectations of future firm productivity, together with credit constraints, lead to reallocation from high- to low-productivity firms, reducing aggregate TFP. The reduction in TFP in turn leads to a recession with aggregate comovements.

¹The two types of policy interventions—borrowing subsidies and credit easing—do not necessarily improve welfare relative to the benchmark, because they both incur a deadweight loss. We use these counterfactual policies to highlight the transmission mechanism of turbulence shocks. We do not study optimal policy here because welfare depends on the calibration of the sizes of the deadweight losses (see also Gertler and Karadi (2011)).

Our model illustrates the importance of financial frictions for propagating turbulence shocks.² Existing studies show that financial frictions are also important for the transmission of uncertainty shocks (Gilchrist et al., 2014; Christiano et al., 2014; Alfaro et al., 2018; Arellano et al., 2019).³ In the model of Arellano et al. (2019), for example, hiring is risky because firms need to finance input costs before they receive revenues, and firms face idiosyncratic productivity shocks between paying for inputs and receiving revenues. An increase in the volatility of idiosyncratic shocks raises default risks, and firms respond by pulling back hiring and reducing production. Since firms are *ex ante* identical, they make identical hiring decisions. Thus, an increase in uncertainty in their model (i.e., firm-level volatility) does not lead to reallocation of capital and labor inputs. In our model, however, reallocation is the central mechanism for propagating turbulence shocks.

Our work is related to the economic development literature on capital misallocation under financial frictions (Midrigan and Xu, 2014; Moll, 2014; Buera and Shin, 2013; Gopinath et al., 2017; Liu et al., 2021). Indeed, our measure of turbulence is similar to the persistence of idiosyncratic productivity in the continuous-time model of Moll (2014). More persistent productivity shocks imply relatively smaller steady-state productivity losses but also slower transitions to the steady-state (Moll, 2014). Other things being equal, the less persistent the idiosyncratic productivity shocks are, the greater the impact of financial frictions on aggregate productivity (Buera and Shin, 2013). We focus on the business cycle implications of turbulence. Thus, our work complements this development literature.

The countercyclical behavior of turbulence that we find is consistent with other empirical studies based on different data and measurements. For example, Aghion et al. (2021) construct a measure of turbulence based on the rate of new product additions and subtractions (i.e., product churn) using US Census of Manufactures data. They find that product churn rises sharply during recessions. Bernard and Okubo (2016) and Dekle et al. (2021) also report evidence of countercyclical product churn based on Japanese manufacturing data. We

²The global financial crisis of 2008-2009 has spurred a large literature that incorporates financial frictions into business cycle models, building on the seminal contributions of Bernanke et al. (1999) and Kiyotaki and Moore (1997). Examples include Jermann and Quadrini (2012), Gertler et al. (2012), Liu et al. (2013), Christiano et al. (2014), Gertler and Kiyotaki (2015), and Lian and Ma (2021). For recent surveys of this literature, see Christiano et al. (2018) and Gertler and Gilchrist (2018).

³There is a large strand of literature on the macroeconomic effects of uncertainty shocks. Examples include Bloom (2009), Bachmann et al. (2013), Fernández-Villaverde et al. (2015), Jurado et al. (2015), Baker et al. (2016), Leduc and Liu (2016), Basu and Bundick (2017), Bansal et al. (2019), Berger et al. (2020), and many others. For recent surveys of the uncertainty literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).

add to this empirical literature by documenting the macroeconomic and reallocation effects of turbulence.⁴

To our knowledge, our paper represents a first attempt to study the transmission mechanism of turbulence shocks over the business cycle using firm-level data and a quantitative business cycle model featuring firm heterogeneity and financial frictions.

III. MEASURING MICRO-LEVEL TURBULENCE AND ITS MACROECONOMIC EFFECTS

This section describes our empirical methods for measuring micro-level turbulence and documents some stylized facts about the macroeconomic effects of turbulence.

III.1. Empirical methodology. Consider the production function for firm j in period t

$$Y_{jt} = z_{jt}F(K_{jt}, N_{jt}), \quad (1)$$

where Y_{jt} denotes value-added output, K_{jt} and N_{jt} denote capital and labor inputs, respectively, and $F(K, N)$ is the production function. The persistent idiosyncratic productivity, z_{jt} , follows the stochastic process

$$z_{j,t+1} = \begin{cases} z_{jt} & \text{with prob } \rho_t, \\ \tilde{z} & \text{with prob } 1 - \rho_t, \end{cases} \quad (2)$$

where \tilde{z} is an i.i.d. random variable with the cumulative density function $\tilde{G}(z)$.

Under the stochastic process of z_t in Eq. (2), a firm's productivity level in period $t+1$ can stay the same as that in period t , in which case the ranking of firm productivity also stays the same. This occurs with the probability ρ_t . With the complementary probability $1 - \rho_t$, the firm's productivity in period $t+1$ is an i.i.d. random variable \tilde{z} , which is independent of the period- t productivity, such that firm productivity in $t+1$ would be uncorrelated with that in t .

The term ρ_t measures the persistence of firm-level TFP. In the extreme case with $\rho_t = 1$ for all t , a firm's productivity level would be permanent: high-productivity firms would remain productive and low-productivity firms would remain unproductive. In the other extreme with $\rho_t = 0$, firm productivity would be an i.i.d. process, with no persistence. In the more general case with $\rho_t \in (0, 1)$, firm productivity is persistent, and the persistence is time varying. A decline in ρ_t implies that high-productivity firms in period t would be less likely to remain productive in period $t+1$ and low-productivity firms in period t would have a chance to draw a high productivity in period $t+1$. Thus a decline in ρ_t increases the churn of firm rankings in the productivity distribution. We measure micro-level turbulence by $1 - \rho_t$.

⁴Similar reallocation effects can arise from labor market churns (Pratap and Quintin, 2011) or supply-chain disruptions (Meier, 2020).

Turbulence is related to but different from the micro-level uncertainty studied by Bloom et al. (2018). An increase in micro-level uncertainty corresponds to a mean-preserving spread of the cross-sectional productivity distribution—an increase in the variance or inter-quartile range (IQR) of productivity. An increase in turbulence also raises the conditional variance of the productivity distribution, as does uncertainty. Thus, turbulence is positively correlated with micro-level uncertainty.⁵ However, unlike uncertainty, turbulence changes not only the conditional variance but also the conditional mean of the firm-level productivity distribution. Through its impact on the conditional mean of firm productivity, a turbulence shock generates between-firm reallocation, which is essential for generating procyclical aggregate productivity and business cycle comovements.

Furthermore, turbulence does not affect the *ex ante* stationary distribution of productivity—it is an *ex ante* distribution-preserving shock, as we show in Proposition 1 below.⁶

Proposition 1. The cross-sectional stationary distribution of idiosyncratic productivity (denoted by $G_t(z)$) is invariant to the realization of ρ_t .

Proof. Under the stochastic process of idiosyncratic productivity specified in Eq. (2), the cumulative density function of productivity is given by

$$\begin{aligned} G_{t+1}(z) &= Pr(z_{t+1} \leq z) \\ &= Pr(z_t \leq z)\rho_t + Pr(\tilde{z} \leq z)(1 - \rho_t) \\ &= G_t(z)\rho_t + \tilde{G}(z)(1 - \rho_t). \end{aligned} \tag{3}$$

Under the stationarity of the distribution of z , we have $G_t(z) = \tilde{G}(z)$ for all t . Thus, the stationary distribution is independent of the realization of ρ_t . \square

When idiosyncratic productivity is perfectly observable, our turbulence measure $1 - \rho_t$ is the inverse of the Spearman rank correlation of firm productivity between adjacent periods, as we formally establish in Proposition 4 in appendix A.1. When there are measurement errors in productivity, however, potential bias may arise in measuring turbulence ρ_t using TFP data.

⁵For example, the correlation between our turbulence measure and the IQR of firm-level TFP from Compustat data is about 0.55.

⁶The distribution-preserving turbulence that we study here can be viewed a discrete-time counterpart to the persistence of idiosyncratic productivity shocks in the continuous-time models of Moll (2014), which is also orthogonal to the stationary productivity distribution.

Consider the setting when measured TFP of firm j at time t , denoted as a_{jt} , is the sum of true productivity z_{jt} and measurement error $\tau_{jt} \sim N(0, \sigma_t^2)$, whose variance can be potentially time-varying. Now the measured TFP follows

$$\begin{aligned} a_{j,t+1} &= \begin{cases} z_{jt} + \tau_{jt+1} & \text{with prob } \rho_t, \\ \tilde{z} + \tau_{jt+1} & \text{with prob } 1 - \rho_t, \end{cases} \\ &= \begin{cases} a_{jt} + \underbrace{\tau_{jt+1} - \tau_{jt}}_{\equiv e_{jt+1}} & \text{with prob } \rho_t, \\ \tilde{z} + \tau_{jt+1} & \text{with prob } 1 - \rho_t, \end{cases} \end{aligned} \quad (4)$$

The presence of measurement errors imposes two challenges in estimating true process of turbulence. The first issue is heteroscedasticity: as the variance of residuals, e_{jt+1} in equation (4), is time-varying due to changing σ_t , OLS estimator of the first-order correlation can be biased and inconsistent. Leveraging on the idea that (percentile) rank distribution of measured productivity is time-invariant regardless of true functional form for its variance, we tackle the heteroscedasticity issue by estimating rank correlation of firm productivity between adjacent years.⁷

The second is a well-known endogeneity problem in dynamic panel analysis: the residual term (e_{jt+1}) in model (4) is correlated with the independent variable a_{jt} due to a confounding factor τ_{jt} . In the spirit of Arellano and Bond (1991), we address the endogeneity issue by using decile ranks of a_{t-2} and a_{t-3} as instrument variable for rank of a_{t-1} in estimating equation, , since $\text{corr}(a_{t-2}, e_t) = 0$ (exogeneity), and $\text{corr}(a_{t-1}, a_{t-2}) > 0$ (relevance).

Building on these insights, we develop the methodology to identify true turbulence at time t by estimating Spearman rank correlation of measured productivity between period $t+1$ and t , using lagged rank at period $t-1$ and $t-2$ as instrument variables for rank at period t .

III.1.1. *Data.* We use firm-level data from Compustat Fundamentals Annual database. To obtain measures of industry-level employment, payroll, and price indices, we use information from the NBER-CES Manufacturing Industry Database.⁸ By combining these two data sources, we obtain an unbalanced panel with 48,197 firm-year observations. This full sample

⁷The non-parametric approach of rank transformation is conceptually similar to parametric approach of using a weighted least squares (WLS) estimator, but bears the advantages of being more robust to misspecification of the variance function and less computationally intensive.

⁸The Compustat database is accessed through Wharton Research Data Service at: <https://wrds-web.wharton.upenn.edu/wrds/>. The NBER-CES database is accessed through <https://www.nber.org/research/data/nber-ces-manufacturing-industry-database>.

TABLE 1. Summary statistics

Variable	Sample 1			Sample 2		
	Mean	SD	N	Mean	SD	N
Log Asset (1m)	5.6	2.0	48197	6.2	2.1	25790
Log Value-Added (1m)	4.7	2.4	48197	5.5	2.3	25790
Log Capital (1m)	3.7	2.5	48197	4.5	2.4	25790
No. of Workers (1000)	4.2	2.0	48197	4.9	2.0	25790
Log Market Value	5.7	2.1	46183	6.1	2.2	24370
Value-Added Growth (%)	7.8	33.0	48197	5.1	22.5	25790
Capital Growth (%)	6.3	30.7	48197	4.7	23.0	25790
Employment Growth (%)	5.0	23.1	48197	3.3	19.0	25790
Market-Value Growth(%)	3.7	50.2	44754	4.1	41.9	23849

Note: Sample 1 covers all listed firms in all manufacturing industries (NAICS code 31 to 33). Sample 2 covers firms with 25+ years of observations in all manufacturing industries.

Source: Compustat, NBER-CES, and authors' calculations.

(Sample 1) includes all listed firms in all manufacturing industries covered by NBER-CES in the years from 1958 to 2016.⁹ Table 1 presents the summary statistics of our samples.

Following Bloom et al. (2018), we focus on the subset of firms with 25+ years of observations in our sample. We use this pseudo-balanced panel as our baseline sample (Sample 2) for estimating firm-level TFP. The baseline sample contains about 25,790 firm-year observations. Since firms in the baseline sample are older than those in the full sample, they are also larger on average in terms of assets, value added, capital, and employment, although their average growth rates of employment and capital are slower.¹⁰

III.1.2. *Measuring turbulence.* We construct a measure of micro-level turbulence based on firm-level TFP. Following the approach in the literature (Syverson, 2004; Foster et al., 2008;

⁹We include firms incorporated in the US (Compustat `fic='USA'`) that trade on major stock exchanges (NYSE, AMEX, and NASDAQ, Compustat `exchg = 11, 12 or 14`), for which the native currency is US dollars (Compustat `cured='USD'`). We exclude firm-year observations with obvious errors: missing or nonpositive values in reported revenue, employment, and capital. We remove a firm if it was involved in a major merger or acquisition that affected its asset by more than 10 percent.

¹⁰In a robustness check, we further narrow down the sample and focus on industries with more than 20 firms in each year. This sample (Sample 3) contains about 19,000 firm-year observations. Firms in Sample 3 have similar characteristics as those in Sample 2.

Bloom et al., 2018), we measure firm-level TFP based on the Solow residual calculated from the constant-returns production function

$$tfp_{ijt} = y_{ijt} - \alpha_{it}k_{ijt} - (1 - \alpha_{it})n_{ijt}, \quad (5)$$

where tfp_{ijt} denotes the TFP (in log units) of firm j in industry i and year t , and y_{ijt} , k_{ijt} and n_{ijt} denote the firm's value added, capital input, and labor input, respectively, all in log units.¹¹ Following Bloom et al. (2018), we assume that the cost share of capital input α_{it} is common for all firms within an industry i , although it can vary across time.¹²

After obtaining firm-level TFP, we construct a measure of turbulence based on the Spearman rank correlation of firm productivity. Specifically, we rank firms within each industry (at the 3-digit level) by deciles of their productivity levels. We then estimate the rank correlation of firm TFP between year $t+1$ and t , using ranking at year $t-1$ and $t-2$ as instrument variables for year- t rank. The time series of the Spearman correlations corresponds to our measure of ρ_t .

Turbulence is measured by $1 - \rho_t$. Intuitively, a decline in ρ_t implies that a high-productivity firm in year t would be less likely to remain as productive in $t+1$, whereas a low-productivity firm in year t might get a high productivity draw in year $t+1$. A reduction in ρ_t reshuffles productivity rankings across firms between t and $t+1$, increasing turbulence.¹³

III.1.3. *Cyclical properties of turbulence.* Figure 1 plots the time series of firm-level turbulence from 1965 to 2015. The mean, standard deviation, and autocorrelation of the estimated turbulence ($1 - \rho_t$) are 0.049, 0.021, and 0.69, respectively. The figure shows that turbulence is countercyclical, rising sharply in recessions.¹⁴

¹¹The cyclical properties and the macroeconomic and reallocation effects of turbulence are similar when we construct a measure of turbulence based on a production function with decreasing returns to scale (see Appendix B).

¹²In our sample, the average value of the cost share of capital (weighted by the value of shipment) is about 0.34. We provide some details of our approach to measuring value added, capital and labor inputs, and the capital share in Appendix A.

¹³Our measure of turbulence is closely related to that in Bloom et al. (2018), who also estimate the Spearman correlation of plant rankings in the TFP distribution across adjacent years using Census Manufacturing (CM) data (see Figure A1 in their online appendix at https://nbloom.people.stanford.edu/sites/g/files/sbiybj4746/f/rubc_appendix_0.pdf). Our measure of ρ_t is positively correlated with theirs, with a correlation coefficient of 0.49.

¹⁴In Appendix A, we show that the baseline estimate of ρ_t is robust to alternative samples and alternative approaches to measuring TFP. Our measured turbulence displays an upward trend. Since an increase in turbulence raises firm-level volatility, the trend increase in turbulence is consistent with the empirical evidence that the volatility for publicly traded firms has increased steadily over time, whereas the volatility of privately held firms has declined (Davis et al., 2006).

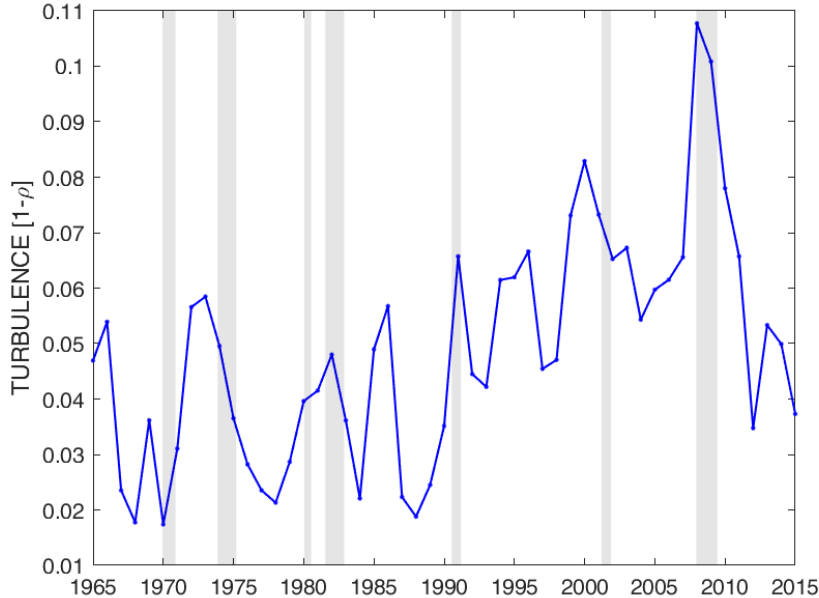


FIGURE 1. Measured micro-level turbulence

Note: Turbulence is measured by $1 - \rho_t$, where ρ_t is the IV estimator of Spearman correlation of firm TFP rankings between year t and year $t + 1$. The gray shaded bars indicate NBER recession dates.

Source: Compustat, NBER-CES, BLS, and authors' calculations.

Our measure of turbulence is negatively correlated with manufacturing TFP, as shown in panel A of figure 2. A rise in turbulence (blue line) is typically associated with a decline in TFP relative to trend (red line), and the correlation between the two series is about -0.28.

Turbulence is also negatively correlated with the stock market value of firms, as shown in panel B of figure 2. The market value of assets is calculated based on firms' stock prices at the end of the fiscal year, multiplied by the shares outstanding and deflated by the consumer price index. The correlation between turbulence with the asset value is negative, at -0.30.¹⁵

III.2. Decomposing churning in firm TFP ranking. Our empirical strategy using instrument variable identifies churning in firm productivity ranking due to turbulence. The fact that τ_{jt} (and its time-varying volatility) can bias OLS estimator of Spearman rank correlation (but not in IV estimator) offers us a way to decompose the churning in firm TFP ranking, i.e. change in the inverse of Spearman rank correlation, into turbulence and measurement error.

¹⁵In Compustat, the firm stock price at the end of the fiscal year is the variable "PRCC_F," the shares outstanding is "CSHO," the firm asset is "AT," and the book equity is "CEQ."

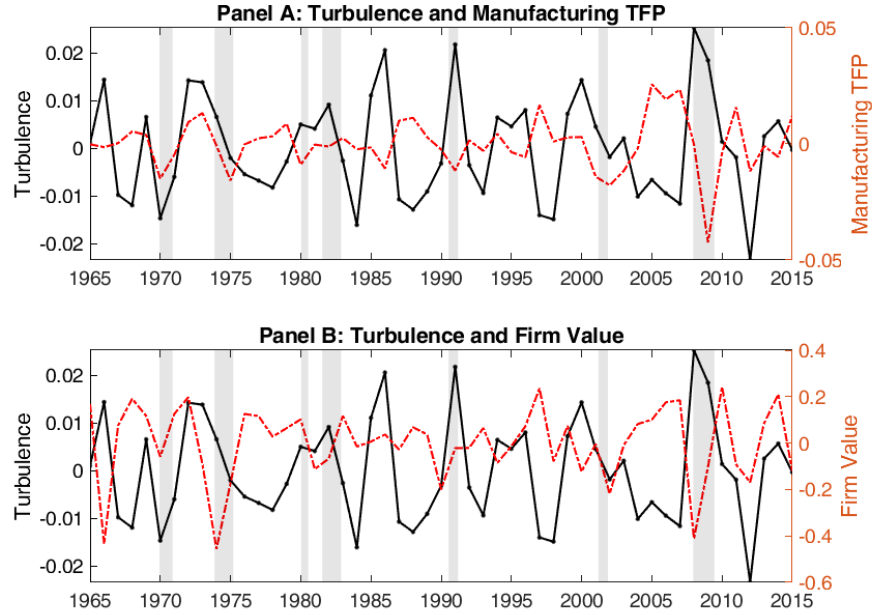


FIGURE 2. Correlation of turbulence with manufacturing TFP and stock market value

Note: The series of manufacturing TFP and firm value are computed as an average of firm-level TFP and stock market value in our benchmark sample. The series are detrended using the HP filter, with a smoothing parameter of 6.25. The gray shaded bars indicate NBER recession dates.

