Increasing Returns and Economic Geography

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April 25, 2018

Introduction: From Krugman (1979) to Krugman (1991)

The award of the 2008 Nobel Prize for economics was given to Paul Krugman for three of his papers (Krugman, 1979, 1980, 1991).

- Krugman 1979: analyses what happens in an economy that is characterized by increasing returns to scale and imperfect competition if countries start to trade.
- Krugman 1980: introduces transport costs and the increasing returns to framework of the 1979 paper, which gives rise to *home market effect*.
- Krugman 1991[1]: endogenizes the spatial allocation of both supply and demand with combination of the home market effect with inter-regional labor mobility, which gives rise to core-periphery equilibria.

[1] Krugman, P. (1991). Increasing returns and economic geography. *Journal of Political Economy*, 99(3), 483-499.

Introduction: Interest of Study

- Question to Address
  Why and when does manufacturing become concentrated in a few regions, leaving others relatively undeveloped?
- Proposed Approach
  A simple model of geographical concentration of manufacturing based on the interaction of economies of scale with transportation costs.
- A country with two kinds of production:
  Agriculture: *constant returns to scale* and intensive use of land.
  Manufacture: *increasing returns to scale* and modest use of land.

Introduction: Questions to Answer

- Where will manufactures production take place?
- Where will demand be large?
- How far will the tendency toward geographical concentration proceed?
- Where will manufacturing production actually end up?

Model: Assumptions

1. **Two Regions**: *region 1* and *region 2*;
2. **Two Sectors**: *A for Agriculture* and *M for Manufacture*;
3. **Utility Function**: (identical preference across individuals in two regions)

$$U = C_M^{\mu} C_A^{1-\mu} \tag{1}$$

C_A is the consumption of the agricultural good and C_M is the consumption of the manufactures aggregate defined as:

$$C_M = [\sum_{i=1}^{N} (c_i)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where N is the number of products in manufacture. Given equation (1), manufactures will receive a share μ of expenditure; agricultural sector will receive a share of $1 - \mu$ of expenditure.

Model: Assumptions

4. **Two Factors**: *Peasants* for Agriculture and *Workers* for Manufacture.

- Peasants are immobile across regions; Workers can move between regions (L_i).
- Peasant supply is exogenously given as $(1-\mu)/2$ in each region.
- Worker supply add up to μ:

$$L_1 + L_2 = \mu \tag{3}$$

5. **Production**:
(1) Agriculture: unit labor requirement is one;
(2) Manufacture: production of good i involves a fixed cost (α) and a constant marginal cost (β): (economies of scale)

$$L_{Mi} = \alpha + \beta x_i \tag{4}$$

Model: Behavior of Firms

1. Price setting

Producer i maximize its profit given by:

$$\max\ p_i x_i - w_i L_{Mi} = p_i x_i - w_i(\alpha + \beta x_i)$$

- *F.O.C. w.r.t* x_t: $p_i + \frac{dp_i}{dx_i} x_i = w_i \beta$
- Equivalently, $p_i(1 + \frac{dp_i}{dx_i}\frac{x_i}{p_i}) = w_i \beta$
- *Price:* As the inverse of elasticity ($\frac{dp_i}{dx_i}\frac{x_i}{p_i} = \frac{1}{-\sigma}$), price setting equation of a representative manufacturing firm in each region is:

$$p_i = (\frac{\sigma}{\sigma - 1})\beta w_i \tag{5}$$

- *Relative Price:*

$$\frac{p_1}{p_2} = \frac{w_1}{w_2} \tag{6}$$

Model: Behavior of Firms

2. Output and Number of Firms

- *Zero Profit Condition:*

$$p_i x_i - w_i L_{Mi} = p_i x_i - w_i(\alpha + \beta x_i) = 0, \text{ or equivalently:}$$

$$(p_i - w_i\beta)x_i = \alpha w_i \tag{7}$$

- *Output per firm*: (Replacing p_i in equation (7) with equation (5): $p_i = (\frac{\sigma}{\sigma-1})\beta w_i$)

$$x_1 = x_2 = \frac{\alpha(\sigma - 1)}{\beta} \tag{8}$$

  The degree of economies of scale: $\sigma/(\sigma - 1)$ (*ratio of the marginal product of labor to its average product*)
  σ: *an inverse index of equilibrium economies of scale* as well.

