Pecuniary Externalities in Economies with Financial Frictions

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Pecuniary Externality

- concept
  - actions of an economic agent $\rightarrow$ market price (*pecuniary*)
  - example: your action of buying a house drives up housing price
  - channel: price (not resources)
- welfare implication
  - complete market: irrelevant (Pareto efficient)
  - incomplete market: relevant (MU/MP of agents $\neq$).
- particularly true for financial economics
  - financial constraint: pecuniary externalities $\Rightarrow \Delta price \Rightarrow$ first-order welfare implications
  - phenomena: fire sales and financial amplification etc.
  - justification for macro-prudential regulation

Pecuniary Externality

- types of pecuniary externality
  - distributive externalities
    - zero-sum in a give state;
    - relevant when MRS between states $\neq$ among agents;
    - example: fire sales and terms of transaction
  - collateral externalities
    - asset price $\rightarrow$ financial constraint
    - example: fire sales and financial constraint

Welfare Implications

- Questions:
  - Is the economy with fire sale always constrained inefficient ?
  - Is the sign of distributive externality always positive/negative?
  - Is collateral externality always consistent with over-borrowing?
    ... with over-investment/under-investment ?

- Quick Answers:
  - fire sales and financial amplification can be constrained efficient
  - distributive externality can flip sign
  - collateral externality $\Rightarrow$ over-borrowing
    ... $\Rightarrow$ over-investment *or* under-investment

A Generalized Theory of Pecuniary Externality

- sign of pecuniary externality:
    - distributive externalities: 3 sufficient statistics
        - difference in MRS of agents
        - trading position of capital and financial assets
        - sensitivity of equilibrium price to changes in sectoral state variables
    - collateral externalities: 3 sufficient statistics
        - shadow value of binding financial constraint (+)
        - sensitivity of financial constraint to asset price (+)
        - sensitivity of equilibrium price to changes in sectoral state variables

A Canonical Model of Kiyotaki and Moore

- two agents $i \in I$ of measure 1
  - borrower (b): productive, financially constrained etc.
  - lender (l)
- two goods
  - consumption good ("numeraire")
  - capital good
- three periods with uncertainty on aggregate state $\omega \in \Omega$
  - date 0
  - date 1 (ω)
  - date 2 (ω)
- preference: time separable utility function

$$U = E_0[\sum_{t=0}^{2} \beta^t u(c_t)] \tag{1}$$

Timeline

- date 0
  - receive endowment e_0
  - consumption: c_0
  - investment: $h(k_1) \rightarrow k_1$
  - contingent security: $E_0[m_1^\omega x_1^\omega]$
- date 1
  - receive $e_1^\omega + x_1^\omega + F_1^\omega(k_1)$ consumption good
  - consumption: c_1^ω
  - buy/sell capital good: $q^\omega \Delta k = q^\omega(k_2 - k_1) \rightarrow k_2$
  - non-contingent security: $m_2^\omega x_2^\omega$ (m_2^ω: market discount factor)
- date 2
  - receive $e_2^\omega + x_2^\omega + F_2^\omega(k_2)$ consumption good
- budget constraint

$$\begin{aligned} e_0^i &= c_0^i + h^i(k_1^i) + E_0[m_1^\omega x_1^{i,\omega}] \\ e_1^{i,\omega} + x_1^{i,\omega} + F_1^{i,\omega}(k_1^i) &= c_1^{i,\omega} + q^\omega(k_2^{i,\omega} - k_1^i) + m_2^\omega x_2^{i,\omega} \\ e_2^{i,\omega} + x_2^{i,\omega} + F_2^{i,\omega}(k_2^{i,\omega}) &= c_2^{i,\omega} \end{aligned} \tag{2}$$

Financial Constraint

- date 0: borrower's security holdings x_1^b s.t. a convex set:

$$\Phi_1^b(x_1^b, k_1^b) \geq 0 \tag{3}$$

interpretation:

  - $\Phi_1^b(x_1^b, k_1^b) = 0$: complete market
  - $\Phi_1^b(x_1^b, k_1^b) := (x_1^{b,\omega})_{\omega \in \Omega}$: no financial trade

- date 1: borrower's security holdings $x_2^{b,\omega}$ s.t. a convex set [1]

$$\Phi_2^{b,\omega}(x_2^{b,\omega}, k_2^{b,\omega}; q^\omega) \geq 0 \tag{4}$$

interpretation:

  - $\Phi_2^{b,\omega}() = x_2^{b,\omega} + \phi^\omega q^\omega k_2^{b,\omega}$: partial collateralization of asset

[1] $\partial \Phi_2^{b,\omega} / \partial q^\omega \geq 0$: a higher price of the capital good *weakly* relaxes the financial constraint.