Proposition 2. If measured TFP contains true productivity and measurement errors *a la* Eq. (4), the Spearman rank correlation of firm productivity between period t and $t + 1$ is given by

$$\rho_t + \frac{\rho_t \text{Cov}[R_t(z_{jt} + \tau_{jt+1} \frac{\sigma_t}{\sigma_{t+1}}) - R_t(z_{jt} + \tau_{jt}), R_t(z_{jt} + \tau_{jt})]}{\text{Var}[R_t(z_{jt} + \tau_{jt})]} \leq \rho_t$$

Proof. Denote by $R_t(a_{jt})$ the ranking of productivity a_{jt} across firms in period t . The Spearman rank correlation of firm productivity between t and $t + 1$ is thus given by

$$\begin{aligned} r_s &\equiv \frac{\text{Cov}[R_{t+1}(a_{jt+1}), R_t(a_{jt})]}{\sqrt{\text{Var}[R_{t+1}(a_{jt+1})]\text{Var}[R_t(a_{jt})]}} = \frac{\text{Cov}[R_{t+1}(z_{jt+1} + \tau_{jt+1}), R_t(z_{jt} + \tau_{jt})]}{\sqrt{\text{Var}[R_{t+1}(z_{jt+1} + \tau_{jt+1})]\text{Var}[R_t(z_{jt} + \tau_{jt})]}} \\ &= \frac{\rho_t \text{Cov}[R_{t+1}(z_{jt} + \tau_{jt+1}), R_t(z_{jt} + \tau_{jt})] + (1 - \rho_t) \text{Cov}[R_{t+1}(\tilde{z} + \tau_{jt+1}), R_t(z_{jt} + \tau_{jt})]}{\text{Var}[R_t(z_{jt} + \tau_{jt})]} \\ &= \frac{\rho_t \text{Cov}[R_{t+1}(z_{jt} + \tau_{jt+1}), R_t(z_{jt} + \tau_{jt})]}{\text{Var}[R_t(z_{jt} + \tau_{jt})]} \end{aligned}$$

$$\begin{aligned}
&= \frac{\rho_t \text{Cov}[R_t(z_{jt} + \tau_{jt}), R_t(z_{jt} + \tau_{jt})] + \rho_t \text{Cov}[R_{t+1}(z_{jt} + \tau_{jt+1}) - R_t(z_{jt} + \tau_{jt}), R_t(z_{jt} + \tau_{jt})]}{\text{Var}[R_t(z_{jt} + \tau_{jt})]} \\
&= \rho_t + \frac{\rho_t \text{Cov}[R_t(z_{jt} + \tau_{jt+1} \frac{\sigma_t}{\sigma_{t+1}}) - R_t(z_{jt} + \tau_{jt}), R_t(z_{jt} + \tau_{jt})]}{\text{Var}[R_t(z_{jt} + \tau_{jt})]} \leq \rho_t
\end{aligned} \tag{6}$$

The last inequality hold in equality if and only if $\sigma_t = 0$. Here, the first two equalities are the definition of the Spearman correlation (denoted by r_s) and measurement equation of a_{jt} . The third equality follows from Eq. (4) and the assumption that the variance of the ranking distribution of a stays constant over time. The fourth equality follows from the fact that $\tilde{z} + \tau_{jt+1}$ is uncorrelated with $z_{jt} + \tau_{jt}$. The sixth equality follows from that the rank of TFP $z_{jt} + \tau_{jt+1}$ in period $t+1$ should equal to the rank of TFP $z_{jt} + \tau_{jt+1} \frac{\sigma_t}{\sigma_{t+1}}$ in period t .

When $\sigma_t = \sigma_{t+1} = 0$, $r_s = \rho_t$ as in the case with perfectly observable productivity (in appendix A.1). When $\sigma_t = \infty$, $\frac{\rho_t \text{Cov}[R_t(z_{jt} + \tau_{jt+1} \frac{\sigma_t}{\sigma_{t+1}}) - R_t(z_{jt} + \tau_{jt}), R_t(z_{jt} + \tau_{jt})]}{\text{Var}[R_t(z_{jt} + \tau_{jt})]} = \rho_t$ thus $r_s = 0$. More generality, larger variance of τ_t imposes a larger downward bias in estimated rank correlation. \square

Figure 3 decomposes the inverse of Spearman rank correlation in firm's productivity between year t and $t+1$ ($1 - \rho_t$) into turbulence and bias following Eq. (6). The blue bars represent the changes due to time-varying ρ_t ("turbulence"), and the red bars represent changes due to measurement errors and its volatility. On average two sources contribute equally to reshuffle of firm productivity ranking, but turbulence plays a more important role in recent years, especially during the financial crisis.

III.3. Reallocation effects of turbulence. To examine the reallocation effects of turbulence, we estimate the empirical specification

$$x_{jt} = \beta_0 + \beta_1 \text{High_TFP}_{jt} + \beta_2 \text{Turb}_t * \text{High_TFP}_{jt} + \mu_j + \eta_t + \epsilon_{jt}, \tag{7}$$

where the dependent variable x_{jt} denotes the growth rate of employment, capital, sales, or market value of firm j in year t from $t - 1$, Turb_t denotes measured turbulence, and High_TFP_{jt} is a dummy variable that equals one if firm j 's TFP level is above the median within its industry and zero otherwise. To mitigate potential biases associated with firm entries and exits, we focus on a pseudo balanced panel, with firms appearing at least 25 years in our sample from 1958 to 2016 (Sample 2), following Bloom et al. (2018). In estimating (7), we control for firm fixed effects (μ_j) and year fixed effects (η_t). The term ϵ_{jt} denotes regression errors.

The relative sensitivity of the firm-level activity of high-productivity firms to turbulence is captured by the coefficient on the interaction term (i.e., β_2). The parameter β_1 measures the average effects of productivity on firm growth absent turbulence shocks. The parameter β_0 is a constant intercept. The estimation results are displayed in Table 2.

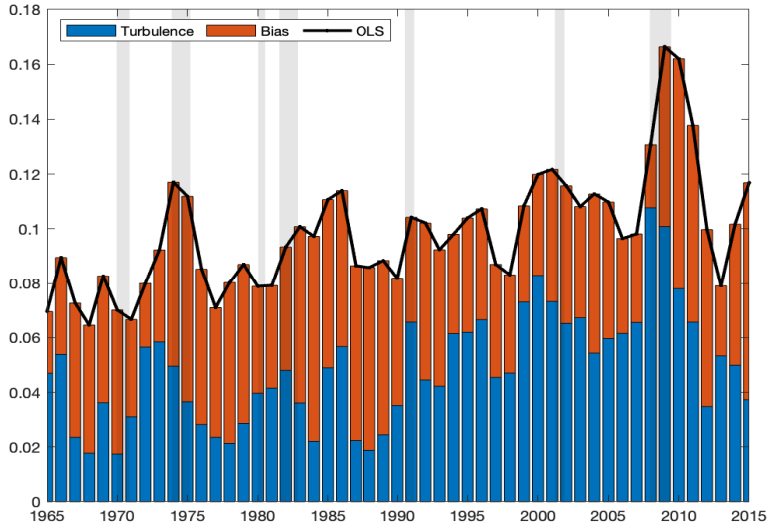


FIGURE 3. Decomposing churning in firm's TFP ranking

This figure plots the times series of estimated $1 - \rho_t$ using OLS and IV. The blue bars is IV estimator of $1 - \rho_t$, representing the effect of turbulence on churning in firm TFP ranking. The sum of blue and red bars is OLS estimator $1 - \rho_t$, representing the total effects from both turbulence and bias

In the baseline regressions, the estimated values of β_2 are negative and statistically significant at the 99-percent confidence level for all measures of firm growth. Thus, an increase in turbulence is associated with declines in the firm-level growth rates for high-productivity firms relative to those for low-productivity firms. The point estimates imply that a one-standard-deviation increase in turbulence reduces the relative growth rates of employment and capital for high-productivity firms by about 6 percent and 2.8 percent, respectively. It also reduces the relative growth rates of sales and the market value for high-productivity firms by about 1.7 and 1.6 percent, respectively.¹⁶ Thus, turbulence has important impact

¹⁶The standard deviation of our measured turbulence is 2.1 percent. The impact of a one-standard-deviation increase in turbulence (from its mean level of 0.049) on the relative employment growth rate for high-productivity firms is thus $(-0.945) \times 0.049 \times 2.1 \approx -0.097$ percentage points, or about 6 percent drop from the average growth rate of 1.62% for high-productivity firms. In our sample, the average growth rates of capital, sales, and market value of the high-productivity firms are 2.8%, 1.7%, and 1.6%, respectively. Thus, the impact of a one standard deviation shock to turbulence on the relative growth rates of capital, sales, and market values are $(-0.969) \times 0.049 \times 2.1/3.6 \approx -2.8\%$, $(-1.220) \times 0.049 \times 2.1/7.3 \approx -1.7\%$, and $(-0.940) \times 0.049 \times 2.1/6.1 \approx -1.6\%$, respectively.

TABLE 2. Impact of turbulence on firms with different levels of productivity

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
<i>High_TFP_{jt}</i>	0.006 (0.006)	0.018** (0.007)	0.094*** (0.006)	0.053*** (0.008)
<i>Turb_t * High_TFP_{jt}</i>	-0.945*** (0.095)	-0.969*** (0.144)	-1.220*** (0.086)	-0.940*** (0.139)
<i>Constant</i>	0.052*** (0.002)	0.060*** (0.003)	0.034*** (0.004)	0.032*** (0.002)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	24,501	24,501	24,501	21,687

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence (*Turb*) for firms with different levels of TFP. The dummy *High_TFP_{jt}* equals one if firm *j*'s TFP is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1965 to 2015. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

on firm-level activity, with greater adverse effects on high-productivity firms than on low-productivity firms. These findings suggest that turbulence reallocates capital and labor from high- to low-productivity firms, contributing to reducing aggregate TFP.

The reallocation effects of turbulence are robust to alternative high-productivity indicators (lagged high-TFP indicators or finer grouping of TFP rankings), alternative samples (with large industries or excluding top firms), and adding controls for potential reallocation effects of business cycle recessions, as we show in the Appendix (see Appendix A.3). The reallocation effects of turbulence are persistent. In appendix we also show that employment, capital, sales, and firm value of high-productivity firms are disproportionately affected by turbulence over an at least five-year horizon.

III.4. Financing constraints and the reallocation effects of turbulence. To examine the extent to which the reallocation effects of turbulence might depend on financial frictions,

we estimate the empirical specification

$$x_{it} = \beta_0 + \beta_1 High_FF_{it} + \beta_2 Turb_t * High_FF_{it} + \mu_i + \eta_t + \epsilon_{it}, \quad (8)$$

where the dependent variable x_{it} denotes interquartile range (IQR) of labor (or capital) of firms in industry i in year t , $Turb_t$ denotes measured turbulence, and $High_FF_{it}$ is a dummy variable that equals one if industry i 's financial constraint is above the median level among all NAICS 4-digit industries in year t . We measure a firm's financial constraint using the Kaplan- Zingales (KZ) index following Kaplan and Zingales (1997) and Lamont et al. (2001)¹⁷. We obtain an industry-level measure of financial constraint by taking the within-industry sales-weighted average of the firm-level KZ indices. To mitigate potential biases associated with firm entries and exits, we focus on the sample with firms appearing 25+ years from 1958 to 2016 (Sample 2). We estimate the empirical specification (8) using the sample with firms appearing 25+ years (Sample 2), controlling for industry fixed effects (μ_i) and year fixed effects (η_t). The term ϵ_{it} denotes regression errors.

Changes in the IQR of employment (or capital) capture reallocations within an industry. For example, a decline in the IQR of employment following an increase in turbulence would indicate reallocation of labor from firms with high levels of employment to those with low levels of employment. We are interested in how financial frictions could affect the reallocation effects of turbulence. This effect is captured by the parameter β_2 in Eq. (8), which measures the relative sensitivity of industry-level employment IQR (or capital IQR) to changes in turbulence for industries with high levels of financial friction. The parameter β_1 measures the average effect of financial constraint on the IQR of employment (or capital). The estimation results are displayed in Table 3.

In the baseline specifications with a contemporaneous indicator of financial constraint (Columns (1) and (3)), the estimated values of β_2 are negative and statistically significant at the 99 percent confidence level. Thus, an increase in turbulence is associated with relative declines the IQRs of both employment and capital in industries with high levels of financial constraint. The positive estimates of β_1 indicate that, absent turbulence, an industry with a high level of financial constraint has also a high within-industry dispersion of employment and capital.

¹⁷We follow Lamont et al. (2001) to construct an index of financial constraint according to Kaplan and Zingales (1997) classification to five readily available accounting variables: cash flow, market-to-book, leverage, dividends, and cash holdings. The coefficients associated with flow variables are adjusted to annual frequency. A higher index value suggests a firm is more constrained.

TABLE 3. Reallocation effect of turbulence: sensitivity to financial frictions

Dep. Var.	IQR of Employment		IQR of Capital	
	(1)	(2)	(3)	(4)
$High_FF_{it}$	0.232*** (0.084)	0.271*** (0.085)	0.335*** (0.097)	0.385*** (0.098)
$Turb_t * High_FF_{it}$	-4.791*** (1.548)	-5.448*** (1.566)	-6.866*** (1.792)	-7.741*** (1.810)
$Constant$	1.869*** (0.024)	1.895*** (0.025)	2.090*** (0.028)	2.122*** (0.028)
Industry Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	3,647	3,552	3,647	3,552

Note: This table shows the regression of interquartile range of employment (or capital) on the measured turbulence ($Turb$) for industries with different levels of external financing dependence. In the baseline specification (Columns (1) and (3)), the dummy $High_FF_{it}$ equals one if industry i 's external financing dependence is above the median. In the alternative specification (Columns (2) and (4)), we use lagged indicator of external financing dependence (i.e., $High_FF_{i,t-1}$ instead of $High_FF_{it}$). All regressions use the pseudo panel of Compustat firms that appear for at least 25 years in the sample from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

The results are robust when we use a lagged indicator of financial constraint (i.e., $High_FF_{i,t-1}$ instead of $High_FF_{it}$), as reported in Columns (2) and (4) of the table. Overall, our estimation suggests that tighter financial constraints are associated with larger declines in the cross-sectional dispersion of employment and capital when turbulence rises.¹⁸

III.5. Macroeconomic effects of turbulence. We now examine the macroeconomic effects of turbulence. For this purpose, we estimate the impulse responses of several key

¹⁸In the Appendix, we show that our results are also robust to finer grouping of industries based on their financial constraint (see Table A.5).

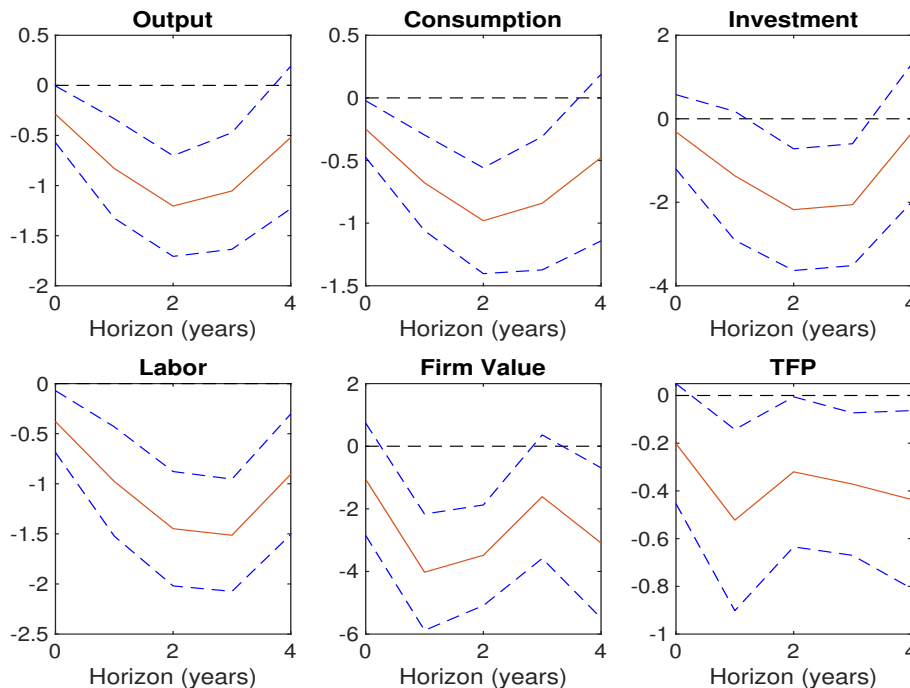


FIGURE 4. Estimated impulse response of macroeconomic variables to a turbulence shock

Note: This figure shows the impulse responses of macroeconomic variables to a one-standard-deviation (34.57%) increase in the log-level of turbulence estimated from the local projections model (9). The solid lines show the point estimates of the impulse responses. The blue dashed lines show the 68% confidence intervals.

Source: BEA, Compustat, NBER-CES, and authors' calculations.

macroeconomic variables to a turbulence shock using the local projections approach of Jordà (2005).¹⁹

We consider the empirical specification

$$x_{t+h} - x_{t-1} = \beta_0^h + \beta_1^h \text{turb}_t + \beta_2^h \text{turb}_{t-1} + \beta_3^h (x_{t-1} - x_{t-2}) + \epsilon_{t+h} \quad h = 0, 1, 2, 3, 4. \quad (9)$$

The dependent variable $x_{t+h} - x_{t-1}$ denotes the cumulative changes in the log-level of the variable of interest from year $t - 1$ to year $t + h$, where h denotes the projection horizons (number of years). The list of dependent variables includes the macroeconomic times series of per capita real consumption, investment, private output (i.e., the sum of consumption and investment), and hours worked, and also the firm value and the manufacture TFP constructed from the firm-level and industry-level data. The independent variable $\text{turb}_t \equiv \log(1 - \rho_t)$

¹⁹As shown by Plagborg-Møller and Wolf (2021), the local projections and vector autoregressions (VARs) estimate the same impulse responses.

denotes the log-level of turbulence in year t . In estimating the local projections, we control for lagged turbulence ($turb_{t-1}$) and the lagged growth rate of the dependent variable ($x_{t-1} - x_{t-2}$). The term ϵ_{t+h} is the regression residual. The parameter β_1^h measures the impulse responses of the macroeconomic variables to a turbulence shock at horizon h .

Figure 4 plots the estimated impulse responses of the macroeconomic variables to a one-standard-deviation turbulence shock (i.e., an increase in turbulence of 0.3457) for horizons up to five years.²⁰ The shock leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, hours worked, and firm value. It also leads to a decline in manufacturing TFP. These macroeconomic effects of turbulence are quantitatively important. For example, a one-standard-deviation increase in turbulence reduces per capita output by about 0.5 percent on impact, and by more than one percent within three years after the shock.

IV. A REAL BUSINESS CYCLE MODEL WITH TURBULENCE SHOCKS

We now construct a real business cycle model to examine the economic mechanism through which turbulence can drive macroeconomic fluctuations and cross-sectional reallocation. In light of the empirical evidence presented in Section III, we incorporate into the model two key ingredients—firm heterogeneity and financial frictions. We show that these ingredients are both important for the transmission of turbulence shocks.

IV.1. The model. The model economy is populated by a continuum of infinitely lived households with measure one. The representative household has the utility function

$$\mathbf{E} \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right\}, \quad (10)$$

where C_t denotes consumption, N_t denotes labor hours, and \mathbf{E} is an expectation operator. The parameter $\beta \in (0, 1)$ is a subjective discount factor, $\psi > 0$ measures the relative weight on the disutility of working, and $\gamma \geq 0$ is the inverse Frisch elasticity of labor supply.

All markets are perfectly competitive. The household takes prices as given and maximizes the utility in Eq. (10) subject to the sequence of budget constraints

$$C_t + K_{t+1} = (R_t + 1 - \delta)K_t + W_t N_t + D_t - T_t, \quad (11)$$

where K_{t+1} denotes the end-of-period capital stock, R_t denotes the capital rental rate, W_t denotes the real wage rate, D_t denotes the dividend income from firms, and T_t denotes a lump-sum tax paid to the government.

²⁰Our measured annual series of (logged) turbulence has a first-order autocorrelation of 0.648 and a standard deviation of 0.4541, implying a standard deviation of the innovation of 0.3457.

There is a continuum of firms, each endowed with a constant-returns technology that produces the final consumption good using capital and labor as inputs.²¹ Firms face idiosyncratic productivity shocks drawn at the beginning of each period, before hiring inputs. The production function for an individual firm is given by

$$y_{jt} = A_t z_{jt} k_{jt}^\alpha n_{jt}^{1-\alpha}, \quad (12)$$

where y_{jt} denotes the output produced by firm j in period t , and k_{jt} and n_{jt} denote the capital and labor inputs, respectively.

The term A_t denotes an aggregate productivity shock that follows the AR(1) process

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \sigma_A \varepsilon_t^A, \quad (13)$$

where the innovation term ε_t^A follows the standard normal process. The parameter ρ_A and σ_A measure the persistence and the volatility, respectively, of the aggregate productivity shock.

