Notes on Krugman (1991)

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April 25, 2018

Abstract: This note covers Krugman's (1991) paper[1] which develops a simple model that shows how a country can endogenously become differentiated into an industrialized "core" and an agricultural "periphery." In order to realize scale economies while minimizing transport costs, manufacturing firms tend to locate in the region with larger demand, but the location of demand itself depends on the distribution of manufacturing. Emergence of a core-periphery pattern depends on transportation costs, economies of scale, and the share of manufacturing in national income.

1 Introduction

- **Question to Address**
  Why and when does manufacturing become concentrated in a few regions, leaving others relatively undeveloped?

- **Proposed Approach**
  This paper develops a very simple model of geographical concentration of manufacturing based on the interaction of economies of scale with transportation costs.
  Imagine a country in which there are two kinds of production, agriculture and manufacturing. Agricultural production is characterized both by *constant returns to scale* and by intensive use of immobile land. The geographical distribution of this production will therefore be determined largely by the exogenous distribution of suitable land. Manufactures, on the other hand, we may suppose to be characterized by *increasing returns to scale* and modest use of land.

[1]Krugman, P. (1991). Increasing returns and economic geography. *Journal of Political Economy*, 99(3), 483-499.

2 Intuition

- Where will manufactures production take place?
  Because of economies of scale, production of each manufactured good will take place at only a limited number of sites. Other things equal, the preferred sites will be those with relatively large nearby demand, since producing near one's main market minimizes transportation costs.

- But where will demand be large?
  Some of the demand for manufactured goods will come from the agricultural sector; and some of the demand for manufactures will come not from the agricultural sector but from the manufacturing sector itself.

- *Positive feedback*: manufactures production will tend to concentrate where there is a large market, but the market will be large where manufactures production is concentrated.

- How far will the tendency toward geographical concentration proceed?
  It depends on the underlying parameters of the economy. The circularity that can generate manufacturing concentration will not matter too much if manufacturing employs only a small fraction of the population and hence generates only a small fraction of demand, or if a combination of weak economies of scale and high transportation costs induces suppliers of goods and services to the agricultural sector to locate very close to their market.
  By contrast, if a higher fraction of income is spent on nonagricultural goods and services, a region with a relatively large non-rural population will be an attractive place to produce both, because of the large local market and because of the availability of the goods and services produced there. This will attract more population, at the expense of regions with smaller initial production, and the process will feed on itself until the whole of the non-rural population is concentrated in a few regions.

- Small changes in some parameters of the economy may have large effects on its qualitative behavior: *transportation costs, economies of scale, and the share of nonagricultural goods in expenditure.*

3 Model

Assumptions

1. Two Regions: *region 1* and *region 2*;
2. Two Sectors: *A for Agriculture* and *M for Manufacture*;
3. Utility Function: (identical preference across individuals in two regions)

$$U = C_M^{\mu} C_A^{1-\mu} \tag{1}$$

C_A is the consumption of the agricultural good and C_M is the consumption of the manufactures aggregate defined as:

$$C_M = [\sum_{i=1}^{N} (c_i)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where N is the number of products in manufacture. Given equation (1), manufactures will receive a share μ of expenditure; agricultural sector will receive a share of $1-\mu$ of expenditure[2]. $\sigma > 1$ *is the elasticity of substitution among the products.*
4. Two Factors: *Peasants* for Agriculture and *Workers* for Manufacture.

- Peasants are immobile across regions; Workers can move between regions (L_i).
- Peasant supply is exogenously given as $(1-\mu)/2$ in each region.
- Worker supply add up to μ:

$$L_1 + L_2 = \mu \tag{3}$$

5. Production:
(1) Agriculture: unit labor requirement is one;
(2) Manufacture: production of good i involves a fixed cost (α) and a constant marginal cost (β): (giving rise to economies of scale)

$$L_{Mi} = \alpha + \beta x_i \tag{4}$$

[2] To see this, the optimization problem of max U s.t. $P_A C_A + P_M C_M = Y$ will give optimal conditions of $P_A C_A = (1-\mu)Y$ and $P_M C_M = \mu Y$.

Behaviors of Firms

1. Price setting

Producer i maximize its profit given by:

$$\max\, p_i x_i - w_i L_{Mi} = p_i x_i - w_i(\alpha + \beta x_i)$$

- *F.O.C. w.r.t* x_t: $p_i + \frac{dp_i}{dx_i} x_i = w_i \beta$
- Or equivalently, $p_i(1 + \frac{dp_i}{dx_i}\frac{x_i}{p_i}) = w_i\beta$
- *Price:* As the inverse of elasticity $(\frac{dp_i}{dx_i}\frac{x_i}{p_i})$ equals to $\frac{1}{-\sigma}$[3], we derive price setting equation of a representative manufacturing firm in region 1 and region 2:

$$p_1 = (\frac{\sigma}{\sigma - 1})\beta w_1 \tag{5}$$

$$p_2 = (\tfrac{\sigma}{\sigma-1})\beta w_2$$

- *Relative Price:* Relative price of representative firms in two regions are:

$$\frac{p_1}{p_2} = \frac{w_1}{w_2} \tag{6}$$

2. Output and Number of Firms

- *Zero Profit Condition:* If there is free entry of firms into manufacturing, the profit must be driven to zero:

$$p_i x_i - w_i L_{Mi} = p_i x_i - w_i(\alpha + \beta x_i) = 0, \text{ or equivalently:}$$

$$(p_i - w_i\beta)x_i = \alpha w_i \tag{7}$$

- *Output per firm*: Replacing p_i in equation (7) with Equation (5): $p_i = (\frac{\sigma}{\sigma-1})\beta w_i$, it implies the output per firm is the same in each region:

$$x_1 = x_2 = \frac{\alpha(\sigma - 1)}{\beta} \tag{8}$$

  It can be seen that in zero-profit equilibrium, $\sigma/(\sigma - 1)$ is the ratio of the marginal product of labor to its average product, that is, the degree of economies of scale. Thus σ can be interpreted as *an inverse index of equilibrium economies of scale* as well[4].

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[3] To see this, simply solve a cost minimization problem subject to equation (2), it can be derived an expression of p_i in terms of c_i (x_i).

[4] Average costs relative to marginal costs is a measure of returns to scale in production. In this model we have $AC/MC = \frac{\alpha w/x + \beta w}{\beta w} = \frac{(\sigma/\sigma-1)\beta w}{\beta w} = \sigma/\sigma - 1$.

- *Number of manufacture firms:* Equation (8) implies that the number of manufacture goods (= number of manufacture firms) in each region is proportional to the number of workers:

$$\frac{n_1}{n_2} = \frac{L_1}{L_2} \tag{9}$$

Short-Run Equilibrium

1. Notations

(1). Consumption. c_{ij}: consumption in region i of a representative region j product. For example, c_{12} is consumption in region 1 of a representative region 2 product.
(2). Iceberg transportation cost: $0 < \tau < 1$.
(3). Price. p_i: the price of a local product in region i. p_i/τ is the price of a product in region i from the other region. For example, for region 1 consumers, the price for region 1 product is p_1, the price for region 2 product is p_2/τ.

2. Relative Demand

The relative demand in region 1 for representative product from both regions is:

$$\frac{c_{11}}{c_{12}} = [\frac{p_1}{p_2/\tau}]^{-\sigma} = [\frac{p_1\tau}{p_2}]^{-\sigma} = [\frac{w_1\tau}{w_2}]^{-\sigma} \tag{10}$$

3. Relative Expenditure

Define z_{11} as the *ratio of region 1 expenditure on region 1's manufactures to that on manufactures from region 2*. Using equation (9), equation (6) and equation (10), the relative expenditure of region 1 is given as:

$$z_{11} = (\frac{n_1}{n_2})(\frac{p_1}{p_2/\tau})(\frac{c_{11}}{c_{12}}) = (\frac{L_1}{L_2})(\frac{w_1\tau}{w_2})^{-(\sigma-1)} \tag{11}$$

Similarly, the relative expenditure of region 2 on region 1's manufactures to region 2's manufactures is:

$$z_{12} = (\frac{n_1}{n_2})(\frac{p_1/\tau}{p_2})(\frac{c_{21}}{c_{22}}) = (\frac{L_1}{L_2})(\frac{w_1}{w_2\tau})^{-(\sigma-1)} \tag{12}$$

4. Total Income

- *Income of workers*: The total income of workers in region 1 equals the total spending from both regions on region 1's manufactures product. Denote Y_1 and Y_2 as total incomes of both regions including peasants' income, recall that μ is the fraction of expenditure on manufactures:

$$w_1 L_1 = \frac{z_{11}}{1+z_{11}}\mu Y_1 + \frac{z_{12}}{1+z_{12}}\mu Y_2 = \mu[\frac{z_{11}}{1+z_{11}}Y_1 + \frac{z_{12}}{1+z_{12}}Y_2] \tag{13}$$

Similarly, the income of region 2 workers is:

$$w_2 L_2 = \frac{1}{1+z_{11}}\mu Y_1 + \frac{1}{1+z_{12}}\mu Y_2 = \mu[\frac{1}{1+z_{11}}Y_1 + \frac{1}{1+z_{12}}Y_2] \tag{14}$$

- *Total income*: Assume that the wage of peasants is the numeraire, i.e., normalized to 1, thus the total income of each region is:

$$Y_1 = \frac{1-\mu}{2} + w_1 L_1 \tag{15}$$

$$Y_2 = \frac{1-\mu}{2} + w_2 L_2 \tag{16}$$

5. Short-Run Equilibrium

Equations (11) - (16) characterize the short-run equilibrium that determines a sequence of variables: $\{w_1, w_2, z_1, z_2, Y_1, Y_2\}$, given the distribution of labor between region 1 and region 2, i.e., L_1 and L_2.

(1). When $L_1 = L_2$, we must have $w_1 = w_2$.

(2). When labor is moving from region 2 to region 1, the relative wage w_1/w_2 can either increase or decrease. There are two opposing effects:

- *Home market effect*: Other things equal, the wage rate tend to be higher in the larger market (Krugman, 1980);
- *Competition effect*: Workers in the region with smaller manufactures labor force will face less competition for local peasant market.

Long-Run Equilibrium

1. Share of Manufacturing Labor force

Recall from equation (3) that $L_1 + L_2 = \mu$, the fraction of manufacturing labor force in region 1 is denoted as f, i.e., $f = L_1/\mu$.

$(1-f)$ is the fraction of workers region 2.

2. Price Index

The true price index of manufacturing goods for consumers residing in region 1 is:

$$P_1 = [f w_1^{-(\sigma-1)} + (1-f)(\frac{w_2}{\tau})^{-(\sigma-1)}]^{-1/(\sigma-1)} \tag{17}$$

And similarly for consumers residing in region 2, the price index is:

$$P_2 = [f(\frac{w_1}{\tau})^{-(\sigma-1)} + (1-f) w_2^{-(\sigma-1)}]^{-1/(\sigma-1)} \tag{18}$$

From equation (17) and (18), when $w_1 = w_2$, or wage rate are equal in two regions, a shift of workers from region 2 to region 1 will lower P_1, the price index in region 1; and raise P_2, the price index in region 2[5] (*price index effect*).

3. Real Wage Rate

The real wage rates, denoted as ω, in each region are:

$$\omega_1 = w_1 P_1^{-\mu} \tag{19}$$

and

$$\omega_2 = w_2 P_2^{-\mu} \tag{20}$$

Following previous discussion, when wage rates are equal, an increase in f will raise real wages in region 1 relative to those in region 2, i.e., ω_1/ω_2 increases with f.

4. Relative Real Wage Rate

How does relative real wage, ω_1/ω_2, change with f?

- *Symmetric Case*: If $f = 0.5$, that is when the two regions have equal number of workers, they offer equal real wage rates, $\omega_1/\omega_2 = 1$.

- *Regional Convergence*: If ω_1/ω_2 decreases with f, i.e., the relative real wage is lower when work force is larger, then workers tend to migrate out of the region with larger worker force. (One force: *degree of competition for local peasant market*)

- *Regional Divergence*: If ω_1/ω_2 increases with f, i.e., the relative real wage is higher when work force is larger, then workers tend to migrate into the region with larger worker force. (Two forces: *home market effect* and *price index effect*)

Question: Which forces dominate?
Answer: Parameters matter: the share of expenditure on manufactured goods, μ; the elasticity of substitution among products, σ; and the fraction of a good shipped that arrives, τ. Depending on the values of these parameters we may have either regional convergence or regional divergence.

[5]When wage rates are equal, as $w_1 > \frac{w_2}{\tau}$, an increase in f implies a higher weight for w_1 in price index P_1, and similarly, in P_2 a higher f implies a higher weight for $\frac{w_1}{\tau}$ such that $\frac{w_1}{\tau} < w_2$.

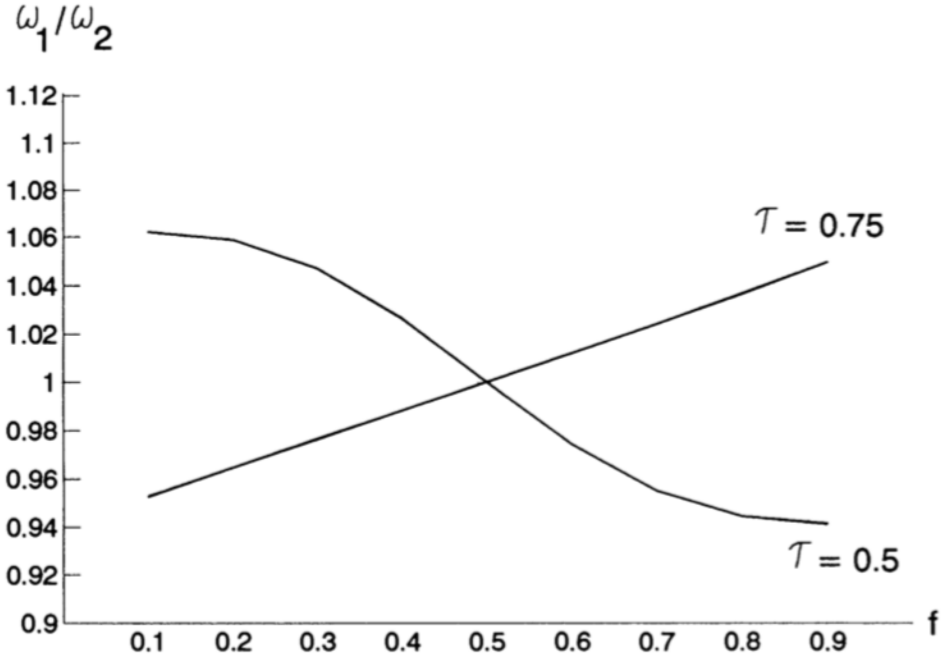

FIG. 1

5. A Simple Numerical Exercise

Given some set of parameter values, i.e., [$\sigma = 4$, $\mu = 0.3$, $\tau = 0.5$] or [$\sigma = 4$, $\mu = 0.3$, $\tau = 0.75$], the model can be easily solved (and compared).

- When $\tau = 0.5$ (high transportation cost), the relative real wage will decline with f. Thus in the case we can expect regional convergence that the geographical distribution of manufacturing resemble that of agriculture.
- When $\tau = 0.75$ (low transportation cost), the relative real wage will decline with f. Thus in the case we can expect regional divergence that manufacturing workers concentrate on one region.
- The numerical analysis on effect of transportation cost can be summarized by the Fig. 1.

Complete Agglomeration Equilibrium

Suppose that all workers are concentrated in region 1.

1. Value of Sales of Region 1 Firm

Since a share of total income μ is spent on manufactures and all this share goes to region 1, we must have:

$$\frac{Y_2}{Y_1} = \frac{1-\mu}{1+\mu} \tag{21}$$

And each manufacturing firm will have a value of sales equal to (let n be the total number of manufacturing firms):

$$V_1 = \frac{\mu(Y_1 + Y_2)}{n} \tag{22}$$

which is just enough to allow each firm to make zero profits *due to free entry*.

Question: Can manufacturing concentration in region 1 be an equilibrium?
Answer: If it is possible for an individual firm to make positive profit by migrating to region 2[6], manufacturing concentration in region 1 is not an equilibrium. If it is not possible, then it is an equilibrium.

When $f = 1$, price index derived from equation (17) and (18) becomes:

$$P_1 = [w_1^{-(\sigma-1)}]^{-1/(\sigma-1)} = w_1$$

and

$$P_2 = [(\tfrac{w_1}{\tau})^{-(\sigma-1)}]^{-1/(\sigma-1)} = w_1/\tau$$

In order to produce in region 2, a firm must be able to attract workers with offered real wage at least as high as in region 1, i.e., $\omega_2 = \omega_1$. As $\omega_i = w_1 P_i^{-\mu}$, the equality of real wage implies that the nominal wage must satisfy:

$$\frac{w_2}{w_1} = (\frac{1}{\tau})^{\mu} \tag{23}$$

2. Value of Sales of Region 2 Firm

From equation (10) and (11) we have shown that relative demand for region 2 manufactures goods from region 1 consumers is

[6]We denote the firm as a *defecting firm*

$$\frac{c_{12}}{c_{11}} = (\frac{p_2/\tau}{p_1})^{-\sigma}$$

and therefore the relative expenditure on region 2 manufactures goods from region 1 consumers is

$$\frac{c_{12}p_2/\tau}{c_{11}p_1} = (\frac{p_2/\tau}{p_1})^{-(\sigma-1)} = (\frac{w_1\tau}{w_2})^{-(\sigma-1)}$$

Similarly, we can show that the relative expenditure on region 2 manufactures goods from region 2 consumers is $(\frac{w_2}{w_1/\tau})^{-(\sigma-1)}$. As fraction of total income μ is spent on manufactures goods, the value of the region 2 firm's sale will be

$$V_2 = \frac{1}{n}[\frac{w_1\tau}{w_2})^{-(\sigma-1)}\mu Y_1 + (\frac{w_2}{w_1/\tau})^{-(\sigma-1)}\mu Y_2] \tag{24}$$

Intuitively, equation (24) show that transportation costs work to the region 2 firm's disadvantage in its sale to region 1 but work to its advantage in sales to region 2.

3. Ratio of the Value of Sales

From equation (22), (23) and (24), the ratio of the value of sales by the defecting firm to the sales of a representative region 1 firm can be derived as

$$\frac{V_2}{V_1} = 0.5\tau^{\mu(\sigma-1)}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)})] \tag{25}$$

4. Defecting Conditions

It's the real value of sales that matters. As price index is higher in region 2, it is profitable for a firm to defect (from region 1 to region 2) only if $V_2/P_2^\mu > V_1/P_1^\mu$, or $V_2/V_1 > P_2^\mu/P_1^\mu$. As $P_2 = P_1/\tau$, or $P_2^\mu/P_1^\mu = \tau^{-\mu}$, we have the following defecting condition that $v > 1$ where v is defined as:

$$v = \frac{V_2/V_1}{\tau^{-\mu}} = 0.5\tau^{\mu\sigma}[(1+\mu)\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}] \tag{26}$$

In other word, when $v < 1$, it is unprofitable for a firm to begin production in region 2 if all other manufacturing production is concentrated in region 1. Thus $v < 1$ is the condition for complete concentration of manufactures production in region 1 being an equilibrium.

Equation (26) is the key equation for analytical results. For a given set of parameter values we can use equation (26) to judge whether concentration is possible or not. Further, equation (26) defines a set of *critical values* of parameters that divide between concentration and non-concentration. It is necessary, then, to examine how v changes with each parameter.

- *Impact of parameter μ*: expenditure share on manufactures goods

$$\frac{\partial v}{\partial \mu} = v\sigma(ln\tau) + 0.5\tau^{\sigma\mu}[\tau^{\sigma-1} - \tau^{1-\sigma}] < 0 \tag{27}$$

  That is, the larger the share of income spent on manufactured goods (μ), the lower the relative sales of the defecting firm, thus the more likely concentration makes an equilibrium. There are two reasons behind this: a). From equation (23) it can been seen that workers demand a larger wage premium in order to move to the second region; b). The larger the share of expenditure on manufactures, the larger the relative size of the region 1 market and hence the stronger the *home market effect.*

- *Impact of parameter τ*: transportation cost
  (1) From equation (26), it can be seen that when $\tau = 1$, $v = 1$: When transportation costs are zero, location is irrelevant.
  (2) From equation (26), it can be been that when τ is small, v approaches $(1-\mu)\tau^{1-\sigma(1-\mu)}$, which must exceed 1 unless σ and μ take extreme (and implausible) values.
  (3) Take derivative w.r.t τ:

$$\frac{\partial v}{\partial \tau} = \frac{v\sigma\mu}{\tau} + 0.5\tau^{\sigma\mu-1}[(1+\mu)\tau^{\sigma-1} - (1-\mu)\tau^{1-\sigma}] \tag{28}$$

  When τ is close to 1, $\frac{\partial v}{\partial \tau}$ is positive. Taken together, above three cases indicate the shape of v as a function τ is similar to *Fig. 2* below. The message is clear that at the critical point when v=1, $\frac{\partial v}{\partial \tau}$ is negative, which suggests that reducing transportation cost (higher τ) from the critical point will lead to manufacturing concentration.

- *Impact of parameter σ*: elasticity of substitution

$$\frac{\partial v}{\partial \sigma} = (ln\tau)\{\mu v + 0.5\tau^{\sigma\mu}[(1+\mu)\tau^{\sigma-1} - (1-\mu)\tau^{1-\sigma}]\} = ln(\tau)(\frac{\tau}{\sigma})(\frac{\partial v}{\partial \tau}) \tag{29}$$

  As we have shown $\frac{\partial v}{\partial \tau}$ is negative around the critical point, this also implies that $\frac{\partial v}{\partial \sigma}$ is positive. Therefore, a lower elasticity of substitution (σ), which also implies larger economies of scale in equilibrium from equation (8), will increase the probability of manufacturing concentration.

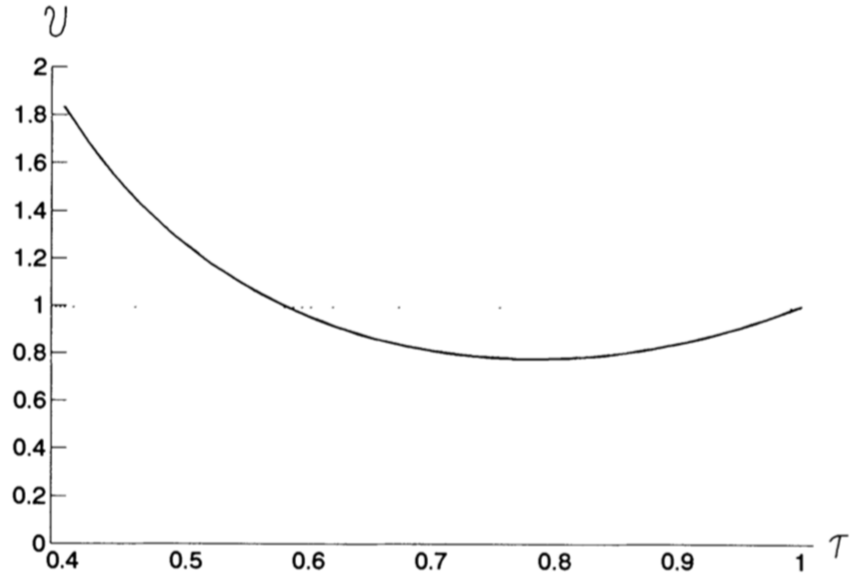

FIG. 2

4 Summary

We have presented a model of possible core-periphery patterns in a two regions economy. There are economies of scale in production of manufactured goods and there are transportation costs.
Because of economies of scale, there is only one producer of each variety. Because of transportation costs, firms will have a tendency to establish in the largest market.

In this model a driving force is mobile labor. Workers move to the region in which real wages are the highest. Firms want to establish in the region where market access is best. Market access is best where firms and workers are already located.
A countervailing force is the incentive to serve distant markets which are populated by land-tied peasants.

For specific parameter values, in particular with low transportation costs(τ), significant economies of scale(σ) and a large share of manufacturing goods(μ) in the economy, a core-periphery pattern is a possible outcome. In such situations, all manufacturing production agglomerates in one region (core) while the periphery becomes de-industrialised.

5 Discussion *

The previous two papers (Krugman, 1979 and 1980) are about international trade, notably intra-industry trade, whereas the last paper extends the analysis by endogenizing the spatial allocation of economic activity, making it the core model of the new economic geography literature.

Krugman (1979) analyses what happens in an economy that is characterized by increasing returns to scale and imperfect competition if countries start to trade. In Krugman (1980) transport costs are introduced and basically added to the increasing returns framework of the 1979 paper. This addition gives rise to the so-called home market effect, which then forms the starting point and backbone of Krugman (1991). In Krugman (1991) the combination of the home market effect with interregional labor mobility endogenizes the location decisions not only of firms but also of footloose workers and hence, unlike his 1980 model, endogenizes the spatial allocation of both supply and demand, and this may give rise to centreperiphery equilibria.

In Krugman (1991), space is deliberately homogeneous and the resulting economic geography is an outcome of the model. By adding interregional labor mobility to his 1980 trade model, Krugman (1991) is a trade model as well as a location model. In Krugman (1991), integrating international trade with intra-national or regional economics is achieved.