Housing Prices, Borrowing Constaints, and Monetary Policy in the Business Cycle

Matteo Iacoviello, *The American Economic Review*, 2005

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"Credit View"

"Deterioriating credit market conditions, [...], are not just passive reflections of a declining economy, but are themselves a major factor depressing economic activity."
—Irving Fisher, *Debt-Deflation Theory of the Great Depression*

"The population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from liquid resources they can command[...]. Business borrowers typically have a strong propensity to hold physical capital [...]. Their desired portfolios contain more capital than their net worth."
—James Tobin, *Asset Accumulation and Economic Activity*

Literature

- Theory[1]
  - Partial Equilibrium: Fisher(1933); Tobin (1980)
  - General Equilibrium: BG (1989); KM (1997); Calstrom and Fuerst (1997); BGG (1999) etc.
- Empirical Studies
  - Empirical Studies: Hubbard (1998);
  - Financial Constraints-Household: Zeldes (1989); Jappelli and Pagano (1989); Campbell and Mankiw (1989); Carrolll and Dunn (1977).
- Explaining Business Cycles: none
- Monetary Policy Analysis: none

[1] For more recent surveys: Quadrini (2011); Gertler and Gilchrist (2018)

This Paper

- Variant of BGG (1999) Set-up
  - New-Keynesian framework of Financial Accelerator
- Collateral Constraints
  - Large proportion of borrowing is secured by real estate.
  - Channels of housing market are not well understood.
- Nominal Debt
  - Debt contracts are in nominal terms in low-inflation countries.
  - The macroeconomic implications are not well understood.
- Capture Business Cycle facts
- Explain Interaction b/w Asset Prices and Real Activity

Intuition: A Positive Demand Shock

- A Positive Demand Shock
  - Consumption Goods Prices $\uparrow$
  - Asset Prices $\uparrow$
- Asset Prices $\uparrow$
  - Borrowing Capacity $\uparrow$
  - Spending and Investment $\uparrow$
- Consumption Goods Prices $\uparrow$
  - Real value of outstanding debt obligation $\downarrow$
  - Net worth of borrowers $\uparrow$
  - MPC of borrowers $>$ MPC of lenders
  - Net effect on demand (consumption): $+$
  - Amplification mechanism on demand shock

$\Rightarrow$ Financial Accelerator of demand shocks.

Intuition: A Negative Supply Shock

- A Negative Supply Shock
  - Consumption Goods Prices ↑
  - Asset Prices ↑
- Asset Prices ↑
  - Borrowing Capacity ↑
  - Spending and Investment ↑
- Consumption Goods Prices ↑
  - Real value of outstanding debt obligation ↓
  - Net worth of borrowers ↑
  - MPC of borrowers > MPC of lenders
  - Net effect on demand (consumption): +
  - Mitigation mechanism on supply shock

⇒ Financial Decelerator of supply shocks.

Outline

- Introduction
  - Literature Review
  - Intuition
- VAR Evidence
- Basic Model
- Full Model
  - Heterogeneous Households
  - Variable capital investment
- Monetary Policy Experiment
- Conclusion