The idiosyncratic productivity shock z_{jt} follows the stochastic process described in Eq. (2), which we rewrite here for convenience of referencing:

$$z_{j,t+1} = \begin{cases} z_{jt} & \text{with prob } \rho_t, \\ \tilde{z} & \text{with prob } 1 - \rho_t. \end{cases} \quad (14)$$

Here, the term \tilde{z} is an i.i.d. random variable with a finite number of states. Specifically, we assume that $\tilde{z} = z_j$ with probability π_j , for $j = 1, 2, \dots, J$. Without loss of generality, we further assume that $z_1 < z_2 < \dots < z_J$. The process features time-invariant cross-sectional distribution of firm productivity such that, regardless of the realization of $\rho_t \in (0, 1)$, there is always a fraction π_j of firms with $z_{jt} = z_j$ in each period. Thus, in a stationary equilibrium, π_j is the measure of firms with productivity z_j .

We measure turbulence by $1 - \rho_t$. If $\rho_t = 1$, then the idiosyncratic productivity z_{jt} would be permanent. If $\rho_t = 0$, on the other hand, then each firm would face i.i.d. shocks to productivity with no persistence. A lower value of ρ_t implies that a high-productivity firm in the current period may not maintain its productivity in the next period, whereas a low-productivity firm in the current period might be able to draw a better productivity in the next period. Thus, a decline in ρ_t reshuffles firms' productivity ranking across time, increasing turbulence.

We assume that the turbulence shock follows the stochastic process

$$\ln(1 - \rho_t) = (1 - \rho_\rho) \ln(1 - \bar{\rho}) + \rho_\rho \ln(1 - \rho_{t-1}) + \sigma_\rho \varepsilon_t^\rho, \quad (15)$$

²¹In Appendix B, we show that the main model mechanism that drives the macroeconomic and reallocation effects of turbulence shocks is similar if the production technology exhibits decreasing returns of scale.

where $\bar{\rho}$ denotes the average level of ρ_t and the innovation term ε_t^ρ follows a standard normal process. The parameter ρ_ρ and σ_ρ measure the persistence and the volatility of the turbulence shock, respectively.

Firms rely on external financing of their working capital. In the beginning of each period, firms need to borrow from a competitive financial intermediary to cover payments for input factors, and these working capital loans are repaid within the period, after firms receive revenues. Following Jermann and Quadrini (2012) and Liu and Wang (2014), we assume that a firm's borrowing capacity is constrained by a fraction θ of its expected equity value in the next period, in line with the empirical evidence of Lian and Ma (2021).

Firms at each level of productivity face idiosyncratic production distortions (denoted by τ_{jt}), reflecting differential policy interventions or government subsidies at the firm level (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). These production distortions drive a wedge between firms' private and social marginal revenue products. We assume that τ_{jt} is drawn from a continuous i.i.d. distribution $F(\tau_{jt})$. Under credit constraints, the presence of idiosyncratic production distortions allows a fraction of firms at each level of productivity to stay active, enabling turbulence shocks to generate reallocation and endogenous fluctuations in aggregate TFP.²²

The firms' optimizing problem is characterized by the Bellman equation

$$V_t(z_{jt}, \tau_{jt}) = \max_{k_{jt}, n_{jt}} \tau_{jt} A_t z_{jt} k_{jt}^\alpha n_{jt}^{1-\alpha} - R_t k_{jt} - W_t n_{jt} + \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}), \quad (16)$$

subject to the working capital constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta \mathbb{E}_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta B_{jt}. \quad (17)$$

Here, the term $V_t(z_{jt}, \tau_{jt})$ denotes the value function of firm j that depends on the firm-level state variables z_{jt} and τ_{jt} . The value function $V_t(z_{jt}, \tau_{jt})$ also depends on aggregate shocks, which are summarized by the time subscript t . The term $M_{t+1} = \beta \frac{C_t}{C_{t+1}}$ denotes the stochastic discount factor determined by the marginal utilities of the representative household who owns all firms. The term B_{jt} denotes the expected present value of a firm with current productivity z_{jt} .

Profit maximizing implies the conditional factor demand functions

$$\alpha \frac{\tau_{jt} y_{jt}}{k_{jt}} = (1 + \mu_{jt}) R_t, \quad (18)$$

²²Including idiosyncratic distortions also serves a technical purpose in our model with a discrete distribution of idiosyncratic productivity. The continuity of the distribution function $F(\tau_{jt})$ implies a well-defined cutoff point τ_{jt}^* that determines the subset of active firms at each level of productivity z_{jt} .

and

$$(1 - \alpha) \frac{\tau_{jt} y_{jt}}{n_{jt}} = (1 + \mu_{jt}) W_t, \quad (19)$$

where μ_{jt} denotes the Lagrangian multiplier associated with the credit constraint (17). Using the factor demand functions, we can write the firm's flow profit as

$$d_{jt} \equiv \left[\tau_{jt} A_t z_{jt} \left(\frac{\alpha W_t}{(1 - \alpha) R_t} \right)^\alpha - \frac{W_t}{1 - \alpha} \right] n_{jt}. \quad (20)$$

Since production subsidies follow an i.i.d. process, a firm would choose to be active in production if and only if its subsidy τ_{jt} is sufficiently high such that $d_{jt} \geq 0$. It follows that there exists a threshold level of production subsidy τ_{jt}^* such that, if $\tau_{jt} \geq \tau_{jt}^*$, then a firm would be active in production, facing binding credit constraints. Otherwise, the firm would remain inactive. At the threshold level of subsidy, a firm earns zero profit and thus it would be indifferent between producing and staying inactive. The indifference condition determines the threshold level of subsidy

$$\tau_{jt}^* = \frac{R_t^\alpha W_t^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha} A_t z_{jt}}. \quad (21)$$

The threshold τ_{jt}^* increases with the factor prices R_t and W_t and decreases with the productivity level z_{jt} . Thus, given the factor prices, the fraction of active firms is larger for firms with higher productivity.

The presence of credit constraints and production distortions creates misallocation of resources. Absent those distortions, all resources would be allocated to the most productive firm (with productivity z_j). However, under those distortions, some low-productivity firms are able to produce because not all high-productivity firms are active. Specifically, at each level of productivity, there is a non-degenerate fraction of firms that are active, with the share of active firms measured by $1 - F(\tau_{jt}^*)$ for all $j \in 1, \dots, J$. Such misallocation opens up a reallocation channel for turbulence shocks, as we show below.

Since active firms face binding credit constraints and inactive firms do not use any input factors, we obtain the conditional demand functions for labor and capital inputs

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_{jt}}{W_t}, & \text{if } \tau_{jt} \geq \tau_{jt}^* \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

and

$$k_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\alpha\theta B_{jt}}{R_t}, & \text{if } \tau_{jt} \geq \tau_{jt}^* \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

Given the factor demand functions, firm j 's value function can be written as

$$V_t(z_{jt}, \tau_{jt}) = \max \left\{ \frac{\tau_{jt}}{\tau_{jt}^*} - 1, 0 \right\} \theta B_{jt} + B_{jt}. \quad (24)$$

Since production subsidies are i.i.d. across time, the average value of a firm with productivity z_{jt} is given by

$$\bar{V}_t(z_{jt}) = \int V_t(z_{jt}, \tau) dF(\tau) = \left[1 + \theta \int_{\tau_{jt}^*}^{\tau} \left(\frac{\tau}{\tau_{jt}^*} - 1 \right) dF(\tau) \right] B_{jt} \equiv \Phi(\tau_{jt}^*) B_{jt}, \quad (25)$$

where the term $\Phi(\tau_{jt}^*) \equiv 1 + \theta \int_{\tau_{jt}^*}^{\tau} \left(\frac{\tau}{\tau_{jt}^*} - 1 \right) dF(\tau)$ is a decreasing function of the threshold subsidy level τ_{jt}^* .

Given the stochastic process of $z_{j,t+1}$ and the definition of B_{jt} in Eq. (17), we have

$$B_{jt} \equiv \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left[\rho_t \bar{V}_{jt+1} + (1 - \rho_t) \sum_{i=1}^J \pi_i \bar{V}_{it+1} \right]. \quad (26)$$

In a competitive equilibrium, markets for labor, capital, and final consumption goods all clear. Labor market clearing implies that

$$N_t = \sum_j \pi_j n_{jt} \equiv \sum_j \pi_j \frac{(1 - \alpha) \theta B_{jt}}{W_t} [1 - F(\tau_{jt}^*)]. \quad (27)$$

Capital market clearing implies that

$$K_t = \sum_j \pi_j k_{jt} \equiv \sum_j \pi_j \frac{\alpha \theta B_{jt}}{R_t} [1 - F(\tau_{jt}^*)]. \quad (28)$$

Goods market clearing implies that

$$Y_t = C_t + K_{t+1} - (1 - \delta) K_t, \quad (29)$$

where aggregate output Y_t is given by

$$Y_t \equiv \sum_j \pi_j y_{jt} = \sum_j \pi_j A_t z_{jt} k_{jt}^\alpha n_{jt}^{1-\alpha}. \quad (30)$$

Given aggregate output, aggregate capital and labor inputs, we define aggregate TFP as

$$Z_t \equiv \frac{Y_t}{K_t^\alpha N_t^{1-\alpha}} = \frac{\sum_j \pi_j A_t z_{jt} k_{jt}^\alpha n_{jt}^{1-\alpha}}{K_t^\alpha N_t^{1-\alpha}}. \quad (31)$$

Definition. A competitive equilibrium consists of the sequence of allocations $\{C_t, Y_t, N_t, K_t\}$ and the sequence of prices $\{W_t, R_t\}$ such that (i) taking all prices as given, the allocations solve the household's utility maximizing problem and the firms' profit maximizing problem; and (ii) markets for labor, capital, and goods all clear.

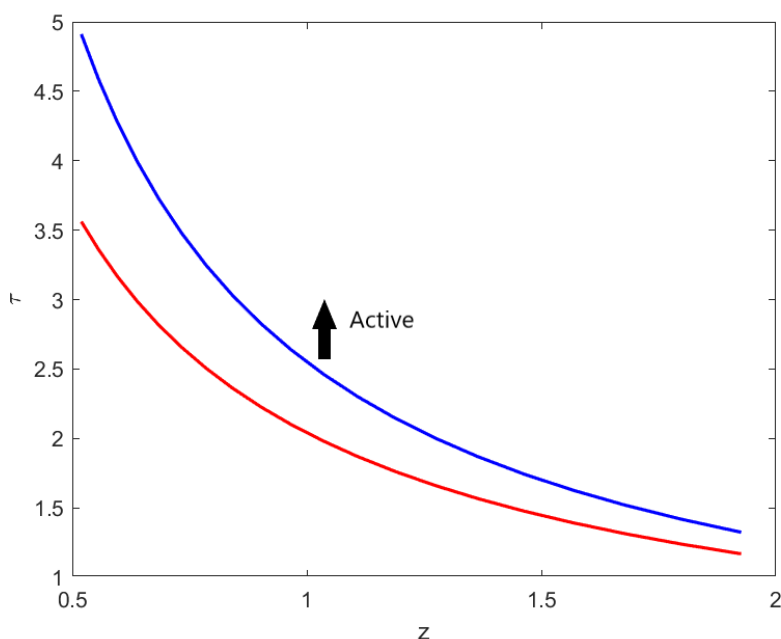


FIGURE 5. Production decisions in the steady-state equilibrium

Note: This figure shows the steady-state relation between the threshold level of subsidy τ^* and firm productivity z (blue line). Firms with (τ, z) lying above the threshold line are active in production and those below the line are inactive. The figure also shows the threshold line (red line) for production decisions in an economy with a higher average level of turbulence (i.e., a lower value of $\bar{\rho}$).

IV.2. Steady-state allocations. We now provide some analytical characterization of the steady-state equilibrium and show how the steady-state allocations vary with the average level of turbulence $(1 - \bar{\rho})$.

In Section IV.1, we have shown that a firm with productivity z_j chooses to produce (i.e., become active) if and only if its subsidy exceeds the threshold τ_j^* . The threshold level of subsidy is given by Eq. (21), and it is a decreasing function of firm productivity z_j .

Figure 5 illustrates the production decisions for firms with different levels of productivity (z) and subsidies (τ). The downward-sloping curve indicates the threshold function $\tau^*(z)$. At each z , a firm with a subsidy $\tau \geq \tau^*(z)$ chooses to produce. Otherwise, it stays inactive. Thus, the region of active firms are those with (τ, z) lying above the threshold curve.

In an economy with a higher average level of turbulence (i.e., with a lower value of $\bar{\rho}$), a high-productivity firm is less likely to remain productive, reallocating resources to low-productivity firms. Such reallocations reduce aggregate TFP, lowering the factor prices. For any given z , the declines in wages and capital rents reduce the threshold level of subsidy

τ^* (see Eq. (21)). Therefore, the threshold curve for production decisions shifts downward, with a flatter slope (red line), indicating that the increase in turbulence expands the active regions for low-productivity firms more than it does for high-productivity firms.

The reallocation effects of turbulence illustrated in Figure 5 can be formalized by the following proposition.

Proposition 3. Given the steady-state factor prices R and W , an increase in average turbulence reduces the share of labor hours allocated to high-productivity firms. Specifically, define the relative share of labor hours as $\eta_{ji} \equiv \frac{N_j}{N_i}$, where N_j and N_i denote labor hours allocated to active firms with productivity z_j and z_i , respectively. Without loss of generality, we assume that $z_j > z_i$. Then, we have

$$\frac{\partial \eta_{ji}}{\partial \bar{\rho}} > 0. \quad (32)$$

Proof. In the steady-state equilibrium, the relative share of labor is given by

$$\eta_{ij} = \frac{\pi_j}{\pi_i} \frac{1 - \beta \bar{\rho} \Phi(\tau_i^*) [1 - F(\tau_j^*)]}{1 - \beta \bar{\rho} \Phi(\tau_j^*) [1 - F(\tau_i^*)]},$$

where we have used aggregated labor demand based on Eq. (22), with the term B_j substituted out using the steady-state version of Eq. (26).

At given values of W and R , the threshold τ_j^* is a function of z_j only. Since $z_j > z_i$, it is easy to show that $\Phi(\tau_j^*) > \Phi(\tau_i^*)$ and that $1 - F(\tau_j^*) > 1 - F(\tau_i^*) > 0$. Thus, we have

$$\frac{\partial \eta_{ji}}{\partial \bar{\rho}} = \frac{\beta [\Phi(\tau_j^*) - \Phi(\tau_i^*)] [1 - F(\tau_j^*)]}{(1 - \beta \bar{\rho} \Phi(\tau_j^*))^2 [1 - F(\tau_i^*)]} > 0 \quad (33)$$

□

When turbulence rises ($\bar{\rho}$ declines), current productivity is less predictive for future productivity, such that productive firms are less likely to stay productive. Thus, an increase in turbulence lowers expected value of productive firms, reducing their borrowing capacity for financing working capital. As a consequence, labor is reallocated to less productive firms.

The analytical results in Proposition 3 are partial equilibrium in nature, because we have assumed that the factor prices W and R are independent of turbulence. However, since the production thresholds for firms with lower productivity are more sensitive to changes in the factor prices, an increase in turbulence that reduces the factor prices would disproportionately expand the active regions for low-productivity firms, reinforcing the misallocation effects of turbulence. We quantify the general equilibrium effects of a turbulence shock in the next section.

IV.3. The calibration. A period in our model corresponds to a year. We set the subject discount factor to $\beta = 0.96$, implying an annualized risk-free interest rate of 4 percent. Based on our estimated firm-level production function parameters using the Compustat and NBER-CES data, we calibrate the cost share of capital to $\alpha = 0.34$. We set the capital depreciation rate to $\delta = 0.10$ to match the average annual investment rate of 10 percent in the U.S. data (Eisfeldt and Rampini, 2006; Clementi and Palazzo, 2016). We assume that labor is indivisible in the sense of Hansen (1985) and Rogerson (1988), implying that $\gamma = 0$. We calibrate the relative utility weight on leisure ψ such that the steady-state labor hours are one-third of the time endowment. We set the parameter θ to 0.35 to match the average ratio of working capital to market equity in the Compustat data. We set the annual persistence of aggregate TFP shocks to $\rho_A = 0.95$, following the real business cycle literature (Cooley and Prescott, 1995; Bloom et al., 2018). We normalize the standard deviation of the aggregate TFP shock to 1 percent (0.01).

The presence of the production distortions τ_{jt} can potentially complicate the calibration of the turbulence shock. As is well-known in the productivity literature (Syverson, 2004), our revenue-based measure of firm-level TFP contains not only true productivity shocks z_{jt} but also information about demand conditions summarized in the production distortion term τ_{jt} . Specifically, the model implies that

$$tfp_{jt} = \log(z_{jt}) + \log(\tau_{jt}), \quad (34)$$

where tfp_{jt} is the firm-level TFP (in log units) that we construct based on the production function using data from Compustat and NBER-CES. For tractability, we assume that τ_{jt} is an i.i.d. process with a constant variance σ_τ and that τ_{jt} is uncorrelated with z_{jt} .

If $\sigma_\tau > 0$, then the estimated average value of the Spearman rank correlation of the observed TFP would understate the true value of $\bar{\rho}$.²³ The presence of τ_{jt} in measured TFP would also distort the estimated value of σ_z . Thus, we need to jointly calibrate the values of σ_τ , $\bar{\rho}$, and σ_z .

We implement this calibration by targeting three moments in the model to their counterparts in the firm-level data. Those three moments in the data include (1) the average value of the Spearman rank correlations of establishment-level TFP (0.72, estimated by Bloom

²³To see this, consider the extreme case with $\bar{\rho} = 1$ (i.e., no changes in firm productivity ranking). Given the noise in observed productivity stemming from τ_{jt} , the estimated Spearman correlation using the observed TFP would be less than one.

TABLE 4. Calibrated parameters

Parameter	Description	Value	Target
β	Subjective discount factor	0.96	Average real interest rate of 4% per year
α	Capital share	0.34	Average cost share of capital (NBER-CES)
δ	Capital depreciation rate	0.10	Capital depreciation rate of 10% per year
γ	Inverse Frisch elasticity	0	Indivisible labor
ψ	Utility weight on leisure	2.15	Average hours of 1/3 of time endowment
θ	Loan to value ratio	0.35	Working capital to equity ratio (Compustat)
$\bar{\rho}$	Firm-level TFP persistence	0.96	Estimated (Compustat and NBER-CES)
σ_z	Volatility of firm-level TFP shock	0.11	Estimated (Compustat and NBER-CES)
σ_τ	Volatility of production distortion (log)	0.35	Estimated (Compustat and NBER-CES)
μ_τ	Average production distortion (log)	-0.045	Normalized
ρ_ρ	Persistence of turbulence shock	0.605	Estimated to match Bloom et al. (2018)
σ_ρ	Volatility of turbulence shock	0.58	Estimated to match Bloom et al. (2018)
ρ_A	Persistence of productivity shock	0.95	Cooley and Prescott (1995)
σ_A	Volatility of aggregate TFP shock	0.01	Normalized

et al. (2018)),²⁴ (2) the standard deviation of the firm-level TFP shock (0.22, based on firm-level TFP constructed using the Compustat/NBER-CES data), and (3) the average IQR of equity values across firms (1.53, also from the Compustat data). This calibration implies that $\bar{\rho} = 0.96$, $\sigma_z = 0.11$, and $\sigma_\tau = 0.35$.

The presence of τ_{jt} can also affect the calibration of the turbulence shock process (i.e., ρ_ρ and σ_ρ). Given our calibration of σ_τ , $\bar{\rho}$, and σ_z , we use Eq. (34) to simulate the true productivity process z_{jt} and calibrate the two parameters ρ_ρ and σ_ρ to target the persistence and the standard deviation of the turbulence measure based on establishment-level TFP (tfp_{jt}) constructed by Bloom et al. (2018). This process leads to our calibration of $\rho_\rho = 0.605$ and $\sigma_\rho = 0.58$.

IV.4. Impulse responses to a turbulence shock. To examine the macroeconomic effects of turbulence shocks, we solve our model based on calibrated parameters. We simulate the model using third-order approximations of the equilibrium conditions around the deterministic steady-state. We then compute impulse responses of several key macroeconomic and

²⁴The calculations of Bloom et al. (2018) are based on establishment-level data from the Census of Manufactures (CM) and the Annual Survey of Manufactures (ASM), which provide much broader and more granular coverage of U.S. businesses than the firm-level data in the Compustat.

distributional variables as deviations of those variables driven by the turbulence shock from their stochastic steady-state levels without the shock.²⁵

Figure 6 displays the impulse responses to a one-standard-deviation shock to turbulence. An increase in turbulence reduces the chance for a current high-productivity firm to remain as productive in the future, and it also increases the chance for a current low-productivity firm to get a higher productivity draw in the future. Thus, the equity values of high-productivity firms declines relative to those of low-productivity firms, leading to a decline in the IQR of firm value and disproportionately tightening the borrowing capacity of high-productivity firms in the current period. Facing tightened credit constraints, high-productivity firms pull back hiring, reallocating labor and capital to low-productivity firms. Since high-productivity firms use more capital and labor in the steady-state than low-productivity firms, the increase in turbulence reduces the IQR of labor and capital across firms, and it also reduces the IQR of sales, in line with the empirical evidence presented in Section III.3.

