Firm Dynamics and Financial Development

Arellano, Bai and Zhang (2012, JME)

Prepared by Ding Dong
Department of Economics, HKUST

Overview

- firm dynamic: size effects
  - size-growth relation: size $\uparrow \Rightarrow$ growth $\downarrow$
  - size-leverage relation: size $\uparrow \Rightarrow$ leverage $\downarrow$
  - frictionless economy: no size effects
  - theory: financial friction [1]; adjustment cost; trade etc.

[1] Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007) etc.

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- effects *conditional* on
  - firm characteristics: age, sector etc.
  - U.S. economy: industry structure, financial development etc

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- this paper: condition of financial development $\Rightarrow$ size effects
  - cross-country variation
  - financial development $\leftrightarrow$ size-growth, size-leverage
  - quantitative model

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Empirical 1

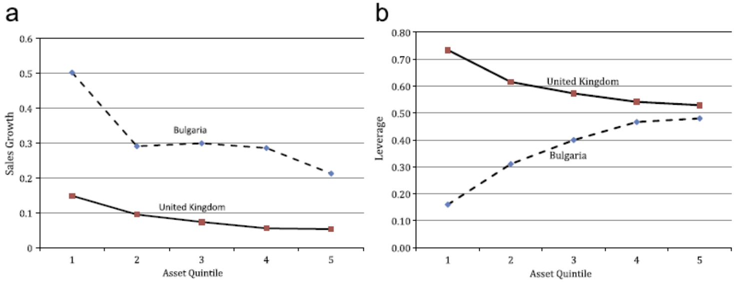

Fig. 1. Firm size, leverage and sales growth. (a) Size and growth. (b) Size and leverage.

- size-growth relation (panel a)
    - small firms grow faster than large firms
    - difference is larger in Bulgaria
- size-leverage relation (panel b)
    - Bulgaria: small firms use less debt financing
    - UK: small firms use more debt financing

Empirical 2

- database: Amadeus
  - 27 European countries
  - 2.6 million firms in non-financial, non-public sectors
- regression:

$$y_{k,c} = \beta_0 + \beta_1 size_{k,c} + \beta_2 size_{k,c} * FD_c + Dummy + v_{k,c} \quad (1)$$

- dependent variables ($y_{k,c}$): growth, leverage
  - growth = growth rates of sales
  - leverage = total debt / total asset
- independent variables: *size*, *FD*, *dummy*
  - size: book value of the firm's total asset
  - FD: development of financial markets
    - average private credit to GDP ratio (+)
    - share of banks' overhead costs in total bank assets (-)
    - coverage of credit bureaus (+)
  - dummy: fixed effects of country, industry and age

Empirical 2

Table 2
Firm leverage, growth and financial development.

	Leverage			Sales growth		
	(1)	(2)	(3)	(1)	(2)	(3)
Size	0.021***	0.014***	0.018***	−0.134***	0.024***	−0.082***
	(0.0002)	(0.0003)	(0.0001)	(0.0016)	(0.0011)	(0.0010)
FD × *Size*	−0.006***	0.050***	−0.005***	0.097***	−1.880***	0.051***
	(0.0002)	(0.0048)	(0.0002)	(0.0013)	(0.0310)	(0.0008)
Adjusted R^2	0.28	0.27	0.28	0.06	0.06	0.06
Observations	2 621 201	2 606 324	2 621 201	2 621 201	2 606 324	2 621 201

Notes: *Size* is measured by the logged asset share of a firm. *FD* denotes financial development, measured by private credit to GDP (1), overhead costs (2) or credit bureau coverage (3). All regressions have a fixed effect at the country × industry × age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

implied y-size coefficient $= \beta_1 + \beta_2 * FD_c$

Country	FD(1)	size-leverage	size-growth
UK	1.42	0.012	0.004
Germany	1.16	0.014	-0.021
Sweden	0.89	0.016	-0.048
Median	*0.47*	*0.018*	*-0.088*
Bulgaria	0.22	0.020	-0.113

Empirical 2

- size–leverage relation
  - median financial market: size $\uparrow \rightarrow$ leverage $\uparrow$
  - financial development $\uparrow \Rightarrow$ size-leverage slope $\downarrow$
- size–growth relation
  - median financial market: size $\uparrow \Rightarrow$ growth $\downarrow$
  - financial development $\uparrow \Rightarrow$ size-growth slope $\uparrow$
- financial development and size effects
  - FD $\uparrow \Rightarrow$ size effects $\downarrow$: small firm $\sim$ large firm
  - FD $\uparrow \Rightarrow$ 'distortion' $\downarrow$ for small firms

Model

- full model
  - idiosyncratic prod shock (permanent and transitory)
  - capital adjustment cost and partial depreciation
  - equity financing: proportional cost
  - debt financing: default risk with partial recovery
  - debt creditor: fixed cost (*proxy for FD*)

- analytical solution w. assumptions

- quantitative solution of full model

Full Model: Technology

Decreasing return to scale technology:

$$y = zK^{\alpha}, \quad 0 < \alpha < 1 \tag{2}$$

- z: idiosyncratic prod
  - z: Markov process, $f(z', z)$
  - $\log(z) = \log(\mu) + \log(\varepsilon)$
  - permanent component (*productivity*): $\{\mu_z^i, i = 1:5\}$
  - stochastic component (*luck*): $\{\varepsilon_l, \varepsilon_h\}$
  - θ: prob of exogenous death
- K: capital stock
  - depreciation: δ
  - net investment: $K' - (1-\delta)K$
  - adjustment cost: $\phi(K' - K)^2/K$
  - degree of friction: ϕ

Full Model: Debt Contract

- debt contract:

$$(B', B'_R) \in \Omega(K', z) \tag{3}$$

B': new loan. B'_R: face value.