- *Number of manufacture firms:* Equation (8) implies:

$$\frac{n_1}{n_2} = \frac{L_1}{L_2} \tag{9}$$

Model: Short-Run Equilibrium

1. Notations

- Consumption. c_{ij}: consumption in region i of a representative region j product. For example, c_{12} is consumption in region 1 of a representative region 2 product.
- Iceberg transportation cost: $0 < \tau < 1$.
- Price. p_i: the price of a local product in region i. p_i/τ is the price of a product in region i from the other region. For example, for region 1 consumers, the price for region 1 product is p_1, the price for region 2 product is p_2/τ.

Model: Short-Run Equilibrium

2. Relative Demand

The relative demand in region 1 for representative product from both regions is:

$$\frac{c_{11}}{c_{12}} = [\frac{p_1}{p_2/\tau}]^{-\sigma} = [\frac{p_1\tau}{p_2}]^{-\sigma} = [\frac{w_1\tau}{w_2}]^{-\sigma} \tag{10}$$

3. Relative Expenditure

Define z_{11} as the ratio of region 1 expenditure *on region 1's manufactures to that on manufactures from region 2*. Using equation (9), equation (6) and equation (10), the relative expenditure of region 1 is given as:

$$z_{11} = (\frac{n_1}{n_2})(\frac{p_1}{p_2/\tau})(\frac{c_{11}}{c_{12}}) = (\frac{L_1}{L_2})(\frac{w_1\tau}{w_2})^{-(\sigma-1)} \tag{11}$$

Similarly, the relative expenditure of region 2 on region 1's manufactures to region 2's manufactures is:

$$z_{12} = (\frac{n_1}{n_2})(\frac{p_1/\tau}{p_2})(\frac{c_{21}}{c_{22}}) = (\frac{L_1}{L_2})(\frac{w_1}{w_2\tau})^{-(\sigma-1)} \tag{12}$$

Model: Short-Run Equilibrium

4. Total Income

- *Income of workers*: The total income of workers in region 1 equals the total spending from both regions on region 1's manufactures product:

$$w_1 L_1 = \frac{z_{11}}{1+z_{11}}\mu Y_1 + \frac{z_{12}}{1+z_{12}}\mu Y_2 = \mu[\frac{z_{11}}{1+z_{11}}Y_1 + \frac{z_{12}}{1+z_{12}}Y_2] \quad (13)$$

  Similarly, the income of region 2 workers is:

$$w_2 L_2 = \frac{1}{1+z_{11}}\mu Y_1 + \frac{1}{1+z_{12}}\mu Y_2 = \mu[\frac{1}{1+z_{11}}Y_1 + \frac{1}{1+z_{12}}Y_2] \quad (14)$$

- *Total income*:

$$Y_1 = \frac{1-\mu}{2} + w_1 L_1 \quad (15)$$

$$Y_2 = \frac{1-\mu}{2} + w_2 L_2 \quad (16)$$

Model: Short-Run Equilibrium

5. Short-Run Equilibrium

Equations (11) - (16) characterize the short-run equilibrium that determines a sequence of variables: $\{w_1, w_2, z_1, z_2, Y_1, Y_2\}$, given the distribution of L_1 and L_2.

(1). When $L_1 = L_2$, we must have $w_1 = w_2$.

(2). When labor is moving from region 2 to region 1, the relative wage w_1/w_2 can either increase or decrease. There are two opposing effects:

- *Home market effect*: Other things equal, the wage rate tend to be higher in the larger market (Krugman, 1980);
- *Competition effect*: Workers in the region with smaller manufactures labor force will face less competition for local peasant market.

Model: Long-Run Equilibrium

1. Share of Manufacturing Labor force
Recall that $L_1 + L_2 = \mu$ (*equation 3*)
$f = L_1/\mu$: the fraction of manufacturing labor force in region 1.
$1 - f = L_2/\mu$: the fraction of workers region 2.
2. Price Index
The true price index of manufacturing goods for consumers residing in region 1 is:

$$P_1 = [fw_1^{-(\sigma-1)} + (1-f)(\frac{w_2}{\tau})^{-(\sigma-1)}]^{-1/(\sigma-1)} \tag{17}$$

