Two Paper on Sovereign Defaults

Arellano (2008) and Arellano and Ramanarayanan (2012)

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Introduction

- Sovereign default is more likely in recession or boom?
- Economies issue more short-term or long-term bond in recession?

A framework for sovereign debt and its business cycle implication

International Risk Sharing

- Countries receive i.i.d income shock $\{y_L, y_H\}$, how to smooth consumption?
- 4 common contract arrangements
  - Arrow-Debreu asset market, full commitment
  - Arrow-Debreu asset market, no commitment
  - One-period debt market, full commitment
  - One-period debt market, no commitment

Arrow-Debreu Asset Market, Full Commitment

- Asset market opens once at time 0
- Assets: claims contingent on all possible realization of shock
- Equilibrium: all countries consume a constant path of consumption
- Contract: $y_H \rightarrow$ transfer out, $y_L \rightarrow$ receive transfer

Arrow-Debreu Asset Market, No commitment

- Country can walk away at the cost of exclusion form the contract
- Suppose today income is y_H, contract regulates "contribute"
- Default decision balances
    - benefit: no "contribute" today
    - cost: go to Autarky and get a bumpy future consumption
- **More likely to default in boom**
    - default benefit: benefit(y_H) $>$ benefit(y_L)
    - default cost: cost(y_H) $=$ cost(y_L)

One-period Debt Market, Full Commitment

- One-period bond market opens at the beginning of each period
- State: country wakes up each period with a bond or debt position
- Equilibrium: today income$\rightarrow$ forecast future income $\rightarrow$ consumption plan
- Consumption: less volatile than Autarky, more volatile than Arrow-Debreu

One-period Debt Market, No Commitment

- Suppose today NIIP is negative (debt), contract regulates "repay"
- Default decision balances
  - benefit: hold up more resource today
  - cost: go to Autarky + current fractional output loss
- **More likely to default in recession**
  - default benefit: $MU(y_H) < MU(y_L) \rightarrow \text{benefit}(y_H) < \text{benefit}(y_L)$
  - default cost: $\text{cost}(y_H) = \text{cost}(y_L)$

One-period Debt Market, No Commitment (Cont'd)

- But in business cycle literature, shock is usually persistent
- **Default in recession or boom becomes a quantitative problem**
    - default benefit: $MU(y_{H,t}) < MU(y_{L,t}) \rightarrow \text{benefit}(y_{H,t}) < \text{benefit}(y_{L,t})$
    - default cost: $y_{H,t} \rightarrow Pr(y_{H,t+1}) > Pr(y_{L,t+1}) \rightarrow \text{cost}(y_H) < \text{cost}(y_L)$
    - with $y_{H,t}$ less precautionary motive
- Literature includes current fractional output loss to reduce $\text{benefit}(y_H)$

Part II: Default Risk and Income Fluctuations

- Arellano, 2008
- A framework to discuss sovereign bond with limited commitment

Several Questions about Default

- Debt and default: positive correlated?
- Country spread and default probability
- Default in good time or bad time?

Some Basic Definitions

- Without default risk, two equivalent ways of pricing bond
  1. today obtain: 1 tomorrow repay: $1 + r^*$
  2. today obtain: q tomorrow repay: 1

$$\frac{1 + r^*}{1} = \frac{1}{q} \quad \rightarrow \quad q = \frac{1}{1 + r^*}$$

- With default risk
  1. today obtain: 1 tomorrow repay: $1 + r^*$
  2. today obtain: q tomorrow repay: $1 - Pr(\text{default})$

$$\frac{1 + r^*}{1} = \frac{1 - Pr(\text{default})}{q} \quad \rightarrow \quad q = \frac{1 - Pr(\text{default})}{1 + r^*}$$

- Interest rate spread

$$r - r^* = \frac{1}{q} - (1 + r^*) = (1 + r^*)\left(\frac{1}{1 - Pr(\text{default})} - 1\right)$$

$$Pr(\text{default}) \uparrow \ \rightarrow q \downarrow \ \rightarrow r - r^* \uparrow$$

Empirical Facts

- Time series for GDP, consumption, spreads in Argentina
- Argentina defaults in December 2001
- Interest rate spread soars in low income periods

Empirical Facts

- Business cycle statistics for Argentina
- Compensation is more volatile than output
- Interest rate spread is countercyclical

Model Elements

- DSGE model, small open economy
- Endogenous default risk from limited commitment problem
- Asset incompleteness:
  - reflects actual credit markets, contracts at noncontingent r
  - deliver countercyclical default risk