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- recovery value if firms default:

$$R(K') = max\{(1-\psi)(1-\delta)K' - \phi K', 0\} \tag{4}$$

- parameters
  - recovery rate: $1-\psi$

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- break-even condition

$$B' + \xi = \frac{B_R(1 - \int \tilde{d} f(z', z)dz') + R(K') \int \tilde{d} f(z', z)dz'}{1 + r} \quad (5)$$

- parameters
    - recovery rate: $1 - \psi$
    - financial intermediation cost: ξ (proxy for financial development)
    - binary default decision: $\tilde{d} = d(K, B_R, z)$

Full Model: Equity

- dividend:

$$D = zK^{\alpha} - B_R + B` - K' + (1-\delta)K - \phi(K'-K)^2/K \quad (6)$$

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- value function:

$$V(K, B_R, z) = \max_{\tilde{d}\in\{0,1\}} (1-\tilde{d})V^c(K, B_R, z) \quad (7)$$

- value function conditional on repayment:

$$V^c(K, B_R, z) = \max_{D, K', (B', B'_R)\in\Omega} (1+\gamma 1_{D<0})D + \beta E_z V(K', B'_R, z') \quad (8)$$

Analytical Solution

- assumptions
  - idiosyncratic prod shock (permanent ~~and transitory~~)
  - ~~capital adjustment cost~~ and ~~partial~~ full depreciation
  - ~~equity financing: proportional cost~~
  - debt financing: default risk with ~~partial~~ no recovery
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$$V^c(K, B_R, z) = \max_{K', B'} zK^{\alpha} - B_R + B' - K' + \beta V(K', B'_R, z) \quad (9)$$

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- assumption: $\beta(1 + r) < 1$ and ξ sufficiently small:

$$K' = K_{fb}(z) : z\alpha K_{fb}^{\alpha-1} = 1 + r \quad (10)$$

Analytical Solution

- debt limit and repayment denoted as $\bar{B}(z)$ and $\bar{B}_R(z)$

$$\bar{B}(z) + \xi = \frac{\bar{B}_R(z)}{1 + r} \tag{11}$$

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- no default at debt limit: $V(K_{fb}, \bar{B}_R, z) = V^c(K_{fb}, \bar{B}_R, z)$

$$V^c(K_{fb}, B_R, z) = [zK_{fb}^{\alpha} - K_{fb} - r\bar{B}(z) - (1+r)\xi]/(1-\beta) \tag{13}$$

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- debt limit derived from:

$$V^c(K_{fb}, B_R, z) = 0 \tag{14}$$

Analytical Solution

- debt limit:

$$\bar{B}(z) = \frac{(1 + r - \alpha)}{r\alpha} K_{fb}(z) - \frac{1 + r}{r} \xi \tag{15}$$

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- leverage ratio:

$$\frac{\bar{B}(z)}{K_{fb}(z)} = \frac{(1+r-\alpha)}{r\alpha} - \frac{1+r}{r}\frac{\xi}{K_{fb}(z)} \tag{16}$$

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- size-leverage relation
  - larger firm $\leftrightarrow$ higher leverage
  - fixed credit cost ξ affects small firm disproportionately
- fixed credit cost $\rightarrow$ size-leverage relation
  - $\xi = 0$: no size effect on leverage
  - $\xi \uparrow$: size effect on leverage $\uparrow$

Full Model: Entrants

- entrant:

$$V^e(K_0, 0, z) = \max_{D, K', (B', B'_R)} (1 + \gamma_e 1_{D<0}) D + \beta E[V(K', B'_R, z')] \quad (17)$$

  subject to

$$D = B' - K' - \phi(K' - K_0)^2 / K_0 \quad (18)$$

  and $z' \sim g(z')$

- mass of project = 1
  - project: exit firms → potential entrants

Full Model: Distribution

- distribution: $s \equiv (K, B_R, z)$

$$\Gamma(s') = \int [1 - d(s)] Q(s, s') f(z', z) \Gamma(s) d(K \times B_R \times z) + \int d(s) Q_e(s') g(z') \Gamma(s) d(K \times B_R \times z) \quad (19)$$

where transition functions are:

$$Q(s', s) = \begin{cases} 1, & \text{if} \quad K'(K, B_R, Z) = K', B'_R(K, B_R, Z) = B'_R \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

and for entrants

$$Q_e(s') = \begin{cases} 1, & \text{if} \quad K'(K_0, 0) = K', B'_R(K_0, 0) = B'_R \\ 0, & \text{otherwise} \end{cases} \quad (21)$$

Calibration

Table 6
Benchmark parameters and target moments.