VAR Evidence

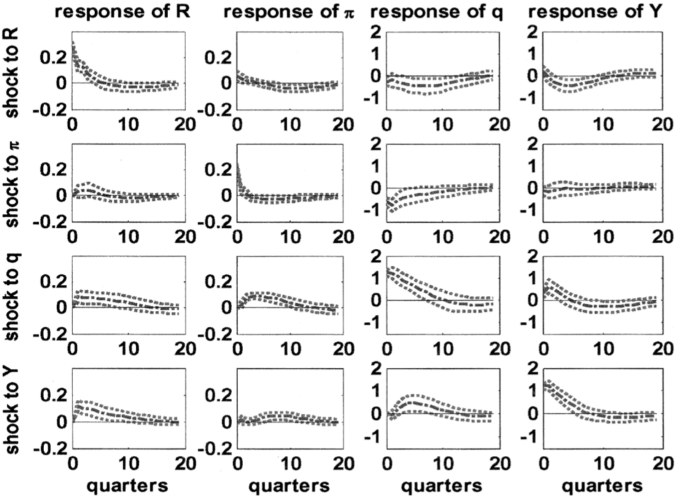


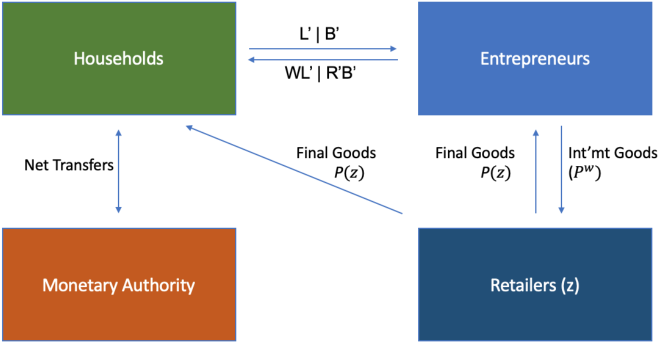
FIGURE 1. VAR EVIDENCE, UNITED STATES

VAR Evidence

A good model has to deliver:

- If interest rate (R) $\uparrow$
  - Nominal prices $(\pi)\downarrow$
  - Real housing prices $(q)\downarrow$
  - Output $(Y)\downarrow$
- If inflation $(\pi)\uparrow$
  - Real housing prices $(q)\downarrow$
  - Output $(Y)\downarrow$ (*small*)
- Positive co-movement of asset prices (q) and output (Y)
  - to Asset price shocks
  - to Output shocks

Basic Model



full

Homogeneous Household

- Infinitely lived, Patient, and Homogeneous[2]
- Utility: consumption, housing service, leisure, real balance

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln c'_t + j \ln h'_t - (L'_t)^\eta / \eta + \chi \ln(M'_t / P_t)]$$

  - β: discount factor of household
  - c'_t, h'_t, L'_t: consumption, house holding, labor supply
  - M'_t / P_t: Real money balance

[2] We will relax the assumption in the full model where households are heterogeneous in patience.

Homogeneous Household

- Budget Constraint (Nominal):

$$P_t c_t' + Q_t h_t' - B_t' + M_t' = W_t' L_t' + Q_t h_{t-1}' - B_{t-1}' R_{t-1} + M_{t-1}' + P_t F_t + P_t T_t'$$

- B_t': the amount household borrow
- Q_t: Nominal housing price
- F_t: Real lump-sum profit from retailers
- T_t': Net transfer from central bank
- Budget Constraint (Real):

$$c_t' + q_t h_t' - b_t' + \Delta M_t'/P_t = w_t' L_t' + q_t h_{t-1}' - b_{t-1}' R_{t-1}/\pi_t + F_t + T_t' \quad (1)$$

- $q_t' = Q_t'/P_t$: Real housing price
- $b_t' = B_t'/P_t$: Real net borrowing

Homogeneous Household

Household's Problem:

$$max \quad E_0 \sum_{t=0}^{\infty} \beta^t [\ln c'_t + j \ln h'_t - (L'_t)^\eta / \eta + \chi \ln(M'_t / P_t)]$$

$$s.t. \quad c'_t + q_t \Delta h'_t - b'_t + \Delta M'_t / P_t = w'_t L'_t - b'_{t-1} R_{t-1} / \pi_t + F_t + T'_t$$

F.O.C.s:

$$w.r.t. \quad c'_t : \quad \frac{1}{c'_t} = \beta E_t \frac{R_t}{\pi_{t+1} c'_{t+1}} \tag{2}$$

$$w.r.t. \quad L'_t : \quad w'_t = (L'_t)^{\eta-1} c'_t \tag{3}$$

$$w.r.t. \quad h'_t : \quad \frac{1}{c'_t} = \frac{1}{q_t} [\frac{j}{h'_t} + \beta E_t q_{t+1} \frac{1}{c'_{t+1}}] \tag{4}$$

Entrepreneurs

- Produce intermediate goods using real estate and labor;
- Sell intermediate goods to retailers at wholesale price P_t^w.

$$max \quad E_0 \sum_{t=0}^{\infty} \gamma^t [\ln c_t], \quad \gamma < \beta$$

Subject to:

$$Y_t = A(h_{t-1})^{\upsilon}(L_t)^{1-\upsilon} \tag{5}$$

$$Y_t P_t^w / P_t + b_t = w_t' L_t + q_t \Delta h_t + b_{t-1} R_{t-1}/\pi_t + c_t \quad (^3) \tag{6}$$

$$b_t R_t \leq m E_t q_{t+1} h_t \pi_{t+1} \quad (^4)$$

[3] $Y_t P_t^w + B_t + Q_t h_{t-1} = W_t' L_t + Q_t h_t + B_{t-1} R_{t-1} + P_t c_t$ (nominal budget)