Through reallocation, a turbulence shock reduces aggregate TFP and leads to a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. The recessionary effects of turbulence are sizable and persistent. For example, a one-standard-deviation turbulence shock leads to a drop in aggregate output of about 2 percent on impact, and output stays below its steady-state level for more than five years after the shock.

IV.5. The role of financial frictions. Our empirical evidence suggests that financial frictions are important for the reallocation effects of turbulence (Section III.4). We now illustrate the quantitative importance of financial frictions for propagating turbulence shocks to driven macroeconomic fluctuations. For this purpose, we consider a counterfactual version of our model, in which firms' borrowing capacity does not vary with the expected equity value. Specifically, we replace the working capital constraint with

$$R_t k_{jt} + W_t n_{jt} \leq \theta \beta E_t \frac{C_t}{C_{t+1}} \left[\rho_t \bar{V}_j^{ss} + (1 - \rho_t) \sum_{i=1}^J \pi_i \bar{V}_i^{ss} \right] \equiv \theta \bar{B}_{jt}, \quad (35)$$

²⁵We follow the approach in Fernández-Villaverde et al. (2011) and Leduc and Liu (2016) to compute the impulse responses. In particular, the model is first simulated for a large number of periods to compute the ergodic mean of each variable. It is then simulated using the ergodic means as a starting point. Finally, impulse responses to a turbulence shock are computed as the differences between the simulated path with the turbulence shock and the path with no shocks. This solution approach helps capture potential non-linear effects of the shock. Since turbulence shocks in our model have first-moment impact, the impulse responses generated from the third-order approximations are essentially the same as those from first-order approximations (and we have verified this).

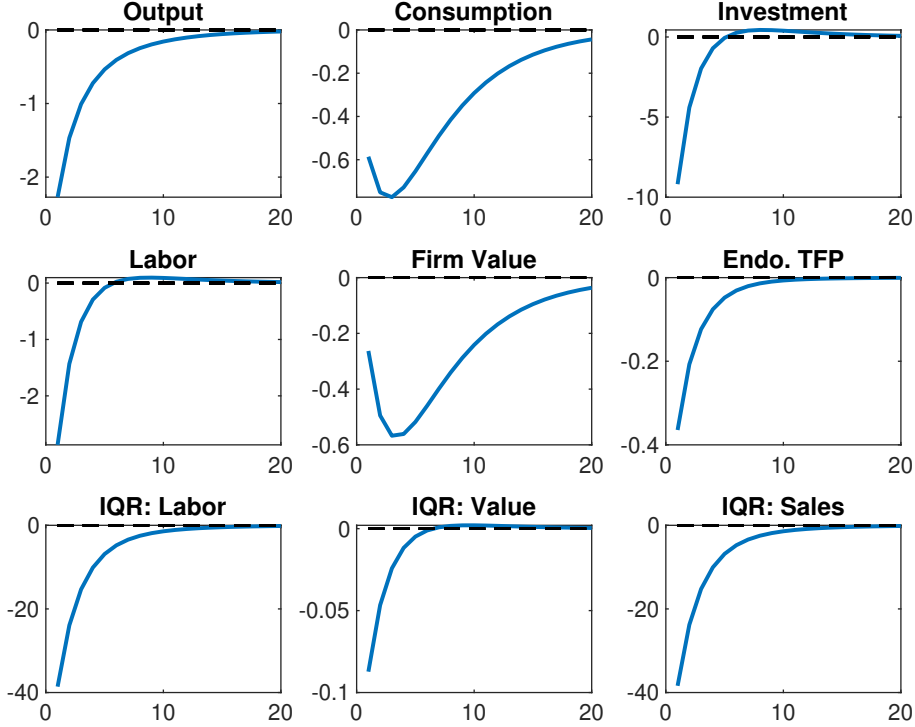


FIGURE 6. Impulse response to a turbulence shock in the benchmark model

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the calibrated benchmark model. The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

where \bar{V}_j^{ss} denotes the steady-state equity value for firms with productivity z_{jt} . In this counterfactual, a turbulence shock can still influence firms’ borrowing capacity by changing the transition probability (ρ_t) of the future productivity distribution, but changes in firms’ expected equity value following a turbulence shock would have no effect on the borrowing capacity.

Figure 7 shows the impulse responses in the benchmark model (blue solid lines) and those in the counterfactual under this “quasi-fixed” borrowing capacity (red dash-dotted lines). The figure shows that the recession effects and the reallocation effects of a turbulence shock would be substantially dampened if firms’ borrowing capacity could not vary with the expected equity value. This counterfactual illustrates the importance of financial frictions—and in particular, the endogenous variations of the borrowing capacity with expected firm values—for propagating turbulence shocks.

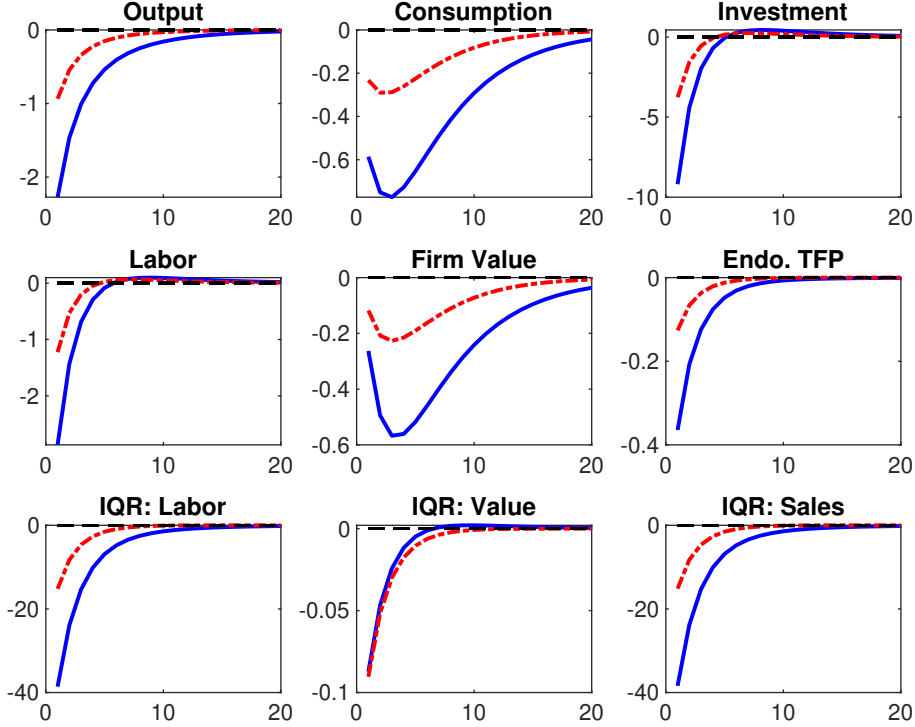


FIGURE 7. Impulse response to a turbulence shock: Benchmark model vs. counterfactual with quasi-fixed borrowing capacity

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the benchmark model (blue lines) and in the counterfactual with quasi-fixed borrowing capacity (red dash-dotted lines). The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

IV.6. Impulse responses to a micro-level uncertainty shock. Turbulence is related to but different from micro-level uncertainty such as that studied by Bloom et al. (2018). Uncertainty is a mean-preserving spread of the firm-level productivity distribution. Similar to uncertainty, turbulence reshuffles firms’ rankings in future productivity distributions, which also raises the *conditional* variance of the idiosyncratic shocks. Different from uncertainty, turbulence also changes the conditional mean of the productivity distribution. By changing the conditional mean, a turbulence shock generates between-firm reallocation, which is amplified through financial frictions and leads to procyclical aggregate TFP and business cycle comovements.

To illustrate the relation between turbulence and uncertainty, we consider a micro-level uncertainty shock. Specifically, we assume that the i.i.d. production distortion τ_{jt} follows

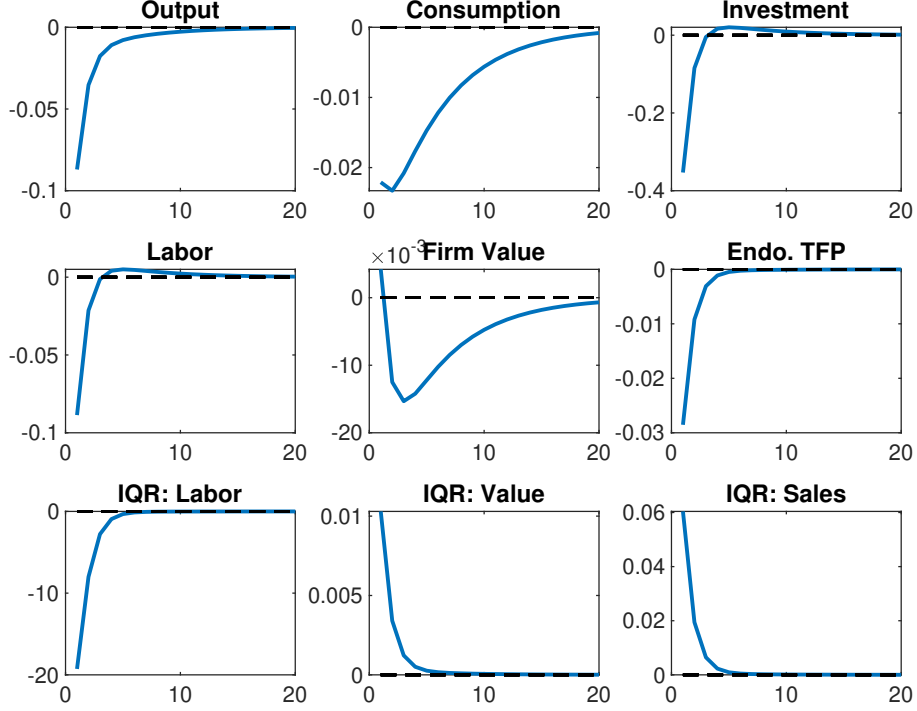


FIGURE 8. Impulse responses to a micro-level uncertainty shock in the benchmark model

Note: This figure shows the impulse responses to a one-standard-deviation shock to micro-level uncertainty in the benchmark model. Uncertainty is measured by the time-varying standard deviation of the production subsidies ($\sigma_{\tau,t}$). The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

a log-normal distribution with a time-varying volatility $\sigma_{\tau,t}$ that follows the stationary stochastic process

$$\ln(\sigma_{\tau,t}) = (1 - \rho_\sigma) \ln(\sigma_\tau) + \rho_\sigma \ln(\sigma_{\tau,t-1}) + \sigma_\sigma \varepsilon_t^\sigma, \quad (36)$$

where the innovation term ε_t^σ follows the standard normal process. The parameters ρ_σ and σ_σ measure the persistence and the standard deviation of the volatility shock, respectively. We normalize the mean value of $\log(\tau_{jt})$ to $\mu_{\tau,t} = -0.5\sigma_{\tau,t}^2$ such that the unconditional mean of τ_{jt} is always one (i.e., $E(\tau_{jt}) = 1$). Our uncertainty measure $\ln(\sigma_{\tau,t})$ here parallels the micro-level uncertainty in Bloom et al. (2018). We set the persistence of the micro-level uncertainty to $\rho_\sigma = 0.75$, consistent with Leduc and Liu (2016) and Bloom et al. (2018). We normalize the standard deviations of the uncertainty shock to 1 percent (0.01).

Figure 8 shows the impulse responses of the key macroeconomic and distributional variables following an increase in micro-level uncertainty. Similar to turbulence, uncertainty also

leads to misallocation and reduces aggregate productivity, but through a different channel. An increase in uncertainty expands the right tails of the subsidy distribution, raising the average subsidies for active firms at each productivity level (i.e., those with $\tau_j \geq \tau_j^*$).²⁶ Since the threshold level of subsidy (τ_j^*) decreases with firm-level productivity (z_j), the expansion of the right tail of the subsidy distribution increases the mass of active low-productivity firms by more than that of high-productivity firms. Thus, uncertainty reallocates capital and labor towards low-productivity firms, reducing aggregate TFP. The recessionary effect of uncertainty shock is amplified through general equilibrium channels due to lower wage and rent. The decline in aggregate TFP can generate synchronized declines in output, consumption, investment and labor hours, which is in line with empirical estimates of the impulse response using local projection method (Figure A3 in Appendix).

An uncertainty shock disproportionately raises the average subsidy received by high-productivity firms, and thus, it raises the IQR of firm value and sales (inclusive of subsidies), consistent with empirical evidence (see Figure A3 in the Appendix). In contrast, a turbulence shock reduces the IQR of both firm value and sales.²⁷

IV.7. Impulse responses to an aggregate productivity shock. The macroeconomic and reallocation effects of turbulence are also different from those associated with other aggregate shocks such as a TFP shock.

Figure 9 displays the impulse responses to a negative TFP shock. Similar to a turbulence shock, a negative TFP shock generates a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. The shock also reduces average firm value.

However, the TFP shock has different reallocation effects than turbulence. A decline in aggregate TFP raises the threshold level of subsidy for active firms at each level of productivity, shrinking the set of active firms. Since labor and capital are perfectly mobile across firms, all firms face the same wages and capital rents. A decline in aggregate TFP would therefore force more low-productivity firms into the inactive regions than high-productivity firms. This “cleansing effect” reallocates resources to more productive firms, raising the endogenous component of aggregate TFP, mitigating the recession. The negative TFP shock also reduces firm value dispersion because, among active firms, lower-productivity firms need

²⁶Uncertainty also expands the left tail of the distribution, but that does affect production because firms with subsidies in the left tails are inactive.

²⁷Here, the IQR of labor is computed based on the share of labor allocated to high-productivity firms (the top quartile) times the mass of high-productivity firms relative to that allocated to low-productivity firms (the bottom quartile). The IQRs of firm value and sales are measured by the ratio of the *average* value of high-productivity firms to that of low-productivity firms (excluding the effects from the mass of producers at each level of productivity), which is comparable to the empirical measures.

to be compensated by higher average subsidies for them to remain active. Thus, the average subsidy for low-productivity firms increases relative to that for high-productivity firms, reducing the dispersion in firm value.²⁸

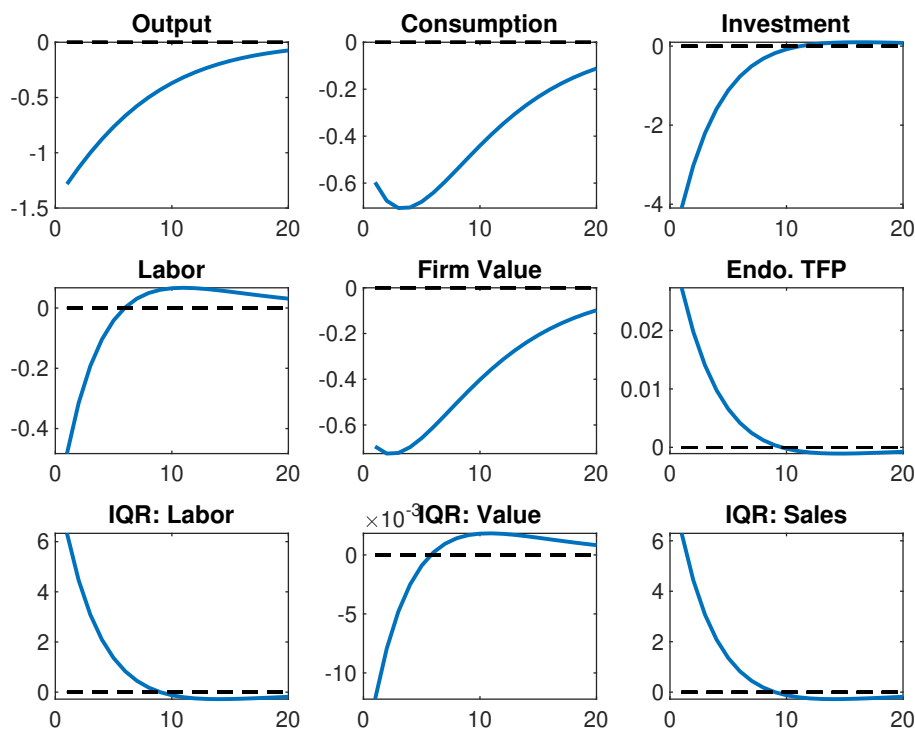


FIGURE 9. Impulse response to a negative aggregate TFP shock in the benchmark model

Note: This figure shows the impulse responses to a one-standard-deviation negative shock to aggregate TFP. The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

IV.8. Quantitative importance of turbulence shocks. Turbulence has quantitatively important recessionary effects, both in the model and in the data. Figure 10 compares the model-implied impulse responses of aggregate output (blue solid line) with the empirical estimates of the impulse response (black solid line). A one-standard-deviation turbulence shock reduces aggregate output by around 1 percent after one year, both in the data and in the model. The shock has persistent recessionary effects on aggregate output both in the model and in the data, although the theoretical impulse responses miss the hump estimated

²⁸For distributional impacts of a TFP shock, see Figure A7 in Appendix A.6.3.

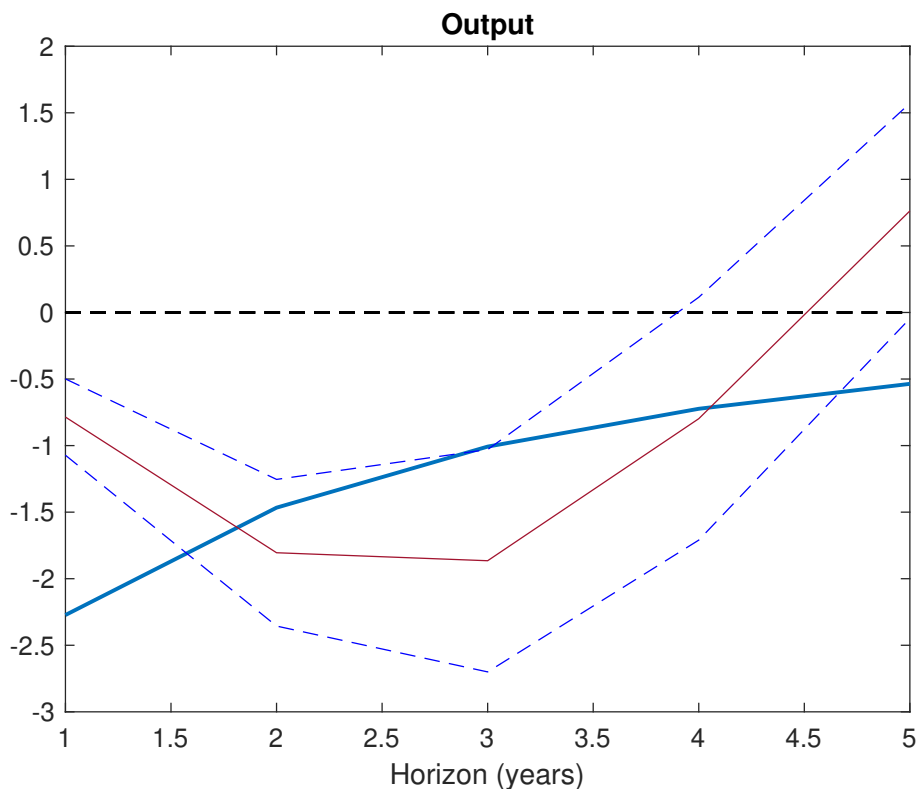


FIGURE 10. Impulse response to turbulence shock: Model vs. data

Note: This figure shows the impulse responses of private aggregate output to a one-standard-deviation shock to turbulence in the data (red solid line) and in the calibrated annual version of the model (blue solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percent deviations of output in the model from its steady-state level and the percentage changes in output in the data relative to its pre-shock level.

from the data. These findings suggest that turbulence plays an important role in driving business cycles.²⁹

V. POLICY INTERVENTIONS

Appropriate policy interventions can potentially undo the financial frictions, stabilizing aggregate output and improving allocative efficiency. To illustrate this point, we consider

²⁹We have also compared the theoretical impulse responses of investment, consumption, and labor hours with those in the data (see Appendix A.6.4). We find that, similar to the case of output, the model-implied responses of investment and consumption are both in line with those estimated from the data, although the calibrated model fails to generate the turbulence-driven hump in the data.

two alternative policy interventions in response to a recession driven by a turbulence shock. One policy is a borrowing subsidy that reduces firms' borrowing costs. The other is a credit-easing policy that expands firms' borrowing capacity.

Under the borrowing subsidy policy (Policy I), the government subsidizes wages and rents for active firms at an exogenous rate ω_{1t} , such that firms with productivity z_{jt} face the effective credit constraint

$$\tilde{R}_t k_{jt} + \tilde{W}_t n_{jt} \leq \theta B_{jt}, \quad (37)$$

where $\tilde{R}_t = (1 - \omega_{1t})R_t$ and $\tilde{W}_t = (1 - \omega_{1t})W_t$ denote the after-subsidy capital rental rate and real wage rate, respectively. The subsidies thus reduce the amount of working capital loans that firms need to borrow. The government finances the borrowing subsidies $\Omega_{1t} = \omega_{1t}(W_t N_t + R_t K_t)$ by imposing lump-sum taxes on the representative household. We assume that operating this policy incurs a resource cost of $\lambda_1 \Omega_{1t}$, where $\lambda_1 \geq 0$ reflects potential deadweight losses associated with the government program.