And similarly for consumers residing in region 2, the price index is:

$$P_2 = [f(\frac{w_1}{\tau})^{-(\sigma-1)} + (1-f)w_2^{-(\sigma-1)}]^{-1/(\sigma-1)} \tag{18}$$

When $w_1 = w_2$, an increase in f (shift of workers from region 2 to region 1) will lower P_1 and raise P_2. (*Price Index Effect*)

Model: Long-Run Equilibrium

3. Real Wage Rate

The real wage rates, denoted as ω, in each region are:

$$\omega_1 = w_1 P_1^{-\mu} \tag{19}$$

and

$$\omega_2 = w_2 P_2^{-\mu} \tag{20}$$

When wage rates are equal, an increase in f will raise real wages in region 1 relative to those in region 2, i.e., ω_1/ω_2 increases with f. How does relative real wage, ω_1/ω_2, vary with f generally?

Model: Long-Run Equilibrium

4. Relative Real Wage Rate

How does relative real wage, ω_1/ω_2, change with f?

- *Symmetric Case*: If $f = 0.5$, that is when the two regions have equal number of workers, they offer equal real wage rates, $\omega_1/\omega_2 = 1$.
- *Regional Convergence*: If ω_1/ω_2 decreases with f, i.e., the relative real wage is lower when work force is larger, then workers tend to migrate out of the region with larger worker force. (One force:*degree of competition for local peasant market*)
- *Regional Divergence*: If ω_1/ω_2 increases with f, i.e., the relative real wage is higher when work force is larger, then workers tend to migrate into the region with larger worker force. (Two forces:*home market effect* and *price index effect*)

Model: Long-Run Equilibrium

4. Relative Real Wage Rate (cont.)

How does relative real wage, ω_1/ω_2, change with f?

- Convergence force: *degree of competition for local peasant market*)
- Divergence forces: *home market effect* and *price index effect*)

Question: Which forces dominate?

Answer: Parameters matter: the share of expenditure on manufactured goods, μ; the elasticity of substitution among products, σ; and the fraction of a good shipped that arrives, τ. Depending on the values of these parameters we may have either regional convergence or regional divergence.

5. A Simple Numerical Exercise

Given some set of parameter values, i.e., [$\sigma = 4$, $\mu = 0.3$, $\tau = 0.5$] or [$\sigma = 4$, $\mu = 0.3$, $\tau = 0.75$], the model can be easily solved (and compared).

- When $\tau = 0.5$ (high transportation cost), the relative real wage will decline with f. Thus in the case we can expect regional convergence that the geographical distribution of manufacturing resemble that of agriculture.
- When $\tau = 0.75$ (low transportation cost), the relative real wage will decline with f. Thus in the case we can expect regional divergence that manufacturing workers concentrate on one region.
- The numerical analysis on effect of transportation cost can be summarized by the Fig. 1

Model: Long-Run Equilibrium

5. A Simple Numerical Exercise

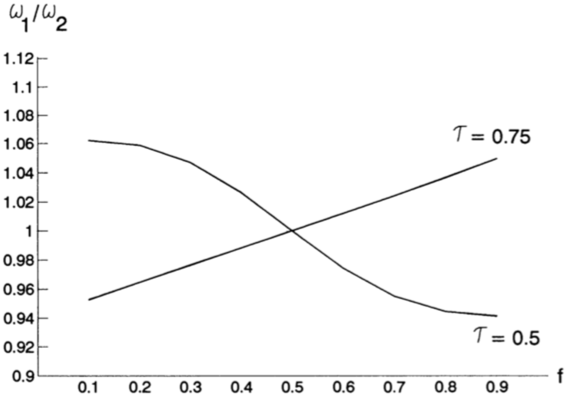

FIG. 1

Model: Complete Agglomeration Equilibrium

Suppose that all workers are concentrated in region 1. (f=1)
1. Value of Sales of Region 1 Firm
Since a share of total income μ is spent on manufactures and all this share goes to region 1, we must have:

$$\frac{Y_2}{Y_1} = \frac{1-\mu}{1+\mu} \tag{21}$$

And each manufacturing firm will have a value of sales equal to (let n be the total number of manufacturing firms):

$$V_1 = \frac{\mu(Y_1 + Y_2)}{n} \tag{22}$$

which is just enough to allow each firm to make zero profits due to free entry.