[4] $B_t R_t \leq m E_t Q_{t+1} h_t$ (nominal credit constraint)

Entrepreneurs

F.O.C. w.r.t.: detail

$$c_t : \quad \frac{1}{c_t} = \gamma E_t \frac{R_t}{\pi_{t+1} c_{t+1}} + \lambda_t R_t \tag{7}$$

$$h_t : \quad \frac{1}{c_t} = \frac{1}{q_t} E_t \left[\underbrace{\left(\frac{\upsilon Y_{t+1}}{h_t} P^w_{t+1} + Q_{t+1} \right) \frac{1}{P_t} \frac{1}{c_{t+1}} \gamma}_{\text{return to housing investment}} + \underbrace{\lambda_t m q_{t+1} \pi_{t+1}}_{\text{liquidity premium}} \right] \tag{8}$$

$$L_t : \quad \underbrace{w'_t}_{\text{MC (real)}} = \underbrace{(1-\upsilon) \frac{Y_t}{L_t} \frac{P^w_t}{P_t}}_{\text{MR (Real)}} \tag{9}$$

Entrepreneurs: Credit Constraint

- Equation (2): $\frac{1}{c'_t} = \beta E_t \frac{R_t}{\pi_{t+1} c'_{t+1}}$
  $\rightarrow$ In the steady state: $1/\beta = R$
- Equation (7): $\frac{1}{c_t} = \gamma E_t \frac{R_t}{\pi_{t+1} c_{t+1}} + \lambda_t R_t$
  $\rightarrow$ In the steady state: $\lambda = (\beta - \gamma)/c$
- By assumption entrepreneurs are less patient than households.
  $\Leftrightarrow \gamma < \beta$
- $\Rightarrow \lambda = (\beta - \gamma)/c > 0$
  $\Leftrightarrow$ Credit constraint is binding around steady state:

$$b_t R_t = m E_t q_{t+1} h_t \pi_{t+1} \tag{10}$$

Retailers

- A continuum of unit mass, indexed by z.
- Buy intermediate goods from entrepreneurs at price P_t^w.
- Transform the goods into final goods $Y_t(z)$ at price $P_t(z)$.
- Subject to monopolistic competition and price rigidity:
  In each period, only $1-\theta$ of retailers can reset their prices.
- Final Goods[5]: $Y_t^f = [\int_0^1 Y_t(z)^{\epsilon-1/\epsilon} dz]^{\epsilon/\epsilon-1}$, $\epsilon > 1$
- Price index: $P_t = [\int_0^1 P_t(z)^{\epsilon-1} dz]^{1/\epsilon-1}$
- Individual demand curve: $Y_t(z) = [P_t(z)/P_t]^{-\epsilon} Y_t^f$

Retailer's Problem: maximize *expected discounted profit*
subject to: *downward sloping demand curve* detail

[5] Around steady state $Y_t^f = Y_t$. We utilize this condition in the analysis.

Retailers

- Optimal Pricing Equation $[P_t^*(z)]$:

$$\underbrace{\sum_{k=0}^{\infty}\theta^k E_t[\beta\frac{c'_t}{c'_{t+k}}\frac{P_t^*(z)}{P_{t+k}}Y_{t+k}^*(z)]}_{\text{Expected discounted marginal revenue}} = \underbrace{\sum_{k=0}^{\infty}\theta^k E_t[\beta\frac{c'_t}{c'_{t+k}}\frac{X}{X_{t+k}}Y_{t+k}^*(z)]}_{\text{Expected discounted marginal cost}} \quad (11)$$

X_t: the markup defined as $X_t = P_t/P_t^w$; $X = \epsilon/\epsilon - 1$
Y_{t+k}^*: expected demand defined as $Y_{t+k}^* = [P_t^*(z)/P_{t+k}]^{-\epsilon}Y_{t+k}$

- Aggregate Price Evolution:

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}]^{1/(1-\epsilon)} \quad (12)$$