Model Setting

- Foreign creditors: risk neutral, require r return
- Households: identical and risk averse

$$\mathbb{E}_0 \sum_{0}^{\infty} \beta^t u(c_t)$$

Income: Markov process y with a transition fn $f(y', y)$, tradable goods

- Government: access to international financial market
  - buy/issue one period bonds B' at price $q(B', y)$
  - if repay next period, resource constraint is

$$c = y + B - q(B', y)B'$$

  - if default next period, government remain in financial autarky with prob θ

$$c = y^{def} = h(y) < y$$

Timing

- Govn't starts with initial asset B, observe income shock y
- Govn't decides repay or default its debt
- If repays, taking as given bond price schedule $q(B', y)$, chooses B'
- Creditors taking q as given choose B'
- Consumption c takes place

Foreign Creditor Problem

- Price the bond contract to break even in expected value

$$q(B', y) = \frac{1 - \delta(B', y)}{1 + r}$$

  $\delta(B', y)$ is probability of default

  - default decision is made tomorrow, depends on B', y'
  - B' is chosen today, y' can be forecast by y

- Govn't also understands this price schedule

Govn't Problem

- Before making default decision, the government value function is

$$v^0(B,y) = \max_{c,d}\{v^c(B,y), v^d(y)\}$$

- Value of default

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^0(0,y') + (1-\theta)v^d(y')\right] f(y',y)dy'$$

- Value of commit

$$v^c(B,y) = \max_{B'} \left[u\left(y - q(B',y)B' + B\right) + \beta \underbrace{\int_{y'} v^0(B',y')f(y',y)dy'}_{\text{incorporate option to defult}} \right]$$

- Default or commit is a period-by-period decision
- No Ponzi scheme $B' \geq -Z$, not binding in eqm

Govn't Policy

- Repay/Commit set

$$A(B) = \{y \in Y : v^c(B, y)) \geq v^d(y)\}$$

- Default set

$$D(B) = \{y \in Y : v^c(B, y)) < v^d(y)\}$$

- Default probability

$$\delta(B', y) = \int_{D(B')} f(y', y) dy'$$

- More debt raised $B' \downarrow$
  - $v^c(B', y') \downarrow \rightarrow D(B') \uparrow \rightarrow \delta(B', y) \uparrow \rightarrow q \downarrow$

Recursive Equilibrium

- Defined as a set of policy functions
    1. consumption $c(s)$
    2. govn't asset holding $B'(s)$, repayment sets $A(B)$, default sets $D(B)$
    3. price function for bonds $q(B', y)$
- such that following conditions are satisfied
    - Taking as given govn't policies, HH $c(s)$ satisfies resource constraint
    - Taking as given $q(B', y)$, gov't $B'(S)$, $A(B), D(B)$ satisfies optimization
    - $q(B', y)$ reflects govn't default prob , consistent with creditor expected return

Default Incentive

- Default incentives are stronger the lower the endowment
- For all $y_1 \leq y_2$, if $y_2 \in D(B)$, then $y_1 \in D(B)$
- Net repayment is more costly when income is low due to higher MU

Total Resource Borrowed

- No further lending

$$B = \sup\{B : D(B) = Y\}$$

- No default risk

$$B = \inf\{B : D(B) = \emptyset\}$$

- The relevant region for "risky borrowing" is $B' \in (B^*, B)$

Quantitative Results

- B is reported as ratio of mean output
- Lager debt induces higher interest rates
- Endogenous countercyclical interest rate schedule $\rightarrow$ more volatile consumption
- Problematic: cannot support high enough debt, steep $q(B', y)$ curve

Quantitative Results

- Low income: borrow less, save less
- Borrower is more often at constraint in recession

Part II: Default Risk and Maturity Structure

- Arellano and Ramanarayanan, 2012
- A framework to introduce long-term debt

Several Questions about debt maturity

- Short-term debt
  - benefit: larger incentive to repay (less sensitive price)
  - cost: roll-over risk
- Long-term debt
  - benefit: hedge against future fluctuations in spreads
  - cost: higher default risk, longer time to exercise default (sensitive price)
- More short-term debt issued when interest rate spread rise (bad time)
- Spread on short-term debt rises more than long-term debt

Some Basic Definitions

- Short-term debt : t obtain q, $t+1$ repay 1
- Long-term zero coupon debt maturing in n years: t obtain q, $t+n$ repay 1

$$q = \frac{1}{(1+r_{t,i})^n} = (1+r_{t,i})^{-n} \sim (\exp r_{t,i})^{-n} = \exp(-nr_{t,i})$$