Under the credit easing policy (Policy II), the government injects liquidity into active firms, such that firms with productivity z_{jt} face the effective credit constraint

$$R_t k_{jt} + W_t n_{jt} \leq \theta(1 + \omega_{2t})B_{jt}, \quad (38)$$

where $\omega_{2t} B_{jt}$ is the amount of government transfers to active firms with expected equity value B_{jt} . The total cost of the credit-easing policy is given by $\Omega_{2t} = \omega_{2t} \sum_j \pi_j \theta B_{jt} [1 - F(\tau_{jt}^*)]$, which is financed by lump-sum taxes on the household. Similar to the borrowing subsidy policy, we assume that credit easing also incurs a resource cost of $\lambda_2 \Omega_{2t}$, where $\lambda_2 \geq 0$ reflects potential deadweight losses in operating the policy.

Following Bloom et al. (2018), we consider transitory and unanticipated policy interventions. A policy would be implemented only if a turbulence shock hits the economy, and the policy intervention has the same persistence as the shock. We evaluate the effectiveness of each of the two alternative policies—borrowing subsidies and credit easing—for mitigating the macroeconomic and reallocation effects of turbulence. For this purpose, we compare the impulse responses of aggregate output and aggregate TFP to a turbulence shock under borrowing subsidies (Policy I) or credit easing (Policy II) to those in the *laissez-faire* benchmark economy without policy intervention (No policy).

In each policy regime, we simulate the model economy for 1000 periods based on third-order approximations of the equilibrium system around the deterministic steady-state. A turbulence shock and a simultaneous policy intervention (if any) are implemented in period 960. The unanticipated policy stimulus has a size of 1 percent of steady-state output, with the same persistence as the turbulence shock. After a policy intervention is implemented, we allow the economy to evolve naturally for the remaining 20 years. We calculate the responses

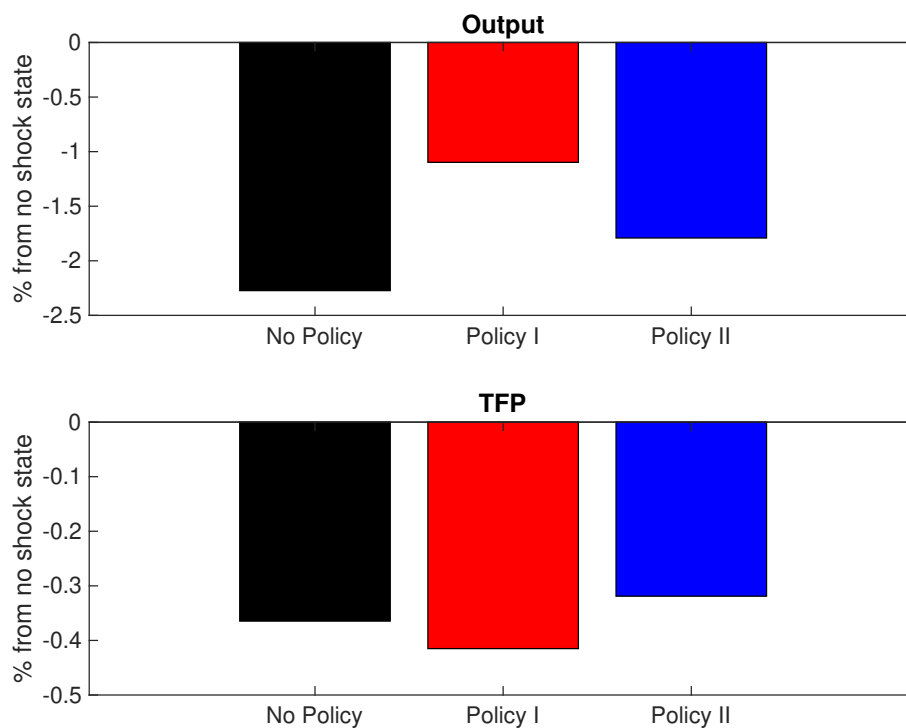


FIGURE 11. Impact effects of a turbulence shock with alternative policy interventions

Note: This figure plots the responses of aggregate output and aggregate TFP in the impact period of a one-standard-deviation turbulence shock. Black bars show the responses in the benchmark economy with no policy interventions. Red bars represent the responses under an unanticipated and temporary borrowing subsidy policy (Policy I). Blue bars represent the responses under an unanticipated and temporary credit easing policy (Policy II).

of each endogenous variable to the turbulence shock (with or without a policy intervention) as percent deviations from the stochastic steady-state.

Figure 11 shows the stabilizing effects of the two alternative policies relative to the benchmark economy, conditional on a one-standard-deviation turbulence shock.³⁰ Compared to the benchmark economy without policy interventions (black bar), Policy I (red bar) is effective for stabilizing the declines in aggregate output (upper panel). By providing borrowing subsidies, the policy stimulates demand for labor and capital, and thus mitigating the declines in equilibrium hours, investment, and output. However, by reducing the effective

³⁰The figure here shows the impact effects of a turbulence shock on aggregate output and TFP. The results are qualitatively the same when we consider the cumulative effects (not reported).

factor prices, Policy I enables a larger fraction of low-productivity firms to produce, exacerbating misallocation and intensifying the decline in aggregate TFP following a turbulence shock (lower panel).

The credit easing policy is also effective for stabilizing turbulence-driven output declines relative to the benchmark economy (blue bar, upper panel). The policy expands the borrowing capacity for all active firms, enabling a larger share of high-productivity firms to finance working capital and produce. The increase in the share of active high-productivity firms pushes up equilibrium wages and rents, shrinking the set of active low-productivity firms. Thus, Policy II reallocates labor and capital to high-productivity firms, improving aggregate TFP relative to the benchmark (blue bar, lower panel).

These policy experiments suggest that temporary borrowing subsidies or credit easing policies are effective for stabilizing turbulence-driven output fluctuations. However, the reallocation consequences of the two policies are different. While borrowing subsidies exacerbate misallocation, credit easing alleviates it.

VI. CONCLUSION

Macroeconomic fluctuations often mask underlying cross currents with important cross-sectional reallocations. We study the implications of turbulence—a form of reallocation shocks—for business cycles. An increase in turbulence changes the conditional distribution of firms' future productivity, leading to reallocations across firms. We measure turbulence based on firm-level TFP data and document evidence that turbulence is countercyclical, rising sharply in recessions. Turbulence has cross-sectional reallocation effects, the magnitude of which depends on financial frictions. Turbulence is negatively correlated with average firm equity values and aggregate TFP. An increase in turbulence is associated with synchronized declines in aggregate output, consumption, investment, and labor hours.

Using a real business cycle model augmented with firm heterogeneity and financial frictions, we have highlighted a quantitatively important reallocation channel, through which a turbulence shock drives macroeconomic fluctuations. An increase in turbulence reduces the likelihood for the current high-productivity firms to maintain their productivity rankings in the future, lowering their expected equity values relative to those of the current low-productivity firms. Facing tightened working capital constraints, high-productivity firms pull back hiring of capital and labor relative to low-productivity firms, leading to reallocation from high- to low-productivity firms and reducing aggregate TFP. Such declines in TFP generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. A one-standard-deviation shock to turbulence leads to a drop in aggregate output of about 0.5 percent, with the recessionary effects persisting for

more than five years, suggesting that turbulence plays an important role in driving business cycles.

Financial frictions are crucial for propagating turbulence shocks in our model. The presence of financial frictions also leads to misallocation. Policy interventions designed to alleviate credit constraints can potentially dampen the impact of turbulence and improve allocative efficiency. However, the particular approach to implementing such policy interventions can produce very different outcomes. For example, borrowing subsidies that reduce the amount of working capital loans that firms need to borrow can effectively boost aggregate output, mitigating the recessionary effects of turbulence. An alternative credit easing policy that expands firms' borrowing capacity can also stimulate aggregate output. However, these two alternative policies have different implications for allocative efficiency. A borrowing subsidy enables more low-productivity firms to stay active, exacerbating misallocation, whereas credit easing allows high-productivity firms to expand production, improving aggregate productivity.

To illustrate the key transmission mechanism of turbulence, we have intentionally kept the model stylized. For example, the model abstracts from firm entries and exits. To the extent that firms rely on external financing and entering (exiting) firms have higher (lower) productivity than incumbent firms, we conjecture that introducing entry and exit decisions could potentially amplify the recessionary effects of turbulence through reallocation. Another direction of generalizing our study is to enrich the model by incorporating other real and nominal frictions such as habit formation, investment adjustment costs, and sticky prices and wages. With these additional frictions, the model can better fit time-series data and it can also be used to examine the role of monetary policy in stabilizing macroeconomic fluctuations driven by turbulence. We leave these important subjects for future research.

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Supplemental Appendices: For Online Publication

APPENDIX A. ROBUSTNESS OF EMPIRICAL RESULTS

This appendix presents some robustness checks on the empirical measurement of turbulence, the macroeconomic and reallocation effects of turbulence.

A.1. Measuring turbulence under full information.

Proposition 4. If idiosyncratic TFP is perfectly observable, the Spearman rank correlation of firm productivity between period t and $t + 1$ is given by ρ_t .

Proof. Denote by $R_t(z)$ the ranking of productivity z across firms in period t . Under the stochastic process of z_t specified in Eq. (2), we have

$$R_{t+1}(z) = \begin{cases} R_t(z) & \text{with prob } \rho_t, \\ R_t(\tilde{z}) & \text{with prob } 1 - \rho_t. \end{cases} \quad (\text{A1})$$

The Spearman rank correlation of firm productivity between t and $t + 1$ is thus given by

$$\begin{aligned} r_s &\equiv \frac{\text{Cov}(R_{t+1}(z), R_t(z))}{\sqrt{\text{Var}(R_{t+1}(z))\text{Var}(R_t(z))}} \\ &= \frac{\rho_t \text{Cov}(R_t(z), R_t(z)) + (1 - \rho_t) \text{Cov}(R_t(\tilde{z}), R_t(z))}{\text{Var}(R_t(z))} \\ &= \frac{\rho_t \text{Var}(R_t(z))}{\text{Var}(R_t(z))} = \rho_t, \end{aligned} \quad (\text{A2})$$

Here, the first equality is the definition of the Spearman correlation (denoted by r_s). The second equality follows from Eq. (A1) and the assumption that the stationary productivity distribution $G(z)$ has a finite, discrete number of realizations such that the variance of the ranking of z stays constant over time. The third equality follows from the fact that $R_t(\tilde{z})$ is uncorrelated with $R_t(z)$. \square

This relation between ρ_t and the Spearman rank correlation of firm-level productivity forms the theoretical underpinning for our empirical measurement of turbulence.

A.2. Measures of turbulence. Our measure of turbulence is robust to alternative measures of value added, capital input, and labor input. It is also robust to the data samples used.

In our benchmark empirical specification, we construct firm-level TFP based on firm-level value added and capital and labor inputs. We measure value added using firm-level sales and the average share of intermediate inputs at the 6 digit industry level, where the intermediate input share is the ratio of costs of materials to total value of shipments. We measure capital

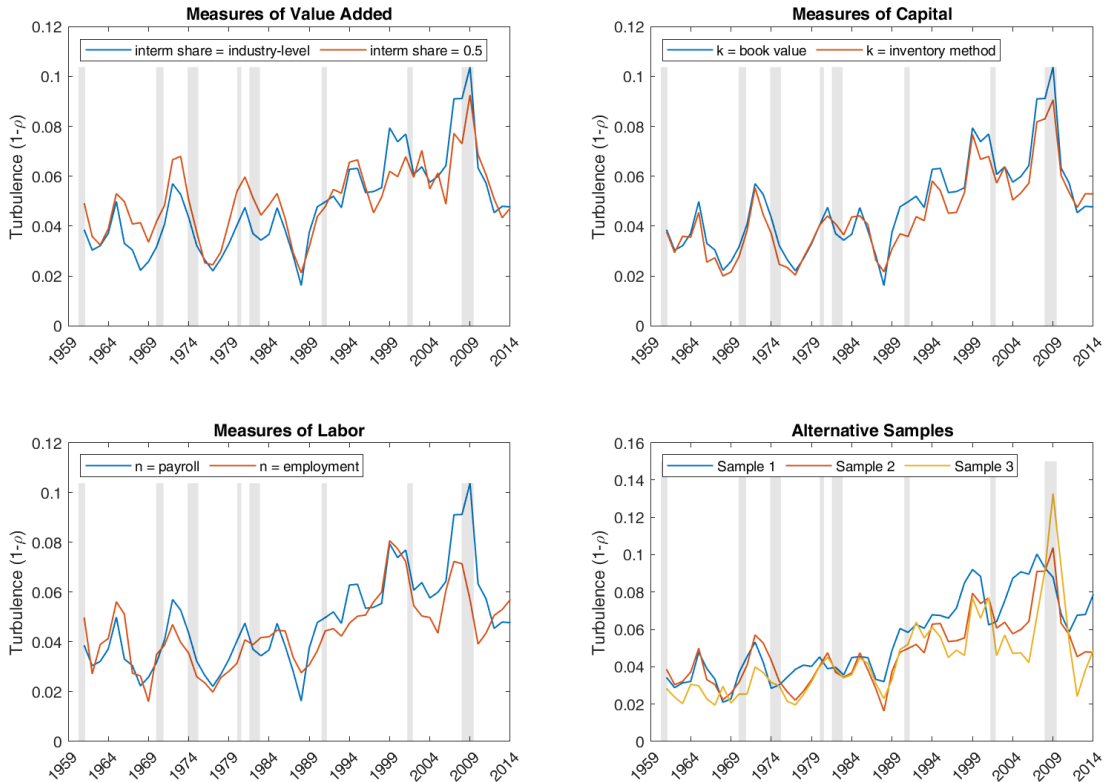


FIGURE A1. Robustness of Turbulence Measure

input using the real book value of a firm and labor input using the number of employees. We also focus on the sample that contains firms with 25+ years of observations (Sample 2). Figure A1 shows that the turbulence measure is robust to alternative measurements of value added, capital and labor inputs, and alternative samples.

The upper left panel of the figure compares the benchmark measure of turbulence (blue line) and the alternative measure using a different measure of value added (red line). In particular, we follow the approach in David et al. (2016) and David and Venkateswaran (2019), and construct value added by assuming a constant intermediate input share of 0.5 for all firms. These two alternative measures of turbulence are highly correlated, with a correlation coefficient of 0.7806. We have also considered another approach to constructing firm-level value added by subtracting the costs of goods sold from reported sales in Compustat (not shown in the figure). The resulting turbulence measure is also highly correlated with our benchmark measure, with a correlation coefficient of 0.7583.

The upper right panel of Figure A1 shows that the measure of turbulence is robust to alternative approaches to measuring capital. Here, instead of using the real book value

of firms, we measure capital input using a perpetual inventory method. Specifically, we first fix the initial real value of capital using $PPEGT$ in the first year of our sample. We then construct a measure of net investment using $PPENT_{it} - PPENT_{it-1}$, deflated by industry-specific investment deflators. Finally we iterate forward the law of motion of the capital stock by adding real net investment to the capital stock in the previous period. With this alternative measure of capital, we obtain a turbulence series (red line) that is highly correlated with the benchmark series (blue line), with a correlation coefficient of 0.8650.

The measure of turbulence is also robust to alternative approaches to measuring labor input, as shown in the lower left panel of Figure A1. Here, instead of measuring labor input by a firm's total payroll, we measure labor input by the number of employees. The turbulence series under this alternative measure of labor input (red line) is highly correlated with the benchmark series (blue line), with a correlation coefficient of 0.7428.

Finally, the lower right panel of the figure shows that the turbulence measure is robust to alternative data samples.

A.3. Reallocation effects of turbulence. The reallocation effects of turbulence are robust to alternative high-TFP indicators, alternative samples, and including controls of the potential reallocation effects of recessions.

Our baseline regression estimates the effects of turbulence on firms with different levels of TFP, based on current-year firm-level TFP ranking. It is possible that a firm's employment, capital or sales growth might affect its contemporaneous TFP ranking. To mitigate this concern, we construct an alternative indicator of high-productivity firms based on their TFP rankings with a one-year lag. We re-estimate the responses of firm growth to turbulence shocks for firms with different levels of productivity. Table A.1, with the lagged high-TFP indicator, turbulence reduces growth for high-productivity firms relative to low-productivity firms, similar to the results obtained under the benchmark specification.

TABLE A.1. Impact of turbulence on firms with different productivity:
Lagged high-TFP indicators

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
<i>High_TFP_{jt-1}</i>	0.068*** (0.007)	0.065*** (0.009)	0.000 (0.008)	0.049*** (0.012)
<i>Turb_t * High_TFP_{jt-1}</i>	-0.954*** (0.125)	-0.809*** (0.148)	-1.309*** (0.087)	-1.146*** (0.170)
<i>Constant</i>	0.019*** (0.003)	0.031*** (0.004)	0.079*** (0.004)	0.041*** (0.004)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	23,802	23,802	23,802	22,377

Note: This table shows the regression of firm-level employment, capital expenditure, sales or firm value growth on the measured turbulence (*Turb*) for firms with different levels of TFP (lagged). The dummy *High_TFP_{jt-1}* equals one if firm *j*'s TFP at year *t* - 1 is above the median and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Our results are robust to alternative sample selections. Table A.2 shows the estimation results from an alternative sample that includes only large industries, i.e., those with at least 20 firms in each industry and each year.

TABLE A.2. Impact of turbulence on firms with different productivity: Sample with large industries

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
<i>High_TFP_{jt}</i>	0.004 (0.004)	0.013 (0.008)	0.099*** (0.007)	0.022** (0.009)
<i>Turb_t * High_TFP_{jt}</i>	-1.046*** (0.066)	-1.027*** (0.144)	-1.343*** (0.104)	-0.374* (0.188)
<i>Constant</i>	0.057*** (0.002)	0.068*** (0.003)	0.039*** (0.005)	0.036*** (0.002)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	25,481	25,481	25,481	23,541

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence (*Turb*) for firms with different levels of TFP. The dummy *High_TFP_{jt}* equals one if firm *j*'s TFP is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015, in industries with at least 20 firms in each year (sample 3). The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Our benchmark empirical results suggest that financial frictions are important for amplifying the reallocation effects of turbulence. However, it is plausible that sufficiently large firms are not as financially constrained as small and medium-sized firms. Including large firms in our sample can thus potentially bias the results against finding important reallocation effects of turbulence. This is indeed the case, as we find using a subsample that excludes the top 10% of firms based on their asset sizes. As shown in Table A.3, excluding the large firms from our sample delivers stronger reallocation effect of turbulence.

TABLE A.3. Impact of turbulence on firms with different productivity: Sample excluding large firms

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
<i>High_TFP_{jt}</i>	0.007 (0.007)	0.008 (0.008)	0.097*** (0.007)	0.056*** (0.010)
<i>Turb_t * High_TFP_{jt}</i>	-0.932*** (0.087)	-0.739*** (0.094)	-1.160*** (0.077)	-0.941*** (0.185)
<i>constant</i>	0.054*** (0.002)	0.062*** (0.003)	0.034*** (0.004)	0.032*** (0.002)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	22,059	22,059	22,059	20,721

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence (*Turb*) for firms with different levels of TFP. The dummy *High_TFP_{jt}* equals one if firm *j*'s TFP is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015, excluding firms with asset sizes at the top 10% in each year. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Since turbulence is countercyclical, the reallocation effects estimated from our benchmark specification might reflect the general impact of recessions instead of that of turbulence. To address this concern, we consider a regression that controls for the direct reallocation effects of recessions. In particular, we add an interaction term between NBER recession year dummy ($NBER_t$) and our high TFP indicator in the regression. As shown in Table A.4, the reallocation effect of turbulence shock remains significant after controlling for the reallocation effect of recessions.

TABLE A.4. Impact of turbulence on firms with different productivity: Control for recession effects

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
$High_TFP_{jt}$	0.007 (0.006)	0.018** (0.007)	0.095*** (0.006)	0.055*** (0.008)
$Turb_t * High_TFP_{jt}$	-0.767*** (0.101)	-0.969*** (0.146)	-0.955*** (0.101)	-0.614*** (0.169)
$NBER_t * High_TFP_{jt}$	-0.043*** (0.009)	0.000 (0.007)	-0.064*** (0.010)	-0.078*** (0.011)
<i>Constant</i>	0.052*** (0.002)	0.060*** (0.003)	0.034*** (0.004)	0.033*** (0.002)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	18,740	18,740	18,740	17,604

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence ($Turb$) for firms with different levels of TFP. The dummy $High_TFP_{jt}$ equals one if firm j 's TFP is above the median within its industry and zero otherwise. The dummy $NBER_t$ equals one if year t contains a recession that lasts for at least 60 days based on the NBER recession dates. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

The reallocation effects of turbulence are persistent. To examine heterogeneous effects of turbulence over a longer horizon, we estimate the following equation using local projection method

$$x_{j,t+h} - x_{j,t-1} = \beta_0^h + \beta_1^h High_TFP_{jt} + \beta_2^h Turb_t * High_TFP_{jt} + \mu_j + \eta_t + \epsilon_{jt} \quad (A3)$$

where x_{jt} denote variables of interest, including capital, employment, sales and value of firm j in year t . The coefficient of interest is on the interaction term (i.e., β_2^h), which captures relative impact of turbulence on growth of high-productivity firms h years after the turbulence shock.