- Combining linearized equation (11) and (12) will yield:

$$\hat{\pi} = \beta E_t \hat{\pi_{t+1}} - \kappa\hat{X}_t$$

Central Bank

The central bank implement a Taylor-type interest rate rule:

$$\ln R_t = r_R \ln R_{t-1} + (1 - r_R)[(1 + r_\pi) \ln \pi_{t-1} + r_y \ln(\frac{Y_{t-1}}{Y}) + \ln \bar{rr}] + \epsilon_{R,t} \quad (13)$$

where rr_t represents real interest rate defined as $rr_t \equiv R_t / E_t \pi_{t+1}$.

Market Clearing Conditions

There are four markets to be cleared in this economy:

- Labor market:

$$L_t = L'_t$$

- Real Estate market:

$$h_t + h'_t = H$$

- Goods market:

$$c_t + c'_t = Y_t$$

- Credit market:

$$b_t + b'_t = 0$$

steady state

linearization

Monetary Policy: Transmission Mechanism

Given a negative monetary shock (interest rate ↑):

- Interest rate channel: i ↑ ⇒ rr ↑ ⇒ c' ↓ ⇒ y ↓

$$\hat{c}'_t = E_t \widehat{c'_{t+1}} - r\hat{r}_t \quad \text{(L2)}$$

- Housing price channel: q ↓ ⇒ b ↓ ⇒ h ↓ ⇒ y ↓

$$\hat{q}_t = \gamma_e E_t \widehat{q_{t+1}} + (1-\gamma_e) E_t(\widehat{Y_{t+1}} - \hat{q}_t - \widehat{X_{t+1}}) - m\beta r\hat{r}_t - (1-m\beta) E_t \Delta \widehat{c_{t+1}} \quad \text{(L4)}$$

$$\hat{q}_t = \hat{c}'_t + \iota \hat{h}_t + \beta E_t \widehat{q_{t+1}} - \beta E_t c'_{t+1} \quad \text{(L5)}$$

$$\hat{b}_t = E_t \widehat{q_{t+1}} + \hat{h}_t - r\hat{r}_t \quad \text{(L6)}$$

- Debt Deflation channel:
  price level ↓ ⇒ cost of debt service ↑ ⇒ *c* and *h* ↓ ⇒ y ↓

Monetary Policy: Transmission Mechanism

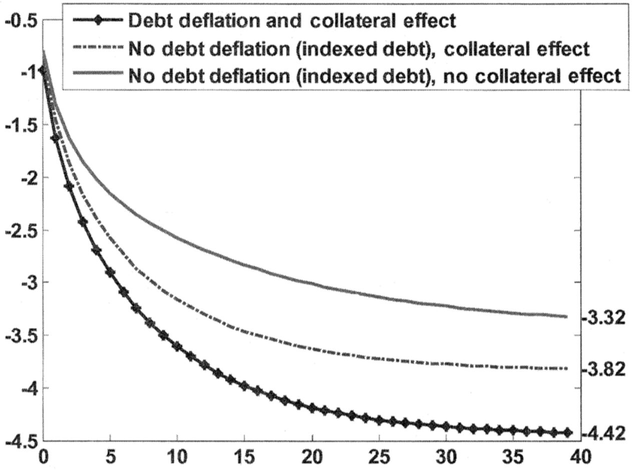
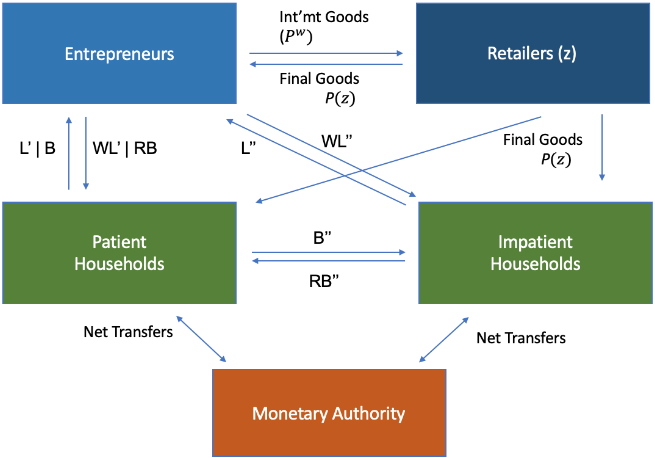