Interpretations of Financial Constraint

- borrower = more productive entrepreneurs:
  financial constraint → inefficient sales of capital
- borrower = financial intermediary + firm:
  financial constraint → external finance ↓ → inefficient sales of capital
- borrower = homeowners holding mortgages:
  financial constraint → foreclosure → house depreciation ↑ → housing price ↓

Decentralized Equilibrium: Date 1

Date 1 Problem[2]:

$$V(n^{i,\omega}, k_1^i; N^\omega, K_1) = \max_{c_1 \geq 0, c_2 \geq 0, k_2, x_2} u(c_1^{i,\omega}) + \beta u(c_2^{i,\omega}) \tag{5}$$

s.t. two budget constraint (multiplier $\lambda_1^{i,\omega}$ and $\lambda_2^{i,\omega}$)

$$n^{i,\omega} \equiv e_1^{i,\omega} + x_1^{i,\omega} + F_1^{i,\omega}(k_1^i) = c_1^{i,\omega} + q^\omega(k_2^{i,\omega} - k_1^i) + m_2^\omega x_2^{i,\omega} \tag{6}$$

$$e_2^{i,\omega} + x_2^{i,\omega} + F_2^{i,\omega}(k_2^{i,\omega}) = c_2^{i,\omega} \tag{7}$$

a financial constraint (multiplier $\kappa_2^{b,\omega}$) [3]

$$\Phi_2^{b,\omega}(x_2^{b,\omega}, k_2^{b,\omega}; q^\omega) \geq 0 \tag{8}$$

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[2] Date 2 problem is trivial: agents consume and capital fully depreciates.

[3] multiplier of lender is $\kappa_2^{l,\omega} = 0$.

Decentralized Equilibrium: Date 1

- F.O.C. on security (debt)

$$m_2\lambda_1^i = \beta\lambda_2^i + \underbrace{\kappa_2^i(\partial\Phi_2^i/\partial x_2^i)}_{\text{shadow value of unit debt}} \tag{9}$$

$\Rightarrow$ If financial constraint is slack, $\frac{\beta\lambda_2^l}{\lambda_1^l} = m_2^\omega$ (market discount).

$\Rightarrow$ ... binding, $\frac{\beta\lambda_2^b}{\lambda_1^b} < m_2^\omega \rightarrow$ capital value $\downarrow \rightarrow$ *fire sale discount*

- F.O.C. on capital

$$q\lambda_1^i = \beta\lambda_2^i F_2'(k_2^i) + \underbrace{\kappa_2^i(\partial\Phi_2^i/\partial k_2^i)}_{\text{benefit of relaxing constraint}} \tag{10}$$

$\Rightarrow$ If ... binding, $\kappa_2^i(\partial\Phi_2^i/\partial k_2^i) > 0 \rightarrow$ *collateral value*

- Equation (9) and (10) define price of bond (m_2) & capital (q)

Decentralized Equilibrium: Date 1

- *un-internalized* welfare effects of sector-wide state N^{ω} [4]

$$\frac{dV^i}{dN^j} = \lambda_1^i \underbrace{[-\frac{\partial q^{\omega}}{\partial N^j}\Delta K_2^i - \frac{\partial m_2^{\omega}}{\partial N^j}X_2^i]}_{\equiv D^i_{N^j}(\text{distributive effect})} + \kappa_2^i \underbrace{[\frac{\partial \Phi_2^i}{\partial q^{\omega}}\frac{\partial q^{\omega}}{\partial N^j}]}_{\equiv C^i_{N^j}(\text{collateral effect})} \tag{11}$$