Figure A2 plots the estimated β_2^h on the marginal effect of turbulence on growth of high-productivity firms, which is significantly negative for each variable and at every year over a five-year horizon. This suggest that that employment, capital, sales and equity value of high-productivity firms are disproportionately affected by turbulence for at least five years.

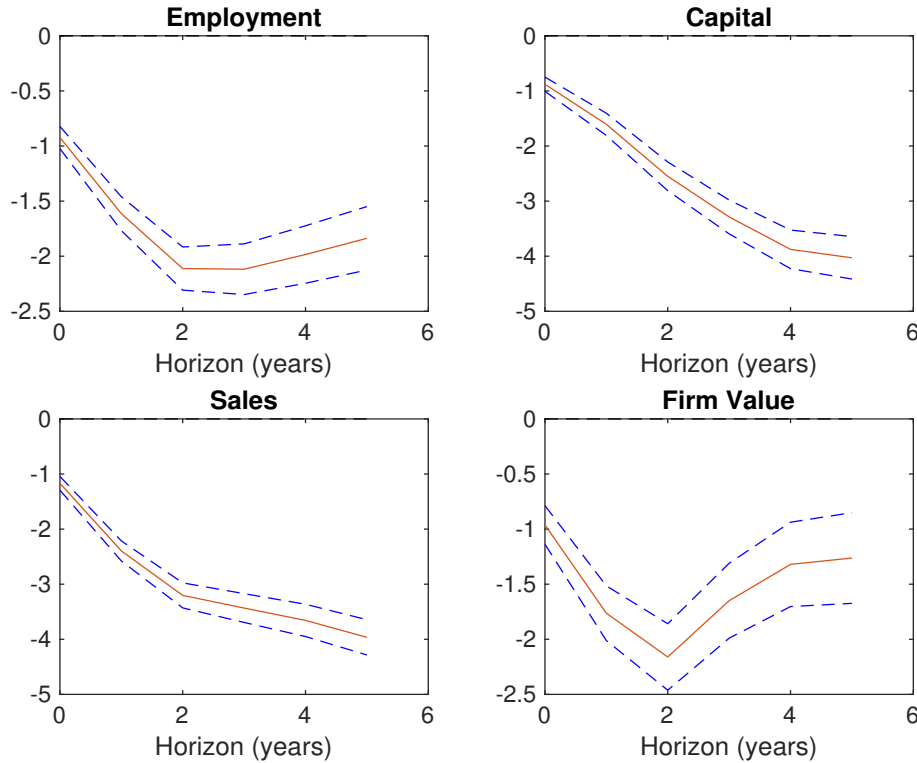


FIGURE A2. Persistent reallocation impact of turbulence

Note: This figure plots the estimated β_2^h in regression (A3). The variables we consider include employment, capital, sales and real equity value.

A.4. Importance of financial frictions for the reallocation effects of turbulence.

To examine the robustness of the impact of financial friction on reallocation effects of turbulence, we also consider an alternative specification in which we sort industry-level external finance dependence into four quartiles and we replace the dummy $High_FF_{it}$ in the baseline specification by the dummy indicators of the top three quartiles of the financial friction distribution (denoted by $FF_{2,it}$, $FF_{3,it}$, and $FF_{4,it}$) (Table A.5). Implicitly, we treat the industry in the first quartile ($FF_{1,it}$) of the external finance dependence distribution (i.e., industries with the lowest external finance dependence) as the reference group. An increase in turbulence is associated with larger declines in interquartile range of employment and capital for industries with higher external finance dependence. These effects are both statistically significant and economically important.

TABLE A.5. Reallocation effects of turbulence: sensitivity to financial friction

Dep. Var.	IQR of Employment	IQR of Capital
	(1)	(2)
$FF_{2,it}$	-0.102 (0.118)	-0.198 (0.136)
$FF_{3,it}$	-0.033 (0.119)	-0.002 (0.138)
$FF_{4,it}$	0.307** (0.120)	0.372*** (0.140)
$Turb_t * FF_{2,it}$	-2.371 (2.184)	-0.809 (2.531)
$Turb_t * FF_{3,it}$	-1.729 (2.198)	-2.557 (2.547)
$Turb_t * FF_{4,it}$	-9.926*** (2.184)	-11.584*** (2.530)
<i>Constant</i>	1.998*** (0.037)	2.231*** (0.043)
Industry Fixed Effect	Yes	Yes
Year Fixed Effect	Yes	Yes
Observations	3,647	3,647

Note: This table shows the regression of inter-quartile range of employment (or capital) on the measured turbulence ($Turb$) for industries with different levels of external finance dependence. We sort the NAICS four-digit industries into four quartiles based on their external finance dependence. The dummy variables $FF_{2,it}$, $FF_{3,it}$, and $FF_{4,it}$ indicate whether a industry i 's KZ index is in the second, third, or fourth quartile in year t . All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

A.5. Macroeconomic effects of uncertainty. We now examine the macroeconomic effects of uncertainty shock. We follow Bloom et al. (2018) to construct uncertainty as IQR of micro-TFP shocks, which is consistent with our model specification³¹. For this purpose, we estimate the impulse responses of several key macroeconomic variables to an uncertainty shock using the local projection regression parallel to Eq. (9):

$$x_{t+h} - x_{t-1} = \beta_0^h + \beta_1^h \text{sigma}_t + \beta_2^h \text{sigma}_{t-1} + \beta_3^h (x_{t-1} - x_{t-2}) + \epsilon_{t+h} \quad h = 0, 1, 2, 3, 4. \quad (\text{A4})$$

The dependent variable $x_{t+h} - x_{t-1}$ denotes the cumulative changes in the log-level of the variable of interest from year $t - 1$ to year $t + h$, where h denotes the projection horizons (number of years). The list of dependent variables includes the macroeconomic times series of per capita real consumption, investment, private output (i.e., the sum of consumption and investment), and hours worked, manufacture TFP and interquartile range of sales growth constructed from the firm-level and industry-level data. The independent variable sigma_t denotes the log-level of uncertainty in year t . In estimating the local projections, we control for lagged uncertainty (sigma_{t-1}) and the lagged growth rate of the dependent variable ($x_{t-1} - x_{t-2}$). The term ϵ_{t+h} is the regression residual. The parameter β_1^h measures the impulse responses of the macroeconomic variables to an uncertainty shock at horizon h .

Figure A3 plots the estimated impulse responses of the macroeconomic variables to a one-standard-deviation uncertainty shock for horizons up to five years³². The shock leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, and hours worked. It also leads to a decline in manufacturing TFP and a rise in dispersion of sales growth. These macroeconomic effects of uncertainty are consistent with predictions from our calibrated model.

A.6. Additional results from the calibrated model. We now report some additional results from the calibrated theoretical model.

A.6.1. Reallocation effects of a turbulence shock. Figure A4 shows that turbulence has heterogeneous effects on firms at different productivity levels. As discussed in the text, turbulence reduces aggregate TFP and leads to a recession. Thus, aggregate factor demand declines, reducing wages and capital rents. At any given productivity level, the decline in the factor prices lowers the threshold level of subsidy for active production. Thus, the mass of

³¹To measure uncertainty, we first estimate TFP shocks (e_{jt}) as the residual from the first-order autoregression after controlling for firm- and year- fixed effects: $\log(TFP_{jt}) = \rho \log(TFP_{jt-1}) + \mu_j + \lambda_t + e_{jt}$. We define microeconomic uncertainty in period t as the cross-sectional dispersion (IQR) of TFP shocks in period $t+1$.

³²Our measured annual series of (logged) uncertainty has a first-order autocorrelation of 0.631 and a standard deviation of 0.1530, implying a standard deviation of the innovation of 0.1187.

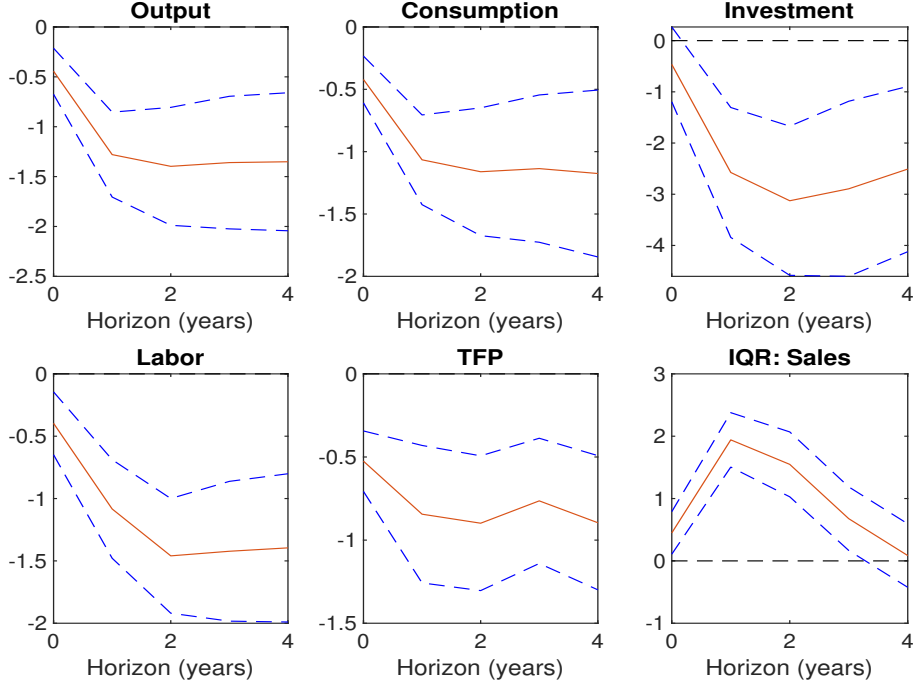


FIGURE A3. Estimated impulse response of macroeconomic variables to an uncertainty shock

Note: This figure shows the impulse responses of macroeconomic variables to a one-standard-deviation (11.87%) increase in the log-level of uncertainty from the local projections model (A4). The solid lines show the point estimates of the impulse responses. The blue dashed lines show the 68% confidence intervals.

Source: BEA, Compustat, NBER-CES, and authors' calculations.

producers at all levels of productivity increases, although the mass of high productivity firms increases less because the shock that reshuffles productivity implies that higher-productivity firms are less likely to remain productive. Since all firms face the same factor prices, firms with lower productivity require higher subsidies to stay active. Therefore, average sales of active low-productivity firms increase relative to those of high-productivity firms, exacerbating misallocation.

A.6.2. Reallocation effects of a micro-level uncertainty shock. An uncertainty shock measured by a mean-preserving spread of the production subsidies (i.e., an increase in $\sigma_{\tau,t}$) expands the tails of the subsidy distribution. At each level of productivity z_{jt} , a firm would be active in production if and only if its actual subsidy exceeds the threshold level τ_{jt}^* . Since firms with subsidies below τ_{jt}^* stay inactive, uncertainty affects production decisions only through the expansion of the right tail of the subsidy distribution. At any given factor prices, the threshold τ_{jt}^* decreases with z_{jt} (see Eq. (21) in the text). Thus, high-productivity

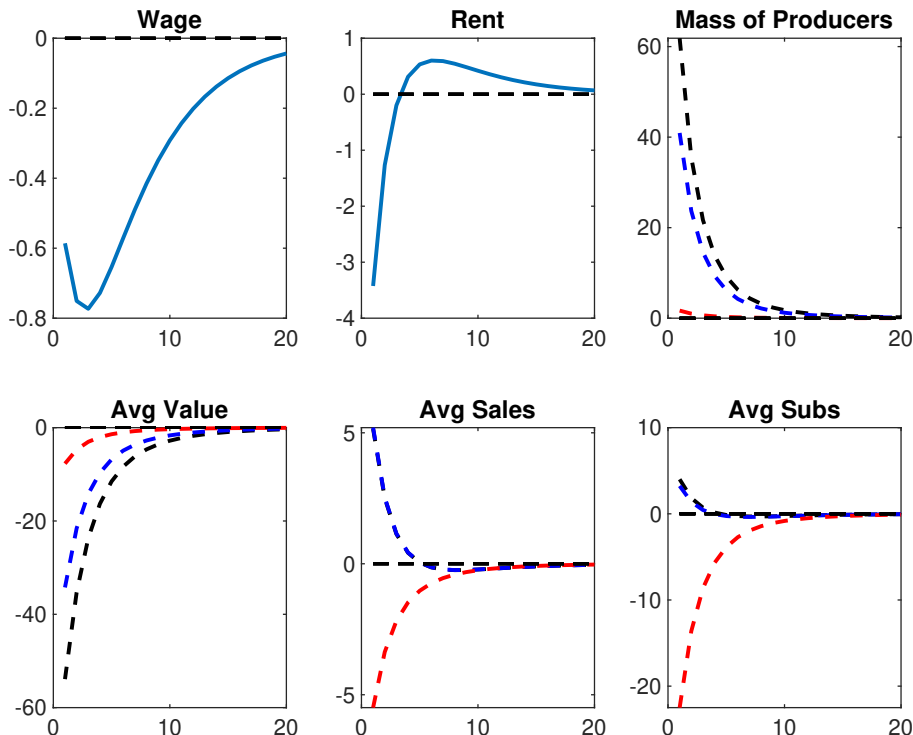


FIGURE A4. Reallocation effects of a turbulence shock

This figure shows the impulse responses to a turbulence shock in the benchmark model. The black, blue and red dashed lines represent, respectively, the responses of firms at the 25, 50 and 100 percentiles of the productivity distribution. The horizontal axis shows the periods (years) after the impact of the shock. The vertical axis measures percent deviations from the stochastic steady-state in response to a one-standard-deviation shock to turbulence.

firms face a lower threshold than low-productivity firms. A right-tail expansion of the subsidy distribution will therefore increase the relative mass of low-productivity firms (relative to high-productivity firms).

This is illustrated in Figure A5. To highlight our point, the figure shows a symmetric distribution, with the threshold for high-productivity firms (τ_h^*) normalized to be the mean of the distribution. The mass of high-productivity firms in this example is 1/2, both before and after the uncertainty shock (because the shock is a mean-preserving spread). Since the threshold for low-productivity firms (τ_l^*) lies to the right of τ_h^* , the increase in uncertainty raises the relative mass of low-productivity firms (upper right panel in Figure A6). This reallocation effect reduces aggregate TFP, leading to a decline in wages and rents, as shown in the impulse responses in Figure A6. Since all firms face the same wages and rents, which

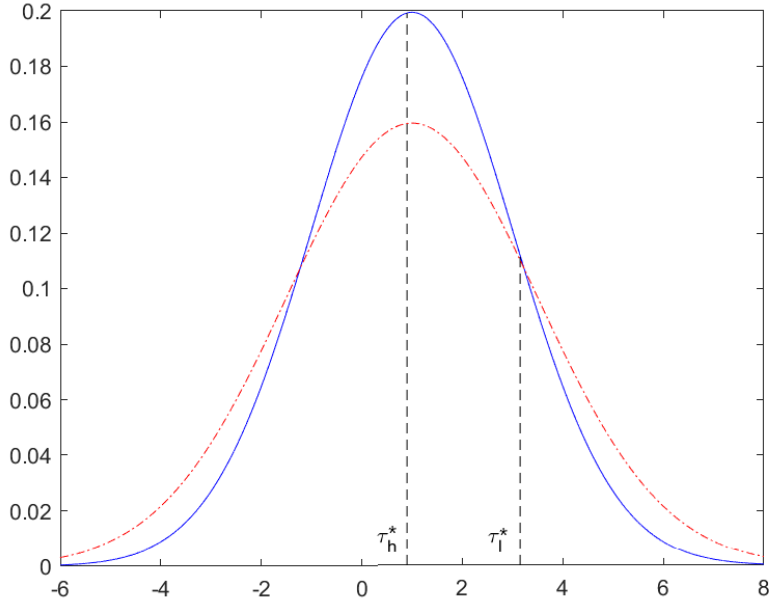


FIGURE A5. Uncertainty and the distribution of active firms

This figure illustrates how an increase in micro-level uncertainty affects the mass of active firms in the benchmark model. The blue solid and red dashed curves represent the distribution of i.i.d. shock τ in the steady state and after a mean-preserving spread shock. τ_h^* and τ_l^* denote the thresholds of active production for high-productivity and low-productivity firms, respectively.

are now lower than in the steady-state, the declines in wages and rents reduce the relative mass of high-productivity firm even further.

Given the size of the right-tail expansion of subsidies, the average subsidy received by each active, low-productivity firm declines relative to that by high-productivity firms. Thus, average sales of active high-productivity firms (inclusive of subsidies) increase relative to those of low-productivity firms, raising the cross-sectional dispersion of average sales, as shown in Figure A6. Through changes in the distribution of average subsidies to active firms, the increase in uncertainty also leads to larger reductions in the average market value of low-productivity firms than that of high-productivity firms.

A.6.3. Reallocation effects of a TFP shock. Figure A7 shows that a negative TFP shock has heterogeneous impacts on firms at different levels of productivity. The shock reduces aggregate labor and capital demand, lowering wages and capital rents. A decline in aggregate TFP raises the threshold of active production for firms at each level of productivity, shrinking the set of active firms, although this effect is partly mitigated by the decline in factor prices.

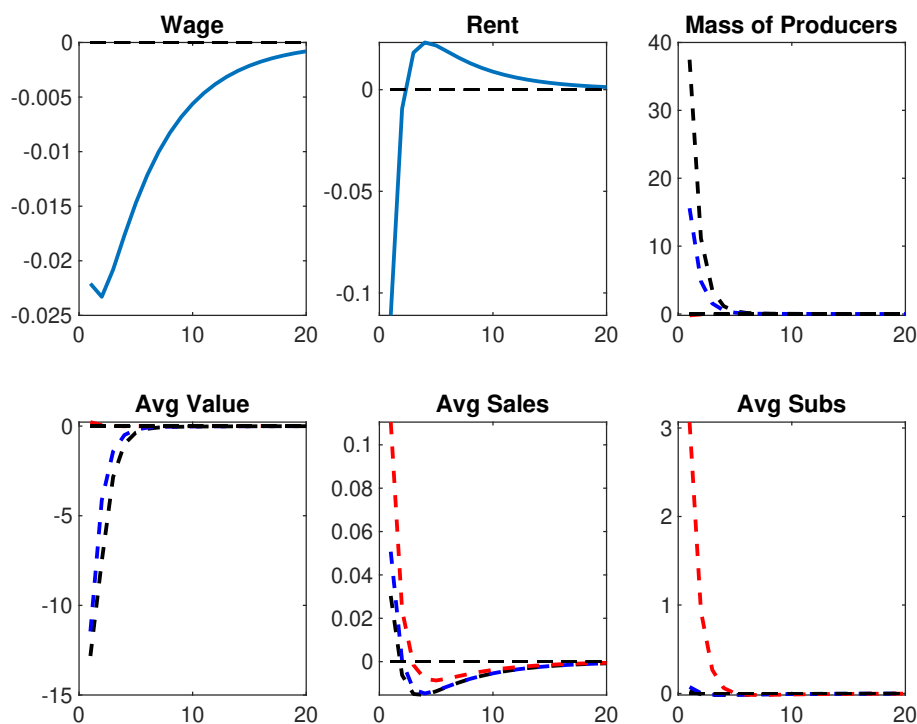


FIGURE A6. Reallocation effects of an uncertainty shock

This figure shows the impulse responses to an uncertainty shock in the benchmark model. The black, blue and red dashed lines represent the responses of firms at the 25, 50, and 100 percentile of the productivity distribution. The horizontal axis shows the periods (years) after the impact of the shock. The vertical axis measures percent deviations from the stochastic steady-state in response to a one-standard-deviation shock to uncertainty.

Since labor and capital are perfectly mobile across firms, all firms face the same wages and capital rents. A decline in aggregate TFP would therefore force more low-productivity firms into the inactive regions than high-productivity firms. The shock reduces the average sales of all active firms, and average sales decline more for firms with higher productivity because, among active firms, lower-productivity firms need to be compensated by higher average subsidies for them to remain active.

A.6.4. Macroeconomic effects of turbulence: Benchmark model vs. data. In the text, we show that an annual version of the benchmark calibrated model generates empirically plausible impulse responses of aggregate output to a turbulence shock. Here, we compare the impulse responses of other macro variables to a turbulence shock from the model against those estimated from the data.