FIGURE 2. TOTAL OUTPUT LOSS IN RESPONSE TO A MONETARY SHOCK IN THE BASIC MODEL: COMPARISON BETWEEN ALTERNATIVE MODELS

Matteo Iacoviello, *The American Economic Review*, 2005

Full Model basic



Heterogeneous Households: Impatient Household

Impatient Household's Problem:

$$max \quad E_0 \sum_{t=0}^{\infty} \beta''^t [\ln c_t^{''} + j \ln h_t^{''} - (L_t^{''})^\eta/\eta + \chi \ln(M_t^{''}/P_t)] \quad (\beta'' < \beta)$$

$$s.t. \quad c_t'' + q_t \Delta h_t'' - b_{t-1}'' R_{t-1}/\pi_t + \Delta M_t''/P_t + \xi_{h,t} = w_t'' L_t'' + b_t'' + T_t''$$

where $\xi_{h,t}$ denotes the housing adjustment cost[6]. And

$$b_t'' R_t \leq m'' E_t(q_{t+1} h_t'' \pi_{t+1})$$

which is a typical lending constraint in mortgage market.
m": the degree of collateralizability. $m'' \to 0$: Households are excluded from financial markets.

[6] $\xi_{h,t} = \phi_h (\Delta h_t''/h_{t-1}'')^2 q_t h_{t-1}''/2$. This adjustment cost applies to patient household as well.

Entrepreneurs

- Produce intermediate goods using real estate, capital and labor;
- Adjustment cost of capital[7] and housing[8] investment.

$$max \quad E_0 \sum_{t=0}^{\infty} \gamma^t [\ln c_t], \quad \gamma < \beta$$

Subject to:

$$Y_t = A_t K_{t-1}^{\mu} h_{t-1}^{\upsilon} (L_t')^{\alpha(1-\mu-\upsilon)} (L_t'')^{(1-\alpha)(1-\mu-\upsilon)} \tag{14}$$

$$\frac{Y_t}{X_t} + b_t = w_t' L_t' + w_t'' L_t'' + q_t \Delta h_t + b_{t-1} R_{t-1}/\pi_t + c_t + I_t + \xi_{e,t} + \xi_{K,t} \tag{15}$$

$$b_t R_t \leq m E_t q_{t+1} h_t \pi_{t+1}$$

[7] Capital adjustment cost: $\xi_{K,t} = \Psi_t (I_t / K_{t-1} - \delta)^2 K_{t-1} / (2\delta)$.

[8] Housing adjustment cost: $\xi_{e,t} = \phi_e (\Delta h_t / h_{t-1})^2 q_t h_{t-1} / 2$.

Housing Price Shock: The Role of Agent Heterogeneity

- Empirical Evidence:
  a. Case et al. (2001): $corr(c, q) > 0$
  b. Davis & Palumbo (2001): $corr(c, qh) > 0$
- Homogeneous Agent $\Leftrightarrow$ homogeneous house holding:
  q $\uparrow \Rightarrow$ homogeneous qh $\uparrow \Rightarrow$ total wealth less qh unchanged
  $\Rightarrow$ non-housing consumption unchanged.
- Heterogeneous Agents $\Leftrightarrow \gamma < \beta$ or $\beta'' < \beta$:
  q $\uparrow \Rightarrow$ b and b" $\uparrow \Rightarrow$ c and c" $\uparrow \Rightarrow$ aggregate demand $\uparrow$

Housing Price Shock: The Role of Agent Heterogeneity

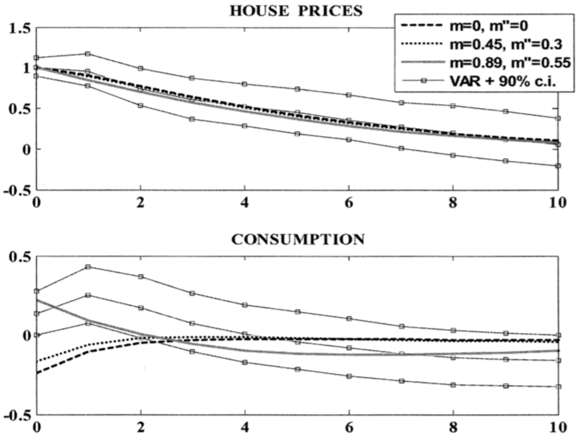


FIGURE 3. RESPONSE OF AGGREGATE CONSUMPTION TO A HOUSING PRICE SHOCK: VARIOUS VALUES OF m AND m''