- *un-internalized* welfare effects of sector-wide state K_1

$$\begin{aligned} \frac{dV^i}{dK_1^j} =& \lambda_1^i \underbrace{[F'(K_1^i)D^i_{N^j} - \frac{\partial q^{\omega}}{\partial K_1^j}\Delta K_2^i - \frac{\partial m_2^{\omega}}{\partial K_1^j}X_2^i]}_{\equiv D^i_{K^j}(\text{distributive effect})} \\ &+ \kappa_2^i \underbrace{[F_1'(K_1^i)C^i_{N^j} + \frac{\partial \Phi_2^i}{\partial q^{\omega}}\frac{\partial q^{\omega}}{\partial K_1^j}]}_{\equiv C^i_{K^j}(\text{collateral effect})} \end{aligned} \tag{12}$$

[4] In symmetric equilibrium, $N^i = n^i$, but individual agents take sector-wide state variable as given. Similarly, $K_1^i = k_1^i$.

Decentralized Equilibrium: Date 1

- distributive effects
  - j sector-wide state variables $N^j/K^j \to$ equilibrium price $\to$ marginal wealth redistribution towards sector i
  - zero-sum across all agents at *given* state

$$\sum_i D^i_{N^j} = 0 \quad \& \quad \sum_i D^i_{K^j} \tag{13}$$

- collateral effects
  - j sector-wide state variables $N^j/K^j \to$ equilibrium price $\to$ tightness of borrowing constraint (faced by i)
  - generally not zero-sum across agents at *given* state
- source of pecuniary externalities
  - individual agents internalize $\partial V^i/\partial n^i \equiv \lambda^i_1$ and $\partial V^i/\partial k^i_1$
  - individual agents do not internalize $\partial V^i/\partial N^i$ and $\partial V^i/\partial K^i_1$

Decentralized Equilibrium: Date 0

- optimization problem of agent

$$\max_{c_0,k_1,x_1} u(c_0) + \beta E_0[V^{i,\omega}(n^{i,\omega}, k_1^i; N^\omega, K_1)] \tag{14}$$

- s.t. budget constraint and financial constraint at date 0

$$e_0^i = c_0^i + h^i(k_1^i) + E_0[m_1^\omega x_1^{i,\omega}] \tag{15}$$

$$\Phi_2^{b,\omega}(x_2^{b,\omega}, k_2^{b,\omega}; q^\omega) \geq 0 \tag{16}$$

- Euler equations (suppressing i, ω)[5]

$$m_1^\omega \lambda_0 = \beta\lambda_1 + \kappa_1[\partial\Phi_1/\partial x_1] \tag{17}$$

$$h'(k_1)\lambda_0 = E_0[\beta\lambda_1(F_1^i)'(k_1) + q^\omega] + \kappa_1[\partial\Phi_1/\partial k_1] \tag{18}$$

[5] Similarly to date 1 problem, $\kappa_1 = 0$ implies $m_1^\omega = \beta\lambda_1/\lambda_0$, i.e. intertemporal marginal rates of substitution of all agents are equalized absent of financial friction.

Application 1

- Is the economy with fire sales always constrained inefficient ?
- An economy with fire sales can be constrained efficient, i.e., when
  - risk markets are complete, and
  - financial constraints do not depend on prices.

Assumptions

- preference and endowment: no time discount
- investment technology at date 0: $h(k_1) \to k_1$
  - borrowers: $h^b(k_1) = \alpha \frac{k^2}{2}$
  - lenders: $h^l(k_1) = +\infty \quad (\to k_1^l = 0)$
- saving at date 0:
  - Arrow securities are available and no financial constraint $\Phi_1^b \equiv 0$
  - risk market is complete
- production technology at date 1:
  - borrowers: $F_t^b(k) = A_t k$
  - lenders: $F_t^{l\prime}(0) = A_t$ and $F_t^{l\prime\prime}(k) < 0$
- financial constraint at date 1: (independent of q^ω)

$$\Phi_2^b(x_2^b, k_2^b) := x_2^b + \phi F_2^b(k_2^b), \quad \phi \in (0,1) \tag{19}$$