Calibrated parameters		
Discount factor	β	0.96
Interest rate	r	0.04
Capital depreciation rate	δ	0.10
Technology	α	0.65
Equity issuance cost	γ	0.30
Capital loss after default	ψ	0.25
Death rate	θ	0.072
Shock persistence	ρ	0.86
Estimated parameters		
Permanent productivity	c	0.550
Stochastic shock variance	σ	0.525
Capital adjustment cost	ϕ	0.001
Credit cost	ξ	0.010
Entrant starting capital	K_0	0.002
Entrant equity issuance cost	γ_e	0.130

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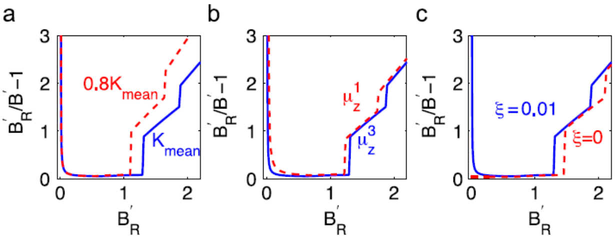
- debt contract: $(B', B'_R) \in \Omega(K', z)$
  - effective interest rate (spread) $= \frac{B'_R}{B'} - 1$
  - spread in U-shape
  - high for small loans: fixed credit cost ξ
  - high for large loans: default risk

Quantitative: Debt Contract



[Figure 2: Sensitivity of Debt Schedule]

- sensitivity to K': collateral effect (panel a)
- sensitivity to μ (panel b)
- sensitivity to ξ (panel c)

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  - high for small loans: fixed credit cost ξ
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- policy rule: $K'(K, B_R, z), D(K, B_R, z), B'(K, B_R, z)$
  - median permanent shock ($\mu = \mu_z^3$)
  - low stochastic shock ($\varepsilon = \varepsilon_l$)
  - average debt level $B = 0.43 * K_{mean}$

Quantitative: Policy Rules

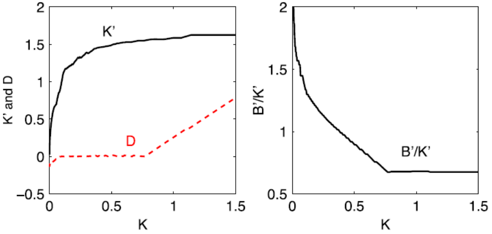

Fig. 3. Policy rules. *Note*: This figure plots the optimal capital choice K', dividends D, and the ratio of the loan choice relative to the capital choice B'/K' as a function of the beginning capital K for a firm with median permanent productivity μ_z^3, stochastic shock ϵ_l and debt at 43% of the average capital across the μ_z^3-firms. All values on the axis are relative to the average capital across the μ_z^3-firms.

2

- smallest firm [0%-20%]
- medium firm [20%-75%]
- largest firm [75%-]

[2]Note: All statistics are normalized by K_{mean}

Quantitative: Model Moments

Table 7
Quantitative model results.

	Bulgaria data		Bulgaria benchmark		Zero credit cost	
	Growth	Leverage	Growth	Leverage	Growth	Leverage
All firms						
Mean	0.32	0.36	0.34	0.48	0.30	0.68
Small firms	0.37	0.26	0.62	0.32	0.34	0.65
Large firms	0.26	0.46	0.05	0.64	0.26	0.71
Difference	0.11	−0.20	0.57	−0.32	0.08	−0.06

- leverage: unproductive vs unlucky
  - unproductive: low permanent shock → high spread → lower leverage
  - unlucky: sequence of low transitory shock → higher leverage
- growth
  - hit by good transitory shock → higher growth → efficient level
- counterfactual: credit cost (ξ)
  - inefficiency: unfavorable debt schedule for small firms

Quantitative: Robustness

- Regression 1:

$$Growth_k = \beta_0 + \beta_1 size_k + e_k$$

  - $\beta_1 < 0$: size-growth relation

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- Regression 2:

$$Leverage_k = \beta_0 + \beta_1 size_k + e_k$$

  - $\beta_1 > 0$: size-leverage relation

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$$Leverage_k = \beta_0 + \beta_1 size_k + e_k$$

  - $\beta_1 > 0$: size-leverage relation

- Regression 3:

$$y_{k,c} = \beta_0 + \beta_1 size_{k,c} + \beta_2 size_{k,c} * (Credit/GDP)_c + e_{k,c}$$

  - y: zero-leverage dummy = 1 if leverage is zero.
  - $\beta_1 > 0$: size-leverage relation
  - $\beta_2 < 0$: financial development $\rightarrow$ size-leverage relation

Conclusion

- benchmark size effects
  - small firms grow faster than large firms
  - small firm use less debt financing than large firms
- as financial development improves
  - growth rate of small firms relative to large firm decreases
  - leverage ratio of small firms relative to large firm increases
- micro-data into macro quantitative model
  - growth and financing patterns
  - across firms and across country

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