Housing Price Shock: The Role of Agent Heterogeneity: Financial Accelerator

- Borrower's Demand:

$$\hat{c}_t = E_t \hat{c_{t+1}} + \frac{1}{1-m\beta}(\hat{q}_t - \underbrace{(1-\gamma_e)E_t \hat{S_{t+1}}}_{E_t \text{ MP of } h_t} - \gamma_e E_t \hat{q_{t+1}}) + \frac{m\beta}{1-m\beta}\hat{rr}_t \quad (16)$$

The multiplier, $\frac{1}{1-m\beta}$ *can be large, and is increasing with m.*

- Lenders' Demand:

$$\hat{c'_t} = \hat{q}_t - \beta E_t \hat{q_{t+1}} - \iota \hat{h}_t + \beta E_t \hat{c_{t'+1}} \quad (17)$$

The effect of q_t *on* c_t *is simply one-for-one.*

- Financial Accelerator: Collateral effects amplify demand-type shocks, i.e. housing price shock.

Inflation Shock: The Role of Nominal Debt

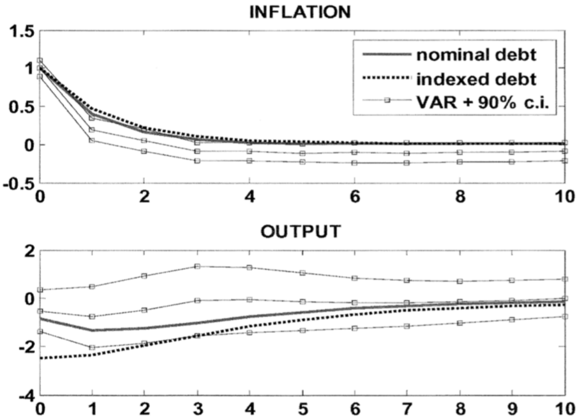


FIGURE 4. RESPONSE OF OUTPUT TO AN INFLATION SHOCK: NOMINAL VERSUS INDEXED DEBT

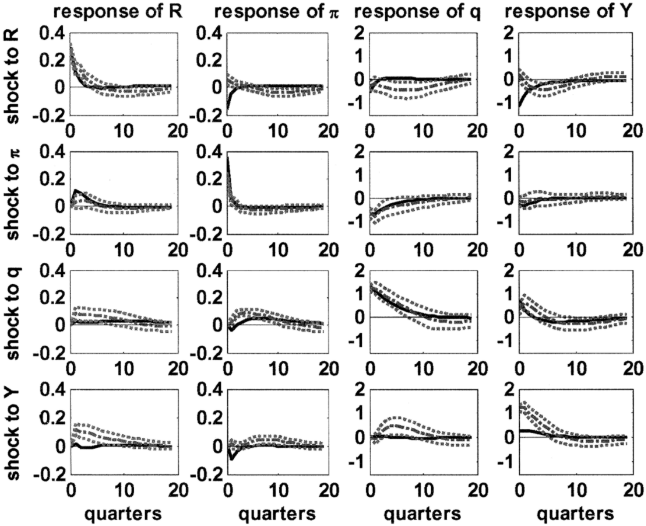
Notes: Ordinate: time horizon in quarters. Coordinate: percent deviation from initial steady state.

Inflation Shock: The Role of Nominal Debt

- Nominal Debt (Two effects):
  1. P $\uparrow \Rightarrow$ desired supply at given price $\downarrow \Rightarrow$ output $\downarrow$
  2. Transfer wealth from lenders to the borrowers (MPC $\uparrow$) $\Rightarrow$ output $\uparrow$

  $\Rightarrow$ Hump-shape response of output
- Indexed Debt (One effect):
  1. P $\uparrow \Rightarrow$ desired supply at given price $\downarrow \Rightarrow$ output $\downarrow$
- Financial Decelerator: Debt-deflation stabilizes supply-type shocks (with negative trade-off between output and inflation), i.e. an inflation shock.