Problem at Date 1

- lenders → borrowers at date 1 via borrowing + fire sales

$$z = m_2 x_2^l + q k_2^l \tag{20}$$

- "supply of fund": lenders gain at date 2 via repayment + production

$$\rho(z) = x_2^l + F^l(k_2^l) \tag{21}$$

- resources given up by borrower:

$$\gamma(z) = x_2^l + A_2 k_2^l \tag{22}$$

- dead-weight loss of fire sales

$$\delta(z) = \gamma(z) - \rho(z) = A_2 k_2^l - F^l(k_2^l) \tag{23}$$

- market prices (pinned down by lender)

$$m_2 = \frac{\lambda_2^l}{\lambda_1^l} = \frac{u'(e_2^l + \rho(z))}{u'(n^l - z)} \tag{24}$$

$$q = m_2 F^{l\prime}(k_2^l) \tag{25}$$

Problem at Date 1

- region 1: unconstrained equilibrium
  - slack financial constraint $\Rightarrow$ no fire sales $\Rightarrow k_2^l = 0$
  - $z = m_2 x_2^l \Rightarrow \rho(z) = \gamma(z) = x_2^l \Rightarrow m_2 = \rho(z)/z \Rightarrow$

$$zu'(n^l - z) = \rho(z)u'(e_2^l + \rho(z))$$

    that defines a supply curve $\rho = \rho(z)$.
  - $\partial\rho/\partial z > 0$ under some conditions.

- region 2: constrained equilibrium (w. fire sales)
  - binding financial constraint $\Rightarrow x_2^l = m_2 \phi A_2(k_1^b - k_2^l) \Rightarrow$

$$zu'(n^l - z) = u'(e_2^l + \phi A_2(k_1^b - k_2^l) + F^l(k_2^l))[\phi A_2(k_1^b - k_2^l) + k_2^l F^{l\prime}(k_2^l)]$$

    that defines a "demand" curve for fire sales $k_2^l = k(z)$
  - $\partial k/\partial z > 0$ under some conditions.
  - $\rho(z)$, $\gamma(z)$ and $\delta(z)$ are strictly increasing with z.

Problem at Date 1

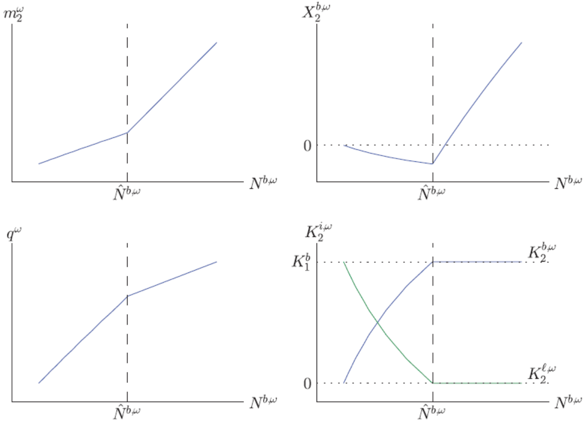

FIGURE 1
Date 1 equilibrium

[6]

[6] Threshold $N^{\hat{b},\omega}$ defines cut-off of net worth of borrowers above which borrowers are unconstrained, holding $N^{l,\omega}$ and K_1 unchanged.

Constrained Efficiency

- problem at date 0: $(MRS^{i,\omega} = \beta\lambda_1^{i,\omega}/\lambda_0^i)$

$$\Delta MRS^{ij,\omega} = MRS^{i,\omega} - MRS^{j,\omega} = 0 \qquad (26)$$

$\rightarrow N^b > \hat{N}^b$: slack financial constraint $\rightarrow$ first-best allocation
$\rightarrow N^b < \hat{N}^b$: binding financial constraint $\rightarrow$ constrained efficient

- collateral externality: absent from financial constraint (19)
- distributive externality: absent due to complete risk market
  - (b/w date 0 and 1) agents optimally share risks before fire sales $\rightarrow \Delta MRS^{ij,\omega} = 0$
  - (b/w date 1 and 2) given ω agent's welfare is monotonic with z $\rightarrow$ any change in flow of resources z hurts one party.
  - no scope for welfare improvement using distributive measures.
- this decentralized equilibrium with fire sales: constrained efficient

Lesson from Application 1

- sign (magnitude) of distributive externalities
  product of 3 (sufficient) variables
  - D1: difference in MRS of agents ($\Delta MRS^{ij,\omega}$)
  - D2: trading position of capital and financial assets ($\Delta k^{i,\omega}$)
  - D3: sensitivity of eqm price to sectoral state variables ($\frac{\partial q^{\omega}}{\partial N^{b,\omega}}, \frac{\partial q^{\omega}}{\partial K^{b}}$)
- In Application 1: signs of D2 and D3 become irrelevant as D1 $= 0$.
- In Application 2: sign of D2 can be either $+$ or -.
- In Application 3: sign of D1 can be either $+$ or -.