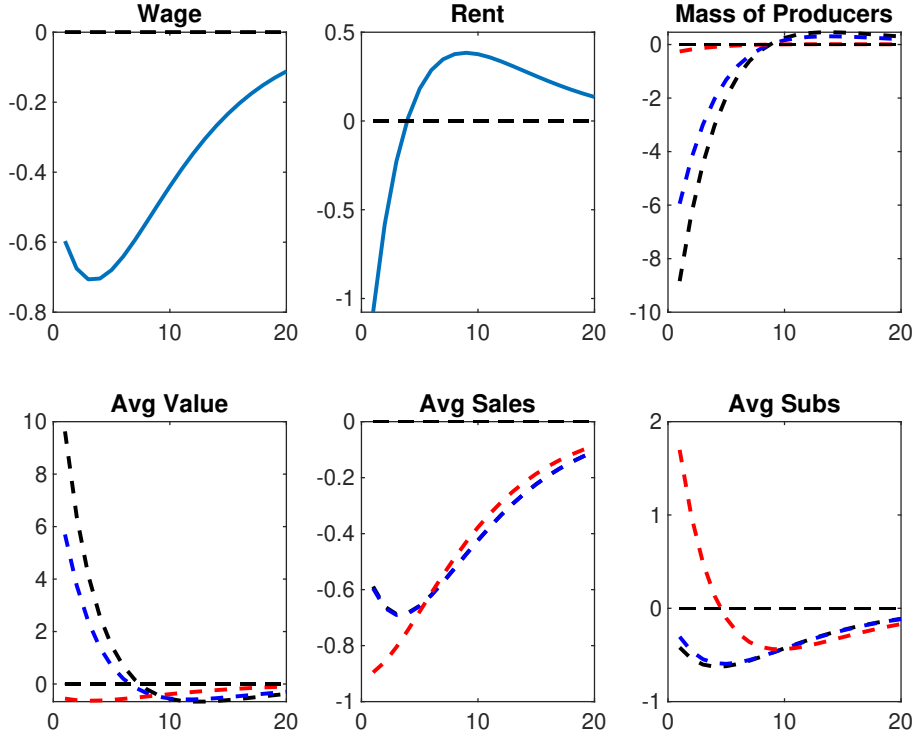


FIGURE A7. Reallocation effects of a TFP shock

This figure shows the impulse responses to a TFP shock in the benchmark model. The black, blue and red dashed lines represent the responses of firms at the 25, 50, and 100 percentile of the productivity distribution. The horizontal axis shows the periods (years) after the impact of the shock. The vertical axis measures percent deviations from the stochastic steady-state in response to a one-standard-deviation shock to TFP.

Figure A8 compares the impulse response of output, consumption, investment and hours worked to a one-standard-deviation turbulence shock in the model (blue lines) vs. those in the data (red lines). The responses of output, investment, and labor hours are close to those in the data. However, under the calibrated intertemporal elasticity of substitution (log utility in consumption) and absence of real friction such as habit persistence and investment adjustment costs, the model does not reproduce the observed persistent, hump-shaped responses of the macroeconomic variables following a turbulence shock.

APPENDIX B. DECREASING RETURNS TO SCALE

Now we consider an alternative version of RBC model with decreasing returns to scale (DRS) and show that a turbulence shock has similar reallocation and macroeconomic effects as in the benchmark model with a constant returns technology.

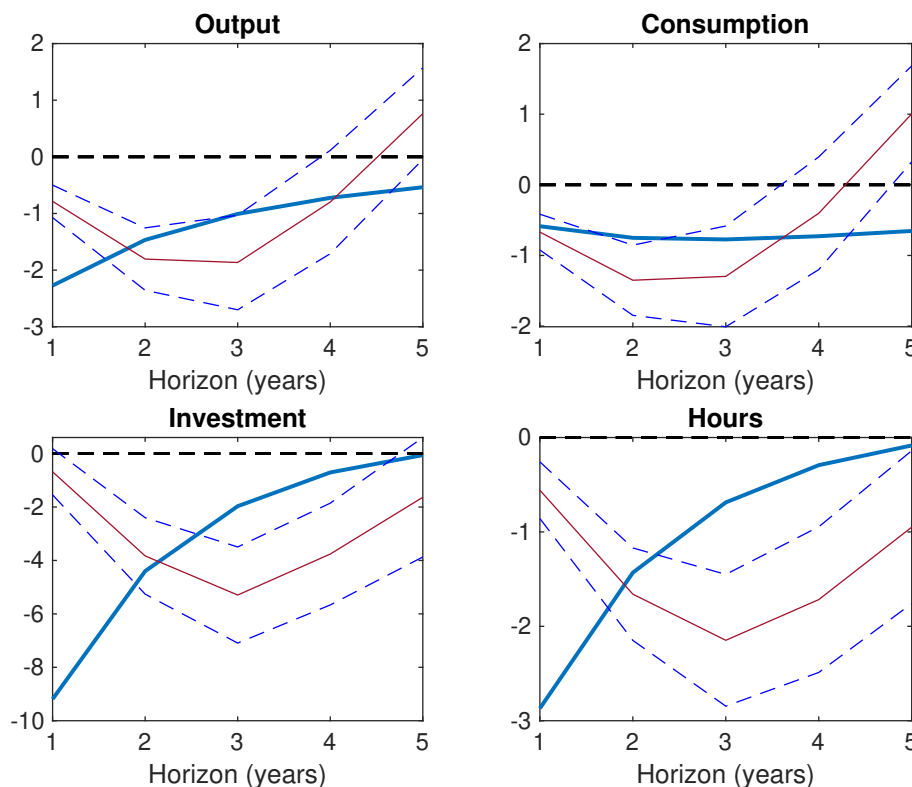


FIGURE A8. Impulse responses of macroeconomic variables to a turbulence shock: Benchmark model vs. data

Note: This figure shows the impulse responses of aggregate output, consumption, investment, and labor hours to a one-standard-deviation shock to turbulence in the data (the red solid line) and in the calibrated annual version of the model (the blue solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percent deviations of output in the model from its steady-state level and the percentage changes in output in the data relative to its pre-shock level.

B.1. The model. The model economy is populated by a continuum of infinitely lived households with measure one, whose preference is characterized by the same utility function as in the benchmark model:

$$\mathbf{E} \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right\}, \quad (\text{A5})$$

All markets are perfectly competitive. The household takes all prices as given and maximizes the utility in Eq. (A5) subject to a budget constraint

$$C_t + I_t = R_t K_t + W_t N_t + D_t - T_t, \quad (\text{A6})$$

where K_{t+1} denotes the end-of-period capital stock, R_t denotes the capital rental rate, W_t denotes the real wage rate, D_t denotes the dividend income from firms, and T_t denotes a lump-sum tax paid to the government.

Household's decision rules are characterized by the following equations:

$$\psi N_t^\gamma = \frac{1}{C_t} W_t \quad (\text{A7})$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (R_{t+1} + (1 - \delta)) \quad (\text{A8})$$

There is a continuum of intermediate good firms, each endowed with a constant-returns technology that produces an intermediate good using capital and labor as inputs. Firms face idiosyncratic productivity shocks drawn at the beginning of each period, before hiring inputs. The production function for an individual firm is given by

$$y_{jt} = A_t z_{jt} k_{jt}^\alpha n_{jt}^{1-\alpha}, \quad (\text{A9})$$

where y_{jt} denotes the output produced by firm j in period t , and k_{jt} and n_{jt} denote the capital and labor inputs, respectively. The term A_t and z_{jt} denote aggregate and idiosyncratic productivity shocks respectively, which are assumed to follow the same stochastic processes as in the benchmark model.

Intermediate goods producers sell competitive monopolistic goods to a final good producer with production function as

$$Y_t = \left(\int_j [y_{jt}]^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}, \quad (\text{A10})$$

from which we can derive demand function for each intermediate good as

$$p_{jt} = Y_t^{\frac{1}{\nu}} y_{jt}^{-\frac{1}{\nu}} \quad (\text{A11})$$

Firms rely on external financing of their working capital, and face idiosyncratic production distortions (denoted by τ_{jt}). The firms' optimizing problem is characterized by the Bellman equation

$$V_t(z_{jt}, \tau_{jt}) = \tau_{jt} p_{jt} y_{jt} - W_t n_{jt} - R_t k_{jt} + E_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \quad (\text{A12})$$

subject to the working capital constraint

$$W_t n_{jt} + R_t k_{jt} \leq \theta E_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta B_{jt} \quad (\text{A13})$$

Profit maximizing implies

$$k_{jt} = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t} n_{jt}. \quad (\text{A14})$$

It follows that there exists a threshold level of production subsidy τ_{jt}^* such that, if $\tau_{jt} \geq \tau_{jt}^*$, then a firm would be facing binding credit constraints in production. Otherwise, the firm would choose an interior optimal size. At the threshold level of subsidy, the optimal size of

production coincides with level implied from binding constraint. The indifference condition determines the threshold level of subsidy, τ_{jt}^* :

$$\tau_{jt}^* = \left[\frac{\theta B_{jt}}{\left(\frac{\nu-1}{\nu}\right)^\nu \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^{\nu-1} (A_t z_{jt})^{\nu-1} Y_t} \right]^{\frac{1}{\nu}}, \quad (\text{A15})$$

The threshold τ_{jt}^* increases with the factor prices R_t and W_t and decreases with the productivity level z_{jt} . Thus, given the factor prices, the fraction of active firms is larger for firms with higher productivity. Idiosyncratic labor demand functions are:

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{1-\alpha}{W_t} \left(\frac{\nu-1}{\nu}\right)^\nu Y_t \tau_{jt}^\nu (A_t z_{jt})^{\nu-1} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^{\nu-1}, & \text{if } \tau_{jt} \leq \tau_{jt}^* \\ \frac{(1-\alpha)\theta B_{jt}}{W_t}, & \text{otherwise} \end{cases} \quad (\text{A16})$$

Idiosyncratic capital demand functions are

$$k_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\alpha}{R_t} \left(\frac{\nu-1}{\nu}\right)^\nu Y_t \tau_{jt}^\nu (A_t z_{jt})^{\nu-1} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^{\nu-1}, & \text{if } \tau_{jt} \leq \tau_{jt}^* \\ \frac{\alpha\theta B_{jt}}{R_t}, & \text{otherwise} \end{cases} \quad (\text{A17})$$

Idiosyncratic output functions are

$$y_t(z_{jt}, \tau_{jt}) = \begin{cases} Y_t \left(\frac{\nu-1}{\nu}\right)^\nu \tau_{jt}^\nu (A_t z_{jt})^\nu \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^\nu, & \text{if } \tau_{jt} \leq \tau_{jt}^* \\ A_t z_{jt} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right] \theta B_{jt}, & \text{otherwise} \end{cases} \quad (\text{A18})$$

Since production subsidies are i.i.d. across time, the average value of a firm with productivity z_{jt} is given by

$$\begin{aligned} \bar{V}_{jt} &= \int V_t(z_{jt}, \tau) dF(\tau) = \frac{1}{\nu-1} \left(\frac{\nu-1}{\nu}\right)^\nu Y_t (A_t z_{jt})^{\nu-1} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^{\nu-1} \int_0^{\tau_{jt}^*} \tau^\nu dF(\tau) \\ &+ Y_t^{\frac{1}{\nu}} (A_t z_{jt})^{\frac{\nu-1}{\nu}} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha z_{jt}\right]^{\frac{\nu-1}{\nu}} (\theta B_{jt})^{\frac{\nu-1}{\nu}} \int_{\tau_{jt}^*}^{\infty} \tau dF(\tau) - \theta B_{jt} [1 - F(\tau_{jt}^*)] + B_{jt} \end{aligned} \quad (\text{A19})$$

Given the stochastic process of $z_{j,t+1}$ and the definition of B_{jt} in Eq. (17), we have

$$B_{jt} \equiv \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left[\rho_t \bar{V}_{jt+1} + (1 - \rho_t) \sum_{i=1}^J \pi_i \bar{V}_{it+1} \right] \quad (\text{A20})$$

In a competitive equilibrium, markets for labor, capital, and final consumption goods all clear. Labor market clearing implies that

$$N_t = \sum_j \pi_j \left\{ \frac{1-\alpha}{W_t} \left(\frac{\nu-1}{\nu}\right)^\nu Y_t (A_t z_{jt})^{\nu-1} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^{\nu-1} \int_0^{\tau_{jt}^*} \tau^\nu dF(\tau) + \frac{(1-\alpha)\theta B_{jt}}{W_t} [1 - F(\tau_{jt}^*)] \right\} \quad (\text{A21})$$

Capital market clearing implies that

$$K_t = \sum_j \pi_j \left\{ \frac{\alpha}{R_t} \left(\frac{\nu-1}{\nu}\right)^\nu Y_t (A_t z_{jt})^{\nu-1} \left[\left(\frac{1-\alpha}{W_t}\right)^{1-\alpha} \left(\frac{\alpha}{R_t}\right)^\alpha\right]^{\nu-1} \int_0^{\tau_{jt}^*} \tau^\nu dF(\tau) + \frac{\alpha\theta B_{jt}}{R_t} [1 - F(\tau_{jt}^*)] \right\} \quad (\text{A22})$$

Final good market clearing condition implies

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t, \quad (\text{A23})$$

where

$$y_{jt} = Y_t \left(\frac{\nu - 1}{\nu} A_t z_{jt} \right)^\nu \left[\left(\frac{1 - \alpha}{W_t} \right)^{(1-\alpha)} \left(\frac{\alpha}{R_t} \right)^\alpha \right]^\nu \int_0^{\tau_{jt}^*} \tau^\nu dF(\tau) + A_t z_{jt} \left[\left(\frac{1 - \alpha}{W_t} \right)^{1-\alpha} \left(\frac{\alpha}{R_t} \right)^\alpha \right] \theta B_{jt} [1 - F(\tau_{jt}^*)] \quad (\text{A24})$$

Definition. A competitive equilibrium consists of the sequence of allocations $\{C_t, Y_t, N_t, K_t\}$ and the sequence of prices $\{W_t, R_t\}$ such that (i) taking all prices as given, the allocations solve the household's utility maximizing problem and the firms' profit maximizing problem; and (ii) markets for labor, capital, and goods all clear.

B.2. Measured turbulence shock. The methodology in estimation of turbulence can be easily extended to the decreasing return to scale (DRS) model. In specific, firm-level TFP is estimated from the following equation:

$$tfp_{ijt} = y_{ijt} - \frac{\nu - 1}{\nu} \alpha_{it} k_{ijt} - \frac{\nu - 1}{\nu} (1 - \alpha_{it}) n_{ijt} \quad (\text{A25})$$

where ν measures elasticity of substitution. We follow standard New Keynesian literature to set $\nu = 10$, implying steady-state mark-up at around 1.1. The firm-level TFP are estimated after controlling for year fixed effect, which captures effect of aggregator Y_t on revenue of each firm. The rest of procedures strictly follow those adopted in constant return to scale model.

Time-varying turbulence in the model is measured by Spearman correlation of firm-level TFP ranking in decile within industry between two consecutive years, similar to measure constructed in figure A1 of Bloom et al. (2018). Figure A9 plots the time series of firm-level turbulence from 1965 to 2015.³³ The mean, standard deviation, and autocorrelation of the estimated turbulence $(1 - \rho_t)$ are 0.041, 0.014, and 0.51, respectively.

B.3. Reallocation and macroeconomic effects of turbulence. The measure of turbulence constructed under the DRS production function here has similar cyclical and reallocation effects as that under the CRS production function in the benchmark model.

To examine the reallocation effects of turbulence, we re-estimate a regression (7) using the turbulence measure constructed from the DRS production function. Table A.6 shows the estimation results. The results show that an increase in turbulence is associated with declines in the firm-level growth rates for high-productivity firms relative to those for low-productivity firms. The reallocation effects of turbulence are robust to alternative indicators of high firm-level TFP (lagged high-TFP indicators), alternative samples (with large industries or

³³Our measure of turbulence is robust to alternative measures of value added, capital input, and labor input, as well as to the data samples used (not reported).

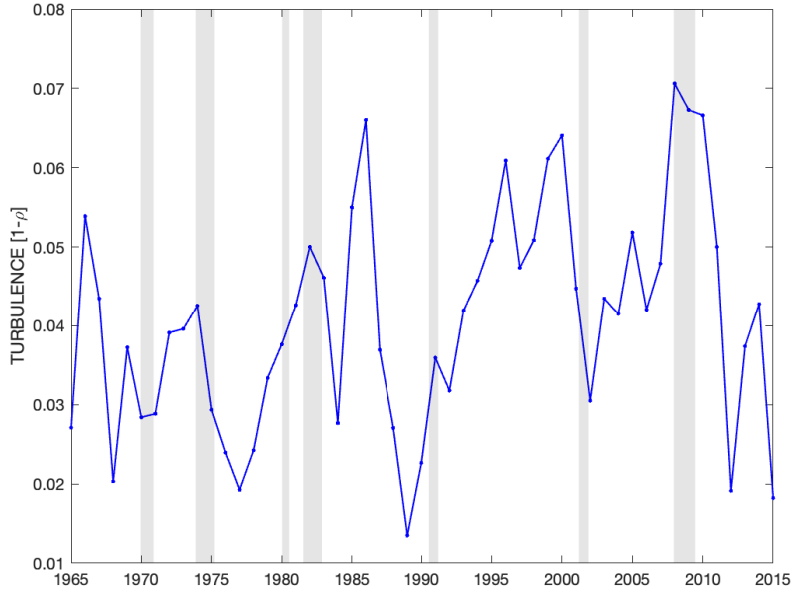


FIGURE A9. Measured micro-level turbulence (decreasing return to scale)

Note: Turbulence is measured by $1 - \rho_t$, where ρ_t is the Spearman correlation of firm TFP rankings between year t and year $t + 1$. The gray shaded bars indicate NBER recession dates.

Source: Compustat, NBER-CES, BLS, and authors' calculations.

excluding top firms), and additional controls for the potential reallocation effects from the business cycle recessions (see Sec. B.6).

The reallocation effects of turbulence also depend on financial frictions, as in our benchmark estimation. Specifically, Table A.7 shows the estimation results from the empirical specification (8) that is identical to that in the benchmark case, except that we use the measure of turbulence under the DRS technology. Again, we find that an increase in turbulence is associated with significant declines in the cross-sectional dispersions of employment and capital in industries with high levels of external financing dependence.

B.4. Macroeconomic effects of turbulence. To examine the macroeconomic effects of a turbulence shock, we re-estimate the local projections specification in Eq. (9) with the turbulence measure constructed under the DRS technology.

Figure A10 plots the estimated impulse responses of the macroeconomic variables to a one-standard-deviation turbulence shock (i.e., an increase in turbulence of 0.3412) for horizons

TABLE A.6. Impact of turbulence on firms with different levels of productivity

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
<i>High_TFP_{jt}</i>	-0.013 (0.011)	-0.019 (0.012)	0.055*** (0.012)	0.010 (0.014)
<i>Turb_t * High_TFP_{jt}</i>	-0.810*** (0.151)	-0.447*** (0.145)	-0.805*** (0.138)	-0.086 (0.250)
<i>Constant</i>	0.055*** (0.003)	0.065*** (0.004)	0.040*** (0.004)	0.033*** (0.002)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	24,501	24,501	24,501	21,687

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence (*Turb*) for firms with different levels of TFP under the DRS production function. The dummy *High_TFP_{jt}* equals one if firm *j*'s TFP is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compu-stat firms that appear for at least 25 years from 1959 to 2015. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

up to five years.³⁴ As in the benchmark case, a turbulence shock leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, hours worked, and firm value. It also leads to a decline in manufacturing TFP. These macroeconomic effects of turbulence are quantitatively important. For example, a one-standard-deviation increase in turbulence reduces per capita output by about 0.5 percent on impact, and by more than one percent within three years after the shock.

B.5. Quantitative impact of a turbulence shock in the RBC model. We calibrate the parameters in the model with DRS using a similar approach as that in our benchmark model with CRS.

³⁴Our measured annual series of (logged) turbulence has a first-order autocorrelation of 0.4514 and a standard deviation of 0.3824, implying a standard deviation of the innovation of 0.3412.