Model: Complete Agglomeration Equilibrium

When $f = 1$, price index derived from equation (17) and (18) becomes:

$$P_1 = [w_1^{-(\sigma-1)}]^{-1/(\sigma-1)} = w_1$$

and

$$P_2 = [(\tfrac{w_1}{\tau})^{-(\sigma-1)}]^{-1/(\sigma-1)} = w_1/\tau$$

In order to produce in region 2, a firm must be able to attract workers with offered real wage at least as high as in region 1, i.e., $\omega_2 = \omega_1$. As $\omega_i = w_i P_i^{-\mu}$, the equality of real wage implies that the nominal wage must satisfy:

$$\frac{w_2}{w_1} = (\frac{1}{\tau})^{\mu} \tag{23}$$

Question: Can such complete agglomeration be an equilibrium?

Model: Complete Agglomeration Equilibrium

Question: Can complete agglomeration in region 1 be an equilibrium?
Answer: If it is possible for an individual firm to make positive profit by migrating to region 2[2], manufacturing concentration in region 1 is not an equilibrium. If it is not possible, then it is an equilibrium.

2. Value of Sales of Region 2 Firm

From equation (10) and (11) we have shown that relative demand for region 2 manufactures goods from region 1 consumers is

$$\frac{c_{12}}{c_{11}} = \left(\frac{p_2/\tau}{p_1}\right)^{-\sigma}$$

and therefore the relative expenditure on region 2 manufactures goods from region 1 consumers is

$$\frac{c_{12}p_1/\tau}{c_{11}p_1} = \left(\frac{p_2/\tau}{p_1}\right)^{-(\sigma-1)} = \left(\frac{w_1\tau}{w_2}\right)^{-(\sigma-1)}$$

[2] We denote the firm as a *defecting firm*

Model: Complete Agglomeration Equilibrium

2. Value of Sales of Region 2 Firm (cont.)

Relative expenditure on region 2 manufactures goods from region 2 consumers: $(\frac{w_2}{w_1/\tau})^{-(\sigma-1)}$.

The value of the region 2 firm's sale will be

$$V_2 = \frac{1}{n}[\frac{w_1\tau}{w_2})^{-(\sigma-1)}\mu Y_1 + (\frac{w_2}{w_1/\tau})^{-(\sigma-1)}\mu Y_2] \tag{24}$$

Intuitively, equation (24) show that transportation costs work to the region 2 firm's disadvantage in its sale to region 1 but work to its advantage in sales to region 2.

Model: Complete Agglomeration Equilibrium

3. Ratio of the Value of Sales

From equation (22), (23) and (24), the ratio of the value of sales by the defecting firm to the sales of a representative region 1 firm can be derived as

$$\frac{V_2}{V_1} = 0.5\tau^{\mu(\sigma-1)}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)})] \tag{25}$$

4. Defecting Conditions

It is profitable for a firm to defect (from region 1 to region 2) when $V_2/P_2^{\mu} > V_1/P_1^{\mu}$, or $V_2/V_1 > P_2^{\mu}/P_1^{\mu}$. As $P_2 = P_1/\tau$, or $P_2^{\mu}/P_1^{\mu} = \tau^{-\mu}$, we have the following defecting condition that $v > 1$ where v is defined as:

$$v = \frac{V_2/V_1}{\tau^{-\mu}} = 0.5\tau^{\mu\sigma}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}] \tag{26}$$

Model: Complete Agglomeration Equilibrium

4. Defecting Conditions (cont.)

$$v = \frac{V_2/V_1}{\tau^{-\mu}} = 0.5\tau^{\mu\sigma}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}]$$

When $v < 1$, concentration of manufactures production in region 1 is an equilibrium.

Equation (26) is the key equation for analytical results. For a given set of parameter values we can use equation (26) to judge whether concentration is possible or not. Further, equation (26) defines a set of *critical values* of parameters that divide between concentration and non-concentration. It is necessary, then, to examine how v changes with each parameter.