Application 4

- Is collateral externality always consistent with over-borrowing?
  Is collateral externality always consistent with over-investment?

- Collateral externality is always consistent with over-borrowing.
  Collateral externality can be consistent with both over-investment and under-investment.

Assumptions

- preference and endowment: no time discount
  - lender's preference: $U^l = c_0^l + c_1^l + c_2^l$
  - lender's endowment: $e_t^l = +\infty \quad (\rightarrow m_2 = 1)$
  - borrower's preference: $U^b = \log c_0^b + \log c_1^b + c_2^b$
  - borrower's endowment: $1 \geq e_0^b > e_1^b = e_2^b = 0$
- investment technology at date 0: $h(k_1) \rightarrow k_1$
  - borrowers: $h^b(k_1) = \alpha \frac{k^2}{2}$
  - lenders: $h^l(k_1) = +\infty \quad (\rightarrow k_1^l = 0)$
- perfect foresight economy with no uncertainty:
  - $\equiv$ complete risk market
- production technology at date 1:
  - borrowers: $F_t^b(k) = A_t k$, with $\alpha > A_1 + A_2 > 0$ and $A_2 > 0$ [7]
  - lenders: $F_t^l(k) = 0 \quad (\rightarrow k_2^l = 0)$

[7] We allow A_1 to be negative, capturing maintenance cost of capital.

Assumptions

- distributive externalities ($D^i_{N^j}$ and $D^i_{K^b}$) are zero by assumption
  - constant bond price: $m_2 = 1$
  - no capital trade *between sectors*: $k^l_t = 0$
- we focus on collateral externalities
  - at date 0 no financial friction
  - at date 1 borrowers can borrow up to ϕ fraction of asset value [8]

$$\Phi^b_2(x^b_2, k^b_2; q) : x^b_2 + \phi q k^b_2 \geq 0 \tag{27}$$

[8] We assume ϕ is less than $\frac{1}{1+A_2}$, a property we use later.

Problem at Date 1

- lenders: $m_2 = 1$
- borrowers:

$$V^b(n^b, k_1^b; N, K_1) = \max_{x_2^b, k_2^b} u(n^b - q\Delta k_2^b - x_2^b) + x_2^b + A_2 k_2^b + \kappa_2^b(x_2^b + \phi q k_2^b) \quad (28)$$

f.o.c. w.r.t k_2^b and x_2^b

$$q[u'(c_1^b - \phi\kappa_2^b)] = A_2 \quad (29)$$

$$u'(c_1^b) = 1 + \kappa_2^b \quad (30)$$

- capital price $(q(C_1^b))$ in equilibrium:

$$q = \frac{A_2 C_1^b}{1 - \phi + \phi C_1^b} \quad (31)$$

Problem at Date 1

- first-best allocation:
  - $C_0^b = C_1^b = 1$ and $K_t^{b*} = \frac{A_1+A_2}{\alpha}$
  - feasible if $X_2^b \geq -\phi q K_2^b$, or $N^b \in [1-\phi A_2 K_1^b, +\infty)$
- we focus on constrained equilibrium:
  - $N^b \in (0, 1-\phi A_2 K_1^b)$
  - binding financial constraint: $X_2^b = -\phi q K_2^b$
  - budget constraint implies a unique $C_1^b = C_1^b(N^b, K_1^b)$ from

$$C_1^b = N^b + \phi q K_1^b = N^b + \phi K_1^b \frac{A_2 C_1^b}{1-\phi+\phi C_1^b} \tag{32}$$

  - consumption $C_1^b(N^b, K_1^b)$ increases in both N^b and K_1^b
  - price of capital $(q(C_1^b(N^b, K_1^b)))$ increases in both N^b and K_1^b.