- Long-term debt paying fixed coupon c in $n_1, n_2, ..., n_J$ year, $t+n$ repay 1

$$q = \sum_{j=1}^{J} \exp(-n_j r_{t,i}^{m_j})c + \exp(-n_J r_{t,i}^{n_J})$$

- Duration

$$d_{t,i}(c) = \frac{1}{q_{t,i(c)}} \left[\sum_{j=1}^{J} \exp(-n_j r_{t,i}^{m_j})c\ n_j + \exp(-n_J r_{t,i}^{n_J}) n_J \right]$$

- Perpetual debt: t obtain q, $t+1$ pay 1, $t+2$ pay δ, ...,$t+n$ pay δ^{n-1} ($\delta = 0$ short-term debt)

Empirical Facts

- Low spreads (good times): gap(spread of LT debt−spread of ST debt) is large
- High spreads (bad times): gap is small even reversed

Empirical Facts

- Regression of average duration of new issuances on spreads
- Larger default premium (long over short debt) $\rightarrow$ longer duration

Model Setting

- Foreign creditors: risk neutral, require r return
- Households: identical and risk averse

$$\mathbb{E}_0 \sum_{0}^{\infty} \beta^t u(c_t)$$

Output: Markov process y with a transition function $f(y', y)$, tradable

- Govn't: issue/buy short and long term bond on international credit market
  - short-term bond: t receive $q_{S_t} b_{S_{t+1}}$, $t+1$ repay $b_{S_{t+1}}$
  - long-term bond: perpetuity contract with coupon that decay geometrically at δ

Long-term Debt

$t-2$	$t-1$	t	$t+1$	$t+2$	...
$q_{Lt-2}\ell_{t-2}$	ℓ_{t-2}	$\delta\ell_{t-2}$	$\delta^2\ell_{t-2}$	$\delta^3\ell_{t-2}$	...
	$q_{Lt-1}\ell_{t-1}$	ℓ_{t-1}	$\delta\ell_{t-1}$	$\delta^2\ell_{t-1}$	...
		$q_{Lt}\ell_t$	ℓ_t	$\delta\ell_t$	...

- Outstanding stock of long-term debt at t

$$b_{Lt} = b_{L0} + \ell_{t-1} + \delta\ell_{t-2} + \delta^2\ell_{t-3} + \ldots + \delta^t\ell_0 = \sum_{j=1}^{t} \delta^{j-1}\ell_{t-j} + b_{L0}$$

$$b_{Lt+1} = \delta b_{L,t} + \ell_t$$

- Only one kind of long-term debt, duration controlled by δ
- No seniority when repay, if default all debt erased
- Vintage does not matter, only stock matters

Model Setting (Cont'd)

- Household resource constraint conditional on not defaulting

$$c_t = y_t - b_{S,t} - b_{L,t} + q_{S,t} b_{S,t+1} + q_{L,t} \ell_t$$

$q_{S,t}, q_{L,t}$ are quoted for each pair of (b_{St+1}, b_{Lt+1})

- Household resource constraint conditional on defaulting

$$c_t = y_t^{def} = \begin{cases} y_t & \text{if } y_t \leq (1-\lambda)y \\ (1-\lambda)y & \text{if } y_t > (1-\lambda)y \end{cases}$$

Govn't Problem

- Before making default decision, the government value function is

$$v^0(b_S, b_L, y) = \max_{c,d}\{v^c(b_S, b_L, y), v^d(y)\}$$

- Value of default

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^0(0, 0, y') + (1-\theta)v^d(y')\right] f(y', y)dy'$$

- Value of commit

$$v^c(b_S, b_L, y) = \max_{b'_S, b'_L, \ell, c} \left[u\left(y - q(B', y)B' + B\right) + \beta \int_{y'} v^0(b'_S, b'_L, y')f(y', y\right.$$

- Resource constraint

$$c_t = y_t - b_{S,t} - b_{L,t} + q_{S,t}(b'_S, b'_L, y')b_{S,t+1} + q_{L,t}(b'_S, b'_L, y')\ell_t$$