Model: Complete Agglomeration Equilibrium

4. Defecting Conditions (cont.): Impact of parameter μ

$$v = \frac{V_2/V_1}{\tau^{-\mu}} = 0.5\tau^{\mu\sigma}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}]$$

$$\frac{\partial v}{\partial \mu} = v\sigma(ln\tau) + 0.5\tau^{\sigma\mu}[\tau^{\sigma-1} - \tau^{1-\sigma}] < 0 \tag{27}$$

The larger the share of income spent on manufactured goods (μ), the lower the relative sales of the defecting firm, thus the more likely concentration makes an equilibrium. There are two reasons behind this:
(1). From equation (23) it can been seen that workers demand a larger wage premium in order to move to the second region;
(2). The larger the share of expenditure on manufactures, the larger the relative size of the region 1 market and hence the stronger the *home market effect*.

Model: Complete Agglomeration Equilibrium

4. Defecting Conditions (cont.): Impact of parameter τ

$$v = \frac{V_2/V_1}{\tau^{-\mu}} = 0.5\tau^{\mu\sigma}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}]$$

(1) when $\tau = 1$, $v = 1$: When transportation costs are zero, location is irrelevant.
(2) when τ is small, v approaches $(1-\mu)\tau^{1-\sigma(1-\mu)}$, which must exceed 1 unless σ and μ take extreme (and implausible) values.
(3) Take derivative w.r.t τ:

$$\frac{\partial v}{\partial \tau} = \frac{v\sigma\mu}{\tau} + 0.5\tau^{\sigma\mu-1}[(1+\mu)\tau^{\sigma-1} - (1-\mu)\tau^{1-\sigma}] \tag{28}$$

When τ is close to 1, $\frac{\partial v}{\partial \tau}$ is positive. Taken together, above three cases indicate the shape of v as a function τ is similar to *Fig. 2*.

Model: Complete Agglomeration Equilibrium

4. Defecting Conditions (cont.): Impact of parameter τ

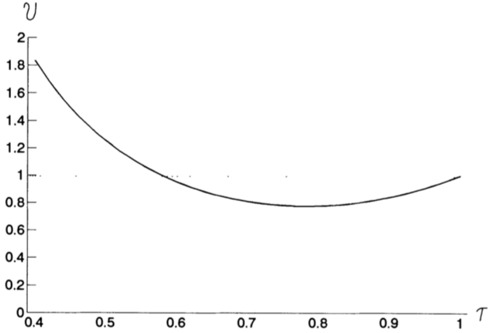

FIG. 2

At the critical point when v=1, $\frac{\partial v}{\partial \tau}$ is negative, which suggests that reducing transportation cost (higher τ) from the critical point will lead to manufacturing concentration.

Model: Complete Agglomeration Equilibrium

4. Defecting Conditions (cont.): Impact of parameter σ

$$v = \frac{V_2/V_1}{\tau^{-\mu}} = 0.5\tau^{\mu\sigma}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}]$$

$$\frac{\partial v}{\partial \sigma} = ln(\tau)(\frac{\tau}{\sigma})(\frac{\partial v}{\partial \tau}) \tag{29}$$

As we have shown $\frac{\partial v}{\partial \tau}$ is negative around the critical point, this also implies that $\frac{\partial v}{\partial \sigma}$ is positive. Therefore, a lower elasticity of substitution (σ), which also implies larger economies of scale in equilibrium from equation (8), will increase the probability of manufacturing concentration.

Summary

- We have presented a model of possible core-periphery patterns in a two regions economy. There are economies of scale in production of manufactured goods and there are transportation costs.
- Because of economies of scale, there is only one producer of each variety. Because of transportation costs, firms will have a tendency to establish in the largest market.
- In this model a driving force is mobile labor. Workers move to the region in which real wages are the highest. Firms want to establish in the region where market access is best. Market access is best where firms and workers are already located.

Summary

- For specific parameter values, in particular with low transportation costs(τ), significant economies of scale(σ) and a large share of manufacturing goods(μ) in the economy, a core-periphery pattern is a possible outcome.
- In such situations, all manufacturing production agglomerates in one region (core) while the periphery becomes de-industrialised.

Discussion: The Merit of Krugman 1991

- The previous two papers (Krugman, 1979 and 1980) are about international trade, notably intra-industry trade, whereas the last paper extends the analysis by endogenizing the spatial allocation of economic activity, making it the core model of the new economic geography literature.
- It is indeed the combination of his contribution to both trade and geography that makes Krugmans work special.
- The real contribution of Krugman (1991) is that the location of both (IRS) firms and workers becomes endogenous, and that Krugman was the first to do this in a fully specified general equilibrium framework.