Impulse Response Functions



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Impulse Response Functions

- A negative monetary shock:
  The drop in inflation: immediate (model) vs. delayed (data)
  House price: initial fall & overshooting
  The drop in output: immediate (model) vs. delayed (data)
- A positive inflation shock:
  Interest rate: positive
  House price: negative
  Output: sluggish
- A positive housing price shock:
  Inflation, Output: positive comovement
- A positive output shock:
  Interest rate, house price: sluggish (model) vs. positive(data)
  Inflation: negative (model) vs. sluggish (data)

Monetary Policy

Assume that volatility of output and inflation are the two goals of central bank. For shocks that generate negative comovement b/w volatility of output and inflation, two questions arise:

- Should monetary policy instrument (interest rate) respond to housing prices?
- How different financing arrangements (nominal vs. indexed debt) affect the volatility of the economy?

Should Central Banks Respond to Housing Prices?

- Specification of policy rule:

$$\hat{R}_t = 0.73\hat{R_{t-1}} + 0.27(r_q \hat{q}_t + (1 + r_\pi)\hat{\pi_{t-1}} + r_Y \hat{Y_{t-1}})$$

- Two efficient frontiers:
  1. $r_q = 0$: No response to asset prices.
  2. r_q free: Allow for response to asset prices.
- Results:
  a. Optimal r_q is positive;
  b. But only marginal gains.
- Literature:
  a.BG(2001) and Gilchrist &Leahy (2002): signal-to-noise ratio of asset prices is too low.
  b.This paper: Asset prices do matter, but the gain is too limited.

Should Central Banks Respond to Housing Prices?

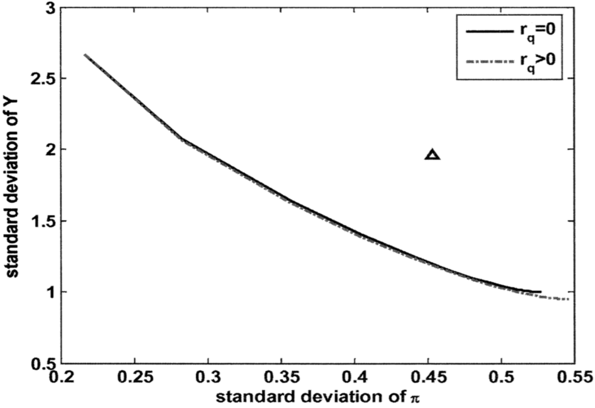

FIGURE 6. POLICY FRONTIERS AND ASSET PRICE RESPONSES

Note: The triangle indicates the performance of the rule estimated for the period 1974Q1–2003Q2.

Does Debt Indexation Reduce Economic Volatility?

- Nominal debt amplifies demand-type shocks and mitigates supply shocks.
  $\Leftrightarrow$ Debt indexation stabilizes only demand-type shocks.
- A demand-type shock (MP shock):
  1. r $\downarrow \Rightarrow$ borrowing limit $\uparrow \Rightarrow$ demand $\uparrow$
  2. r $\downarrow \Rightarrow$ debt service of debtor $\downarrow \Rightarrow$ demand $\uparrow$
- A supply-type shock (inflation shock):
  1. P $\uparrow \Rightarrow$ desired supply at given price $\downarrow \Rightarrow$ output $\downarrow$
  2. wealth transfer from lenders to the borrowers (MPC $\uparrow$) $\Rightarrow$ output $\uparrow$
- For demand-type shocks: Debt indexation can reduce volatility.

Conclusion

- Incorporate financial friction to monetary business cycle model
- Add two dimensions
  a. Collateral effect: match positive co-movement b/w output and housing price.
  b. Nominal debt contract: match the sluggish response of output to inflation shocks.
- Asymmetric financial accelerator / decelerator:
  Debt-deflation amplifies demand shocks and stabilizes supply shocks. (debt deflation channel)
- Unimportance of monetary policys response to asset prices:
  The welfare gains are only marginal.
- Household heterogeneity: Debtor vs. Creditor
  Mian, Rao and Sufi (2013); Baker (2017)