- Law of motion of long-term bonds

$$b'_L = \delta b_L + \ell$$

Govn't Policy

- Repay/Commit set

$$R(b_S, b_L) = \{y \in Y : v^c(b_S, b_L, y)) \geq v^d(y)\}$$

- Default set

$$D(b_S, b_L) = \{y \in Y : v^c(b_S, b_L, y)) < v^d(y)\}$$

- When borrower does not default, new debt issuance rule is

$$b'_S = b_S(b_S, b_L, y)$$
$$b'_L = b_L(b_S, b_L, y)$$

Bond Prices

- Short-term debt

$$q_S(b'_S, b'_L, y) = \frac{1}{1+r^*} \int_{R(b'_S, b'_L)} f(y, y')dy'$$

price only depends on next period repay probability

- Long-term debt:

$$q_L(b'_S, b'_L, y) = \frac{1}{1+r^*} \int_{R(b'_S, b'_L)} \left[1 + \delta q'_L(b''_S, b''_L, y')\right] f(y, y')dy'$$

  - decompose: price $= t+1$ payoff + PV(coupon from $t+1$ on)
  - recursive structure of debt: PV(coupon) = price of new debt at $t+1$
  - **more sensitive to** y: additional hedging effect from q'_L as $corr(q'_L, c') \geq 0$
  - deep reason: long-term debt has longer default option period

Optimal Maturity Structure

- First order necessary conditions:

$$[b'_S] : u'(c)\left[q_S + \frac{\partial q_S}{\partial b'_S} b'_S + \frac{\partial q_L}{\partial b'_S}(b'_L - \delta b_L)\right] = \beta \int_{R'} u'(c') f(y, y') dy'$$

$$[b'_L] : u'(c)\left[q_L + \frac{\partial q_S}{\partial b'_L} b'_S + \frac{\partial q_L}{\partial b'_L}(b'_L - \delta b_L)\right] = \beta \int_{R'} (1{+}\delta q'_L) u'(c') f(y, y') d$$

- marginal gain in util today= marginal reduction in util repay
- Larger debt quantities, lower incentive to repay, $\frac{\partial q_m}{\partial b'_n} \leq 0, m, n \in S, L$

Optimal Maturity Structure (Cont'd)

- First order necessary conditions:

$$[b'_S] : u'(c)\left[1 + \frac{\partial q_S}{\partial b'_S}\frac{b'_S}{q_S} + \frac{\partial q_L}{\partial b'_S}\frac{(b'_L - \delta b_L)}{q_S}\right] = \beta(1+r^*)\mathbb{E}[u'(c')|R']$$

$$[b'_L] : u'(c)\left[1 + \frac{\partial q_S}{\partial b'_L}\frac{b'_S}{q_L} + \frac{\partial q_L}{\partial b'_L}\frac{(b'_L - \delta b_L)}{q_L}\right] = \beta(1+r^*)\mathbb{E}[u'(c')|R'] \underbrace{\frac{\mathbb{E}[(1+\delta q'_L)u'(c')|R']}{\mathbb{E}[(1+\delta q'_L)|R']\mathbb{E}[u'(c')|R']}}_{\text{hedging benefit}, corr(u'(c'), q'_L) < 0}$$

- Long term debt
  - price is more sensitive to income shock
  - hedging effect is why borrowers want to issue long term debt
  - pro: in good times, long-term debt price rise fast
  - con: in bad times when total debt increase, long-term debt price drops rapidly

Optimal Maturity Structure (Cont'd)

- Hedging benefit:

$$\frac{\mathbb{E}[(1+\delta q_L')u'(c')|R']}{\mathbb{E}[(1+\delta q_L')|R']\mathbb{E}[u'(c')|R']}$$

- Relative incentive benefit:

$$\frac{1+\frac{\partial q_S}{\partial b_L'}\frac{b_S'}{q_L}+\frac{\partial q_L}{\partial b_L'}\frac{(b_L'-\delta b_L)}{q_L}}{1+\frac{\partial q_S}{\partial b_S'}\frac{b_S'}{q_S}+\frac{\partial q_L}{\partial b_S'}\frac{(b_L'-\delta b_L)}{q_S}}$$

- Good times:
  - hedging benefit is dominating, issue more long-term debt
- Bad times:
  - incentive benefit is dominating, issue more short-term debt

Bond Price Functions

- As debt increase, debt price drops
- Price of long term debt is more sensitive

Debt Policies and Repay Incentives

- Bad times: issue more b'_s, good times: issue more b'_L
- Repay incentives: short-term $>$ long-term

Takeaways

- Default is more likely in recession, countercyclical spreads
- Maturity shorten in recession
- Short-term debt is for incentivise repayments
- Long-term debt is for hedging fluctuation of interest rate spreads