TABLE A.7. Reallocation effect of turbulence: sensitivity to financial frictions

Dep. Var.	IQR of Employment		IQR of Capital	
	(1)	(2)	(3)	(4)
<i>High_FF_{it}</i>	0.159*	0.189*	0.164	0.181
	(0.096)	(0.098)	(0.111)	(0.113)
<i>Turb_t * High_FF_{it}</i>	-3.954*	-4.497**	-4.031	-4.352*
	(2.196)	(2.224)	(2.541)	(2.570)
<i>Constant</i>	1.868***	1.895***	2.088***	2.122***
	(0.024)	(0.024)	(0.028)	(0.028)
Industry Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	3,647	3,522	3,647	3,522

Note: This table shows the regression of interquartile range of employment (or capital) on the measured turbulence (*Turb*) for industries with different levels of external financing dependence. In the baseline specification (Columns (1) and (3)), the dummy *High_FF_{it}* equals one if industry *i*'s external financing dependence is above the median. In the alternative specification (Columns (2) and (4)), we use lagged indicator of external financing dependence (i.e., *High_FF_{i,t-1}* instead of *High_FF_{it}*). All regressions use the pseudo panel of Compustat firms that appear for at least 25 years in the sample from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

We set the elasticity of substitution between differentiated goods to $\nu = 10$, implying a steady-state mark-up of about 11 percent, in line with the empirical estimates of Basu and Fernald (1997). We calibrate three parameters $\bar{\rho}$, σ_z , and σ_τ in the stochastic processes of firm-level TFP and production distortions to target the three empirical moments, including (1) the average value of the Spearman rank correlations of establishment-level TFP (0.72, estimated by Bloom et al. (2018)), (2) the standard deviation of firm-level TFP shock (0.21, based on firm-level TFP constructed using the Compustat/NBER-CES data), and (3) the average IQR of equity values across firms (1.53, also from the Compustat data). The resulting calibrated values are $\bar{\rho} = 0.93$, $\sigma_z = 0.10$ and $\sigma_\tau = 0.14$. Given these parameter values, we calibrate the persistence and volatility of the turbulence shock such that the model-implied

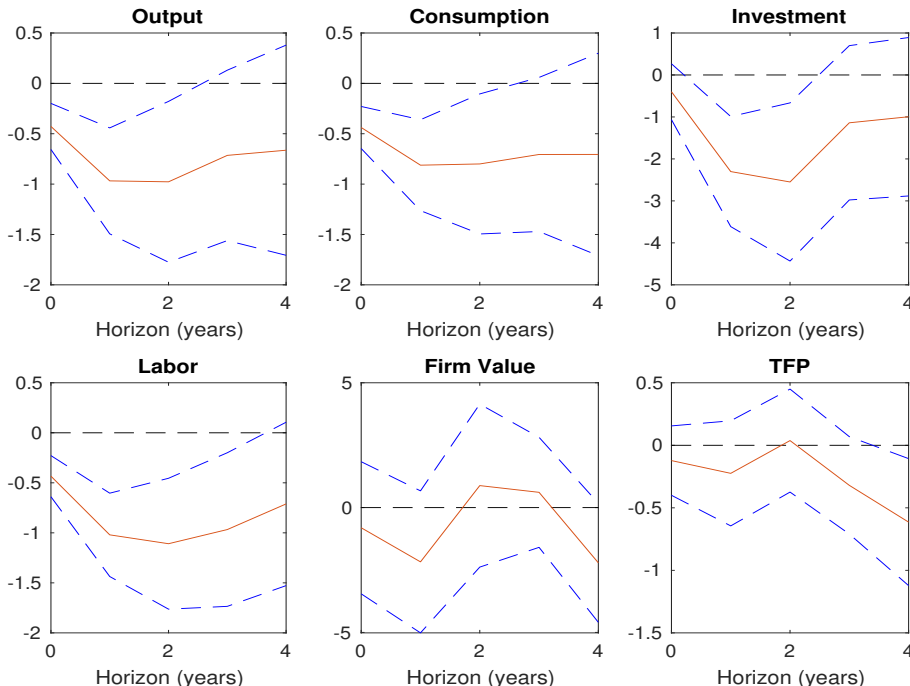


FIGURE A10. Estimated impulse response of macroeconomic variables to a turbulence shock

Note: This figure shows the impulse responses of macroeconomic variables to a one-standard-deviation (38.24%) increase in the log-level of turbulence estimated from the local projections model (9). The solid lines show the point estimates of the impulse responses. The blue dashed lines show the 68% confidence intervals.

Source: BEA, Compustat, NBER-CES, and authors' calculations.

persistence and standard deviation of the Spearman rank correlation of measured firm-level TFP (which include both true TFP z_{jt} and production distortions τ_{jt}) match the same moments in the Spearman correlation estimated by Bloom et al. (2018). This leads to the calibrated values of $\rho_\rho = 0.605$ and $\sigma_\rho = 0.35$. We calibrated the remaining parameters to the same values as those in the benchmark model.

The qualitative impact of firm-level turbulence shock with decreasing return to scale is consistent with that in our baseline model. Figure A11 displays the impulse responses to a one-standard-deviation shock to turbulence. An increase in turbulence reduces the chance for a current high-productivity firm to remain as productive in the future, and it also increases the chance for a current low-productivity firm to get a higher productivity draw in the future. Thus, the equity values of high-productivity firms declines relative to those of low-productivity firms, leading to a decline in the dispersion of firm value and disproportionately tightening the borrowing capacity of high-productivity firms in the current period. Facing

tightened credit constraints, high-productivity firms pull back hiring, reallocating labor and capital to low-productivity firms. Since high-productivity firms use more capital and labor in the steady-state than low-productivity firms, the increase in turbulence reduces the share of labor and capital in high-productivity firms, and it also reduces the dispersion of sales, in line with the empirical evidence presented in Section III.3.

Through reallocation, a turbulence shock reduces aggregate TFP and leads to a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. The recessionary effects of turbulence are sizable and persistent. For example, a one-standard-deviation turbulence shock leads to a drop in aggregate output of more than 1 percent on impact, and output stays below its steady-state level for more than ten years after the shock.

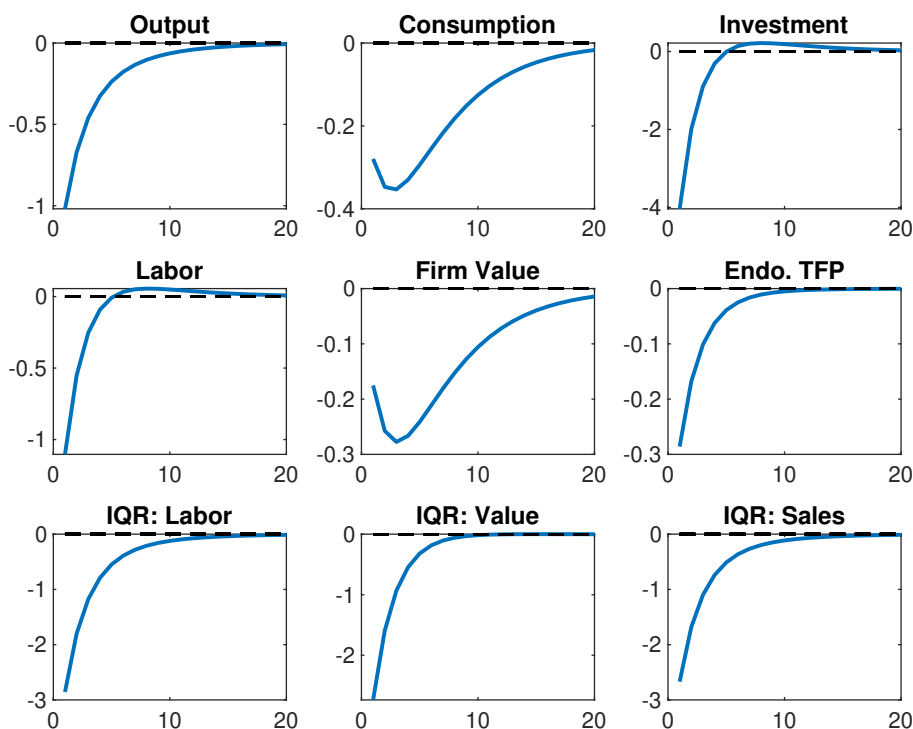


FIGURE A11. Impulse response to a turbulence shock in decreasing-return-to-scale model

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the calibrated decreasing-return-to-scale model. The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

The role of financial frictions. As in the benchmark model, Figure A12 shows that the recession effects and the reallocation effects of a turbulence shock would be substantially

dampened if firms' borrowing capacity could not vary with the expected equity value. This counterfactual illustrates the importance of financial frictions—and in particular, the endogenous variations of the borrowing capacity with expected firm values—for propagating turbulence shocks.

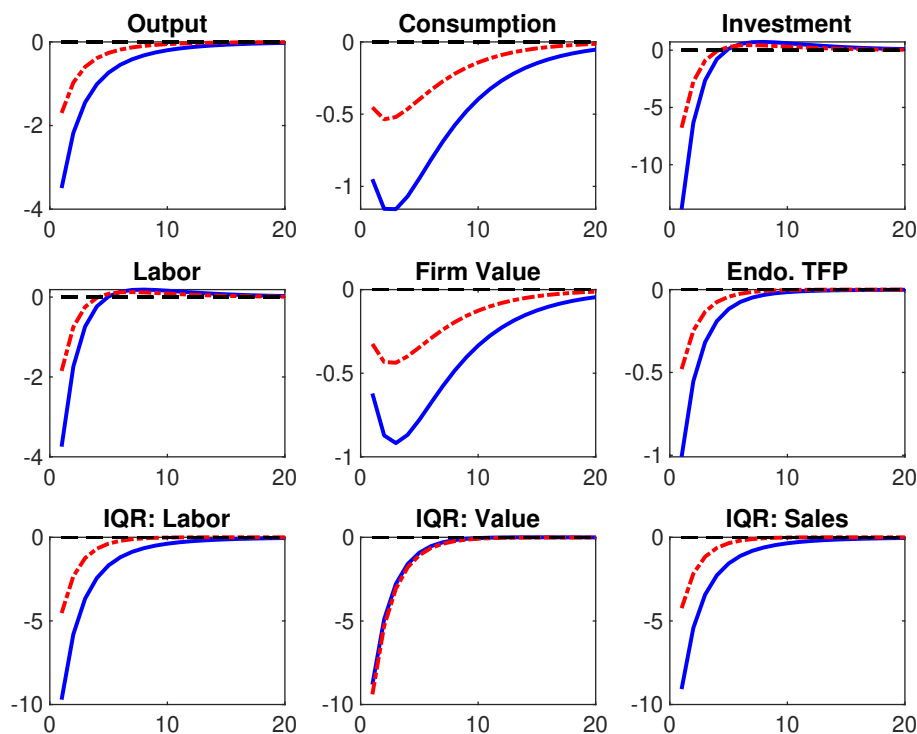


FIGURE A12. Impulse response to a turbulence shock: decreasing-return-to-scale model vs. counterfactual with quasi-fixed borrowing capacity

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the decreasing-return-to-scale model (blue lines) and in the counterfactual with quasi-fixed borrowing capacity (red dash-dotted lines). The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

Turbulence has quantitatively important recessionary effects, both in the model and in the data. Figure A13 compares the model-implied impulse responses of aggregate variables (blue solid lines) with the empirical estimates of the impulse response (red solid lines). A one-standard-deviation turbulence shock reduces aggregate output by around 1 percent on impact, both in the data and in the model. The shock has persistent recessionary effects on aggregate output both in the model and in the data, although the theoretical impulse responses miss the hump estimated from the data. These findings suggest that turbulence

plays an important role in driving business cycles. Similar to the case of output, the model-implied responses of investment and labor are both in line with those estimated from the data, although the calibrated model fails to generate the turbulence-driven large declines in consumption observed in the data.

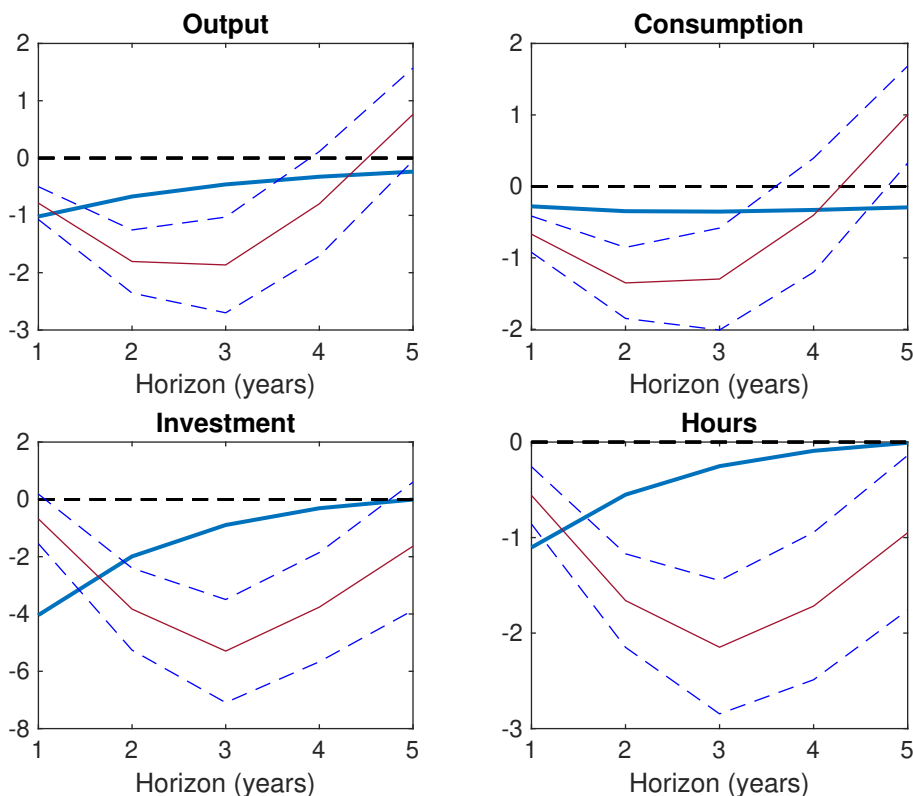


FIGURE A13. Impulse response to turbulence shock: Model vs. data (decreasing-return-to-scale)

Note: This figure shows the impulse responses of private aggregate output to a one-standard-deviation shock to turbulence in the data (red solid line) and in the calibrated annual version of the decreasing-return-to-scale model (blue solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percent deviations of output in the model from its steady-state level and the percentage changes in output in the data relative to its pre-shock level.

B.6. Robustness. This section presents robustness of regressions under DRS specification.

Our baseline regression estimates the effects of turbulence on firms with different levels of TFP, based on current-year firm-level TFP ranking. It is possible that a firm's employment,

TABLE A.8. Impact of turbulence on firms with different productivity:
Lagged high-TFP indicators

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
$High_TFP_{jt-1}$	0.034*** (0.011)	0.017 (0.015)	-0.042** (0.016)	-0.014 (0.020)
$Turb_t * High_TFP_{jt-1}$	-0.784*** (0.143)	-0.312* (0.159)	-0.903*** (0.143)	-0.202 (0.197)
<i>Constant</i>	0.029*** (0.004)	0.042*** (0.005)	0.087*** (0.006)	0.049*** (0.008)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	22,861	22,861	22,861	20,622

Note: This table shows the regression of firm-level employment, capital expenditure, sales or firm value growth on the measured turbulence ($Turb$) for firms with different levels of TFP (lagged). The dummy $High_TFP_{jt-1}$ equals one if firm j 's TFP at year $t - 1$ is above the median and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

capital or sales growth might affect its contemporaneous TFP ranking. To mitigate this concern, we construct an alternative indicator of high-productivity firms based on their TFP rankings with a one-year lag. We re-estimate the responses of firm growth to turbulence shocks for firms with different levels of productivity. Table A.8, with the lagged high-TFP indicator, turbulence reduces growth for high-productivity firms relative to low-productivity firms, similar to the results obtained under the benchmark specification.

Since turbulence is countercyclical, the reallocation effects estimated from our benchmark specification might reflect the general impact of recessions instead of that of turbulence. To address this concern, we consider a regression that controls for the direct reallocation effects of recessions. In particular, we add an interaction term between NBER recession year dummy ($NBER_t$) and our high TFP indicator in the regression. As shown in Table A.9, the

TABLE A.9. Impact of turbulence on firms with different productivity: Control for recession effects

Dep. Var.	Δn_{jt}	Δk_{jt}	Δy_{jt}	Δv_{jt}
	(1)	(2)	(3)	(4)
<i>High_TFP_{jt}</i>	-0.006 (0.011)	-0.018 (0.012)	0.065*** (0.012)	0.023 (0.014)
<i>Turb_t * High_TFP_{jt}</i>	-0.715*** (0.160)	-0.436*** (0.144)	-0.671*** (0.146)	0.059 (0.262)
<i>NBER_t * High_TFP_{jt}</i>	-0.049*** (0.009)	-0.006 (0.006)	-0.068*** (0.008)	-0.082*** (0.010)
<i>Constant</i>	0.055*** (0.003)	0.065*** (0.004)	0.040*** (0.004)	0.034*** (0.003)
Firm Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Observations	25,481	25,481	25,481	25,481

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence (*Turb*) for firms with different levels of TFP. The dummy *High_TFP_{jt}* equals one if firm *j*'s TFP is above the median within its industry and zero otherwise. The dummy *NBER_t* equals one if year *t* contains a recession that lasts for at least 60 days based on the NBER recession dates. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

reallocation effect of turbulence shock remains significant after controlling for the reallocation effect of recessions.

The reallocation effects of turbulence are persistent. To examine heterogeneous effects of turbulence over a longer horizon, we estimate the following equation using local projection method

$$x_{j,t+h} - x_{j,t-1} = \beta_0^h + \beta_1^h High_TFP_{jt} + \beta_2^h Turb_t * High_TFP_{jt} + \mu_j + \eta_t + \epsilon_{jt} \quad (A26)$$

where x_{jt} denote variables of interest, including capital, employment, sales and value of firm j in year t . The coefficient of interest is on the interaction term (i.e., β_2^h), which captures relative impact of turbulence on growth of high-productivity firms h years after the turbulence shock.

Figure A14 plots the estimated β_2^h on the marginal effect of turbulence on growth of high-productivity firms, which is significantly negative for each variable and at every year over a five-year horizon. This suggest that that employment, capital, sales and equity value of high-productivity firms are disproportionately affected by turbulence for at least five years.

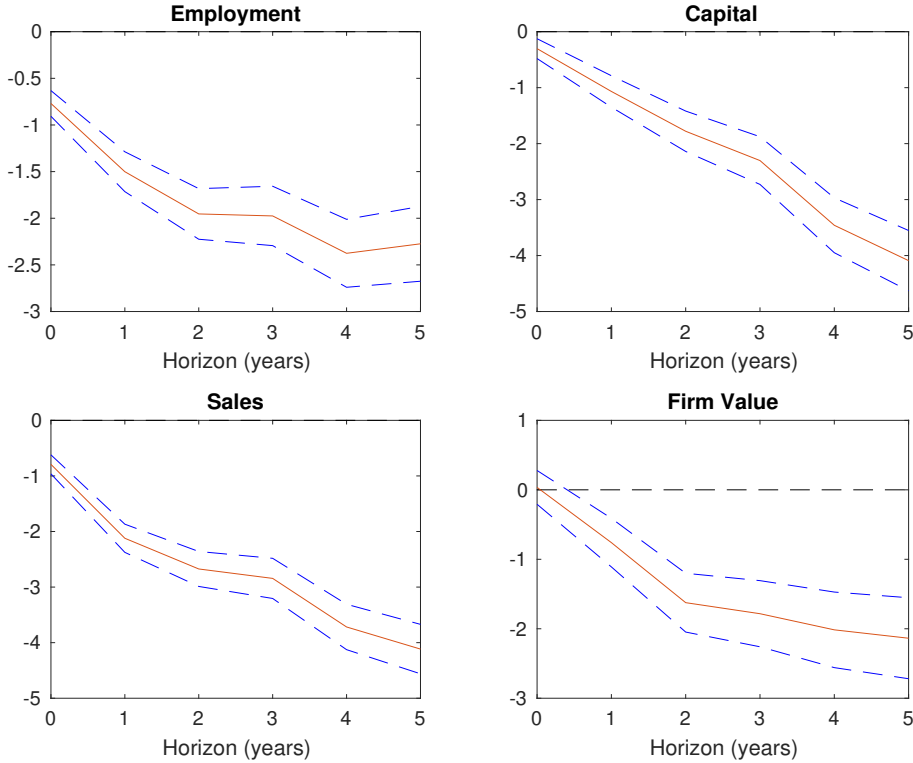


FIGURE A14. Persistent reallocation impact of turbulence

Note: This figure plots the estimated β_2^h in regression (A26). The variables we consider include employment, capital, sales and real equity value.

To examine the robustness of the impact of financial friction on reallocation effects of turbulence, we also consider an alternative specification in which we sort industry-level external finance dependence into four quartiles and we replace the dummy $High_FF_{it}$ in

the baseline specification by the dummy indicators of the top three quartiles of the financial friction distribution (denoted by $FF_{2,it}$, $FF_{3,it}$, and $FF_{4,it}$) (Table A.10). Implicitly, we treat the industry in the first quartile ($FF_{1,it}$) of the external finance dependence distribution (i.e., industries with the lowest external finance dependence) as the reference group. An increase in turbulence is associated with larger declines in interquartile range of employment and capital for industries with higher external finance dependence. These effects are both statistically significant and economically important.

TABLE A.10. Reallocation effects of turbulence: sensitivity to financial friction

Dep. Var.	IQR of Employment	IQR of Capital
	(1)	(2)
<i>High_FF_{2,it}</i>	-0.160 (0.134)	-0.224 (0.156)
<i>High_FF_{3,it}</i>	-0.104 (0.136)	-0.183 (0.157)
<i>High_FF_{4,it}</i>	0.223 (0.137)	0.234 (0.158)
<i>Turb_t * High_FF_{2,it}</i>	-1.402 (3.091)	-0.240 (3.578)
<i>Turb_t * High_FF_{3,it}</i>	-0.221 (3.116)	1.600 (3.607)
<i>Turb_t * High_FF_{4,it}</i>	-9.753*** (3.091)	-10.379*** (3.577)
Constant	1.994*** (0.037)	2.225*** (0.043)
Industry Fixed Effect	Yes	Yes
Year Fixed Effect	Yes	Yes
Observations	3,647	3,647

Note: This table shows the regression of inter-quartile range of employment (or capital) on the measured turbulence (*Turb*) for industries with different levels of external finance dependence. We sort the NAICS four-digit industries into four quartiles based on their external finance dependence. The dummy variables *FF_{2,it}*, *FF_{3,it}*, and *FF_{4,it}* indicate whether a industry *i*'s KZ index is in the second, third, or fourth quartile in year *t*. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.