Entrepreneurs' Problem

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t \{ \ln c_t +$$

$$\mu_t [A(h_{t-1})^{\upsilon} (L_t)^{1-\upsilon} \frac{P_t^w}{P_t} + b_t - w_t' L_t - q_t \Delta h_t - \frac{b_{t-1} R_{t-1}}{\pi_t} - c_t] +$$

$$\lambda_t [m E_t q_{t+1} h_t \pi_{t+1} - b_t R_t] \}$$

FOC. w.r.t. b_t:

$$\mu_t = \gamma E_t \mu_{t+1} R_t / \pi_{t+1} + \lambda_t R_t$$

back

Retailers' Problem

max

$$\sum_{k=0}^{\infty} \theta^k E_{t-1}\left[\Lambda_{t,k} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^*(z)\right],$$

where the discount rate $\Lambda_{t,k} \equiv \beta C_t/(C_{t+k})$

subject to

$$Y_t(z) = [P_t(z)/P_t]^{-\epsilon} Y_t^f$$

back

Steady State

Steady State of the Basic Model. Assuming zero inflation (so that $R = 1/\beta$), the steady state will be described by

$$\frac{h}{H} = \frac{\gamma\nu(1-\beta)}{\gamma\nu(1-\beta) + j((X-\nu)(1-\gamma_e) + \gamma\nu(1-\beta)m)}$$

$$\frac{qh}{Y} = \frac{\gamma\nu}{1-\gamma_e}\frac{1}{X}$$

$$\frac{b}{Y} = \frac{\beta m\gamma\nu}{1-\gamma_e}\frac{1}{X}$$

$$\frac{c}{Y} = \frac{\nu}{X} - (1-\beta)m\frac{qh}{Y} = \nu\frac{(1-\gamma)(1-\beta m)}{1-\gamma_e}\frac{1}{X}$$

$$\frac{c'}{Y} = \frac{X-\nu}{X} + (1-\beta)m\frac{qh}{Y} = \left(X - \nu + \frac{\gamma\nu(1-\beta)m}{1-\gamma_e}\right)\frac{1}{X}$$

where $\gamma_e \equiv (1-m)\gamma + m\beta$ is the *average* discount factor for the returns to entrepreneurial real estate investment.

back

Linearized System

$$\text{(L1)} \quad \hat{Y}_t = (c/Y)\hat{c}_t + (c'/Y)\hat{c}'_t$$

$$\text{(L2)} \quad \hat{c}'_t = E_t\hat{c}'_{t+1} - \widehat{rr}_t$$

$$\text{(L3)} \quad c\hat{c}_t = b\hat{b}_t + Rb(\hat{\pi}_t - \hat{R}_{t-1} - \hat{b}_{t-1}) + (\nu Y/X)(\hat{Y}_t - \hat{X}_t) - qh\Delta\hat{h}_t$$

$$\text{(L4)} \quad \hat{q}_t = \gamma_e E_t\hat{q}_{t+1} + (1-\gamma_e)E_t \times (\hat{Y}_{t+1} - \hat{h}_t - \hat{X}_{t+1}) - m\beta\widehat{rr}_t - (1-m\beta)E_t\Delta\hat{c}_{t+1}$$

$$\text{(L5)} \quad \hat{q}_t = \beta E_t\hat{q}_{t+1} + \iota\hat{h}_t + \hat{c}'_t - \beta E_t\hat{c}'_{t+1}$$

back

Linearized System

$$\text{(L6)} \qquad \hat{b}_t = E_t\hat{q}_{t+1} + \hat{h}_t - \widehat{rr}_t$$

$$\text{(L7)} \qquad \hat{Y}_t = \frac{\eta\nu}{\eta - (1-\nu)}\hat{h}_{t-1} - \frac{1-\nu}{\eta-(1-\nu)}(\hat{X}_t + \hat{c}'_t)$$

$$\text{(L8)} \qquad \hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} - \kappa\hat{X}_t$$

$$\text{(L9)} \qquad \hat{R}_t = (1-r_R)((1+r_\pi)\hat{\pi}_{t-1} + r_Y\hat{Y}_{t-1}) + r_R\hat{R}_{t-1} + \hat{e}_{R,t}$$

where $\iota \equiv (1-\beta)h/h'$, $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$, $\gamma_e \equiv m\beta + (1-m)\gamma$, and $\widehat{rr}_t \equiv \hat{R}_t - E_t\hat{\pi}_{t+1}$ is

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