Collateral Externality

- collateral externality:

$$C^b_{N^b} = \phi K^b_1 \frac{\partial q}{\partial N^b} > 0 \tag{33}$$

$$C^b_{K^b_1} = \phi K^b_1 (A_1 \frac{\partial q}{\partial N^b} + \frac{\partial q}{\partial K^b_1}) = \frac{\phi K^b_1 q'(C^b_1)}{1 - \phi K^b_1 q'(C^b_1)} (A_1 + \phi q) \tag{34}$$

- sign of collateral externality $C^b_{N^b}$: positive [9]
  - $\Rightarrow$ borrowers engage in over-borrowing
  - planner: saving $\uparrow \Rightarrow$ net worth $\uparrow \Rightarrow$ q $\uparrow \Rightarrow$ financial constraint $\downarrow$
- sign of collateral externality $C^b_{K^b_1}$ = sign of $(A_1 + \phi q)$
  - $A_1 < -\phi q$: capital $\uparrow \rightarrow$ liquid net worth of borrower *sector* $\downarrow \rightarrow$ q $\downarrow$ $\rightarrow$ negative collateral effect
  - cut-off $\hat{A}_1 : \hat{A}_1 + \phi q(\hat{A}_1) = 0$

[9] An assumption is made before in the paper that $\partial q / \partial N^b > 0$. Violating this assumption may lead to multiple equilibria.

Comparative Statics

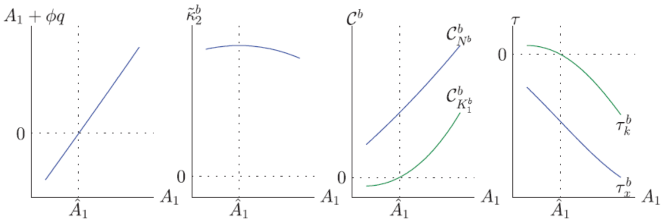

FIGURE 4
Components of optimal taxes τ_x^b, τ_k^b in Application 4

- borrowers over-invest if $A_1 < \hat{A}_1$
- borrowers invest efficiently if $A_1 = \hat{A}_1$
- borrowers under-invest if $A_1 > \hat{A}_1$

Lesson from Application 4

- sign (magnitude) of collateral externalities
  product of 3 (sufficient) statistics
  - C1: shadow value of binding financial constraint ($\kappa > 0$)
  - C2: sensitivity of financial constraint to asset price ($\frac{\partial \Phi_2^{\omega}}{\partial q^{\omega}} > 0$)
  - C3: sensitivity of eqm price to sectoral state variables ($\frac{\partial q^{\omega}}{\partial N^{b,\omega}}, \frac{\partial q^{\omega}}{\partial K^{b}}$)

- In Application 4: signs of C3 vary with A_1 while $C1, C2 > 0$

Policy: Corrective Tax

$$\tau_x^{i,\omega} = -\Delta MRS^{ij,\omega} \mathcal{D}_{N^i}^{i,\omega} - \tilde{\kappa}_2^{b,\omega} \mathcal{C}_{N^i}^{b,\omega}, \forall i, \omega$$

$$\tau_k^i = -\mathbb{E}_0 \left[\Delta MRS^{ij,\omega} \mathcal{D}_{K^i}^{i,\omega} \right] - \mathbb{E}_0 \left[\tilde{\kappa}_2^{b,\omega} \mathcal{C}_{K^i}^{b,\omega} \right], \forall i$$

- positive $\tau_x^{i,\omega}$ tax: agent i should carry less wealth toward ω
- negative τ_k^i tax: agent i should invest less in capital
- examples:
  - distributive externality: $\Delta k^{b,\omega} < 0$, $\Delta MRS^{bl,\omega} > 0$, $\frac{\partial q^\omega}{\partial N^{b,\omega}} > 0$ $\Rightarrow \tau_x^{b,\omega} < 0$: borrowers under-save
  - collateral externality: $\kappa^{b,\omega} > 0$, $\frac{\partial \Phi_2^\omega}{\partial q^\omega} > 0$, $\frac{\partial q^\omega}{\partial N^{b,\omega}} > 0$ $\Rightarrow \tau_x^{b,\omega} < 0$: borrowers under-save

Wrap Up

- General and extensible methodology to characterize pecuniary externalities
- Categorize two distinct types:
  - distributive externalities
  - collateral externalities
- Describe suffcient statistics for optimal taxation
- Externalities can generally go either way in principle, although typical situations lead to over-